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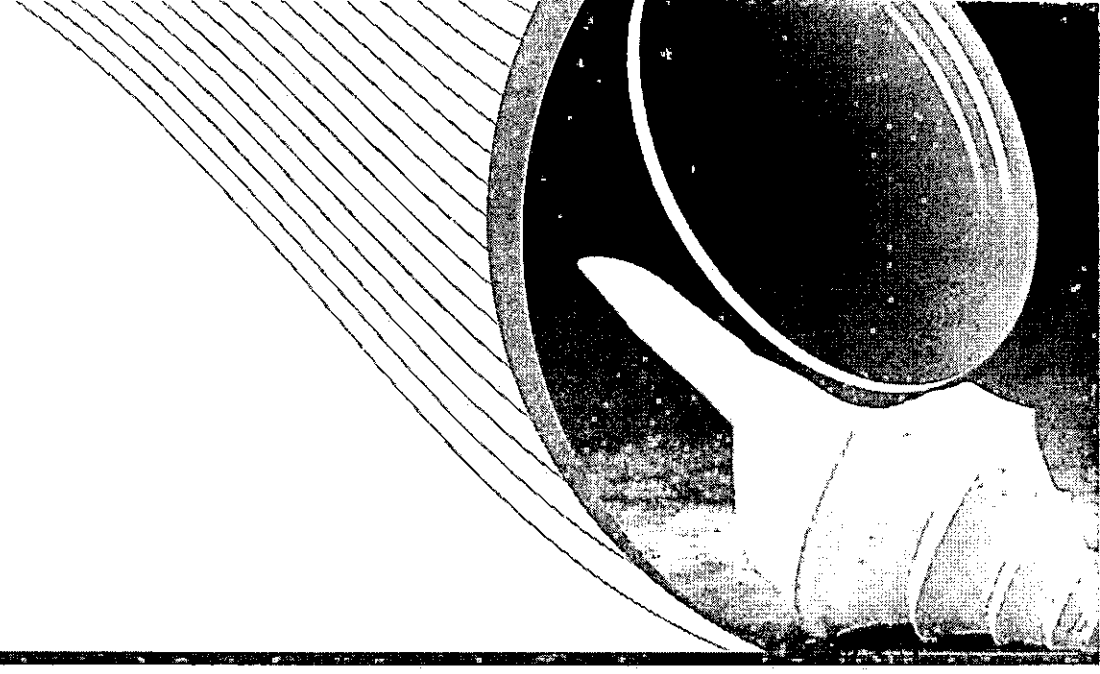

Optics

for **JEE** Main & Advanced

by:

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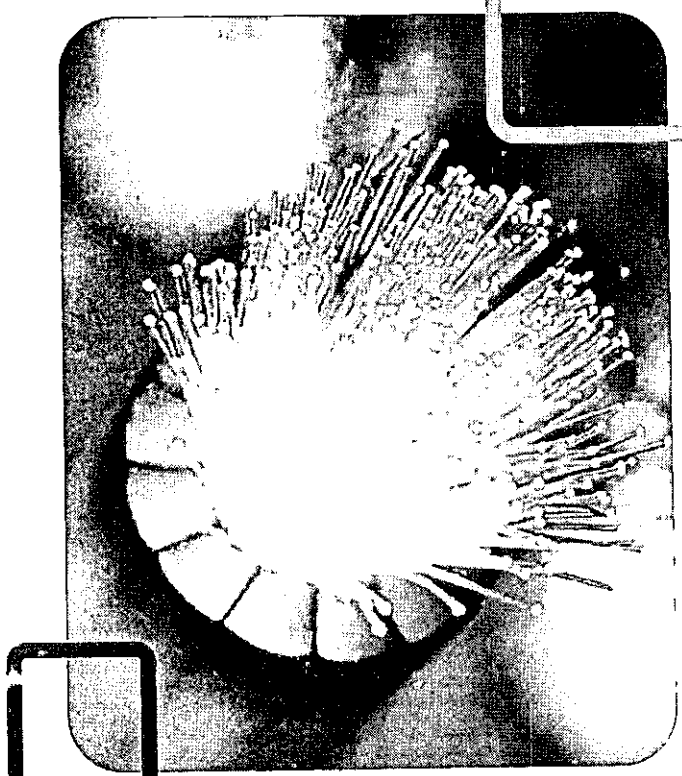

Topic

Preface

My interaction with students, sincerely preparing for IIT-JEE motivated me to write this book on Optics.

Optics is also an important part of physics Like my other books. "Optics for IIT-JEE" is designed to clear the concepts through numerical approach.

This book will help the students in building analytical and quantitative skills, addressing key misconceptions and developing confidence in solving the problems.

I sincerely wish that this book will fulfill all the aspirations of the readers. Although utmost care has been taken to make the book free from errors but some errors inadvertently may creep in. Author and Publisher shall be highly obliged if suggestions regarding improvement and errors are pointed out by readers. I am indebted Neeraj Ji for providing me an opportunity to write a book of this magnitude.

I am indebted to my father Sh. Bhavesh Mishra, my mother Smt. Priyamvada Mishra, my wife Manjari, my sister Parul, my little kids Vrishank and Ira for giving their valuable time which I utilized during the writing of this book and people of Moradabad, who supported me throughout my career.

I am also thankful to **Mr. T. Kondala Rao, Mr. Abhishek Sinha (Ranchi), Mr. Sunil Manohar, Mr. S.P. Sharma, Mr. Sudhir Sharma** and **Mr. P. Narendra Reddy** for their valuable suggestions in improving the book.

In the last, I also pay my sincere thanks to all the esteemed members of **Shri Balaji Publications** in bringing out this book in the present form.

Anurag Mishra

A Few Words to the Students

How to face the challenge ?

Following are some doubts which arise in the mind of almost all the students but may face them by taking some care.

1. *I can not solve numerical because my concepts are not clear.* In fact numerical solving itself is an exercise to learn concepts.

2. *I can not study because I am in depression, I fell into it because I was not studying*! Depression is escape mechanism of people afraid of facing failures. Failure is integral part of learning.

3. *I understand everything in class but can not solve on my own.* WRITING work is vital. It is a multiple activity, initially idea comes in mind then we put into language to express it, we are focussed in hand eye coordination, eyes create visual impression on brain which is recorded there. WRITING WORKS ARE EMBOSSED ON BRAIN LIKE CARVINGS OF AJANTA CAVES.

4. *In exams my brain goes blank, but I can crack them at home.* Home attempt is your second attempt! you are contemplating about it while home back. You do not behave differently in exam you replicate your instincts. Once a fast bowler was bowling no balls. His coach placed a stump on crease, in fear of injury he got it right. CONCEPTUALIZATION, WRITING EQUATION, SOLVING, THEN PROBLEM GETS TO CONCLUSION!

5. *I am an average student.* It is a rationalization used by people afraid of hard work. In their reference frame Newton's first law applies "if I have a misconception I will continue with it unless pushed by an external agent even I will surround him in my web of misconception yielding zero resultant." AVERAGE IS NOT DUE TO CAPACITY LACUANE BUT DUE TO LACK OF DETERMINATION TO SHED INERTNESS.

6. A famous cliche "*I do not have luck in my favour*" PRINCIPLE OF CAUSALITY: CAUSE OF AN EVENT OCCURS IN TIME BEFORE OCCURRENCE OF THAT EVENT *i.e.,* cause occurs first then event occurs. SHINING OF LUCK IS NOT AN INSTANTANEOUS EVENT IT IS PRECEDED BY RELENTLESS HARD WORK. Sow a seed of aspiration in mind, water it with passion, dedication it will bear fruit, luck can give you sweeter fruit.

Useful tips :

1. Do not take study as a burden actually its a skill like singing and dancing. It has to be honed by proper devotion and dedication.

2. Without strong sense of achievement you can't excel. Before entering the competitive field strong counselling by parents is must. Majority do not know what for they are here. No strategic planning, they behave like a tail ender batting in front of Steyn's bouncers.

3. Science is not a subject based on well laid down procedures or based on learning some facts, it involves very intuitive and exploratory approach. Unless their is desire and passion to learn you can not discover new ideas. It requires patience and hard work, whose fruits may be tangible later on.

4. Some students realize very late that they are studying for acquiring skills and honing them. Their is a feeling that they can ride at the back of instructor and achieve excellence. Study comes as torturous exercise enforced on them and their is some mechanism that can take this burden of them.

5. Science is not about gaining good marks, up to Xth by reading key points good marks are achieved but beyond that only those survive who have genuine interest in learning and exploring. Self study habit is must.

6. IF YOU WANT TO GAIN LEAD START EARLY. Majority of successful students try to finish major portion elementary part of syllabus before they enter Coaching Institute. Due to this their maturity level as compared to others is more they get ample time to adjust with the fast pace. They are less traumatized by the scientific matter handed over. For those who enter fresh must be counselled to not get bullied by early starters but work harder initially within first two months initial edge is neutralized.

7. Once a student lags behind due to some forced or unforced errors his mind begins to play rationalization remarks like I am an average student, my mind is not sharp enough, I have low IQ etc. These words are mechanisms used to avoid hard work. These words are relative terms a person who has early start may be intelligent relative to you.

   Intelligence means cumulative result of hard work of previous years, that hard work has eventually led to a development of instinct to crack things easily.

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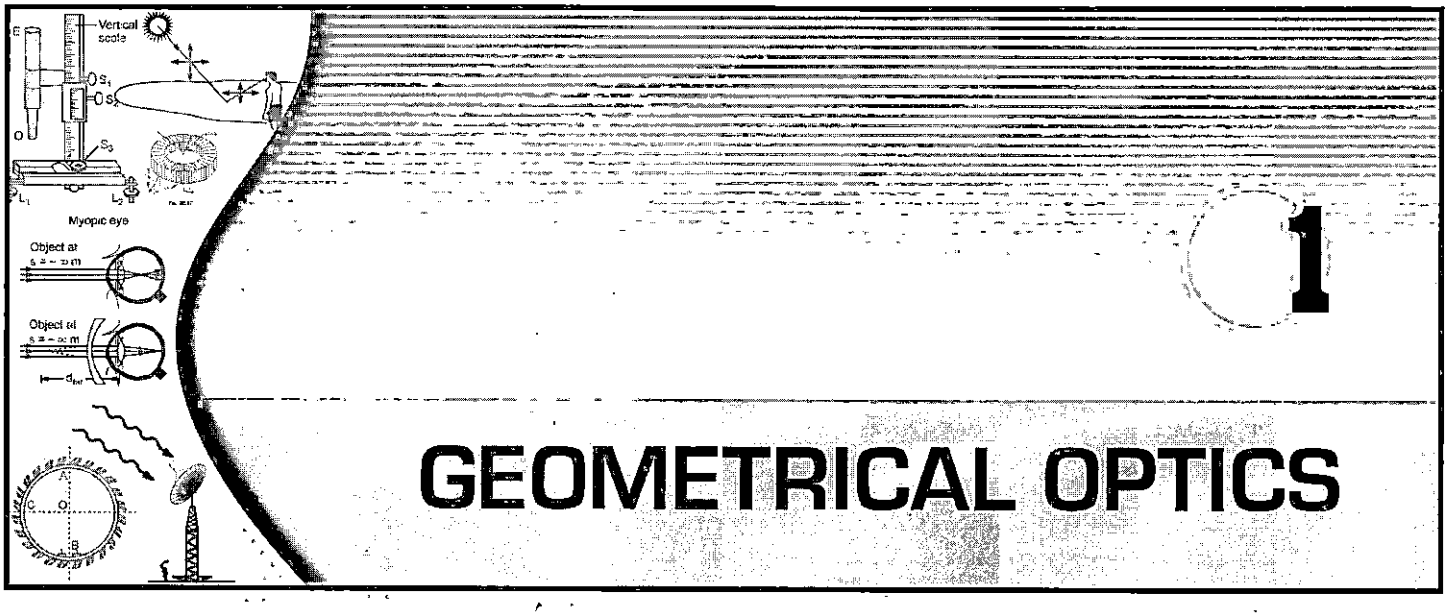
GROW GREEN

Save Nature



1

GEOMETRICAL OPTICS

OPTICS

Optics is the branch of physics that studies the phenomena and laws associated with the generation, propagation, and interaction with a substance of electromagnetic light waves. Light waves occupy an enormous range on the scale of electromagnetic waves. It is after the ultrashort millimetre radio waves and extends up to the shortest electromagnetic waves known at present-gamma rays with a wavelength λ less than one angstrom (Å) ($1\text{Å} = 10^{-10}\text{m}$).

Geometrical (ray) optics treats the laws of propagation of light in transparent media on the basis of notions of light as a combination of light rays-lines along which the energy of light electromagnetic waves propagates. Geometrical optics does not take into consideration the wave properties of light and the diffraction phenomena associated with them. For instance, **when light passes through a lens with a mount diameter of $d \geq \lambda$, where λ is the length of the light wave, we may ignore the diffraction at the edges of the lens.**

NATURE OF LIGHT

Light is a transverse electromagnetic wave. Light waves spread out in space in three dimensions; this makes visualization and representation of wave difficult. Fig. 1.1 (a) shows representation of simplest light wave, a monochromatic plane wave.

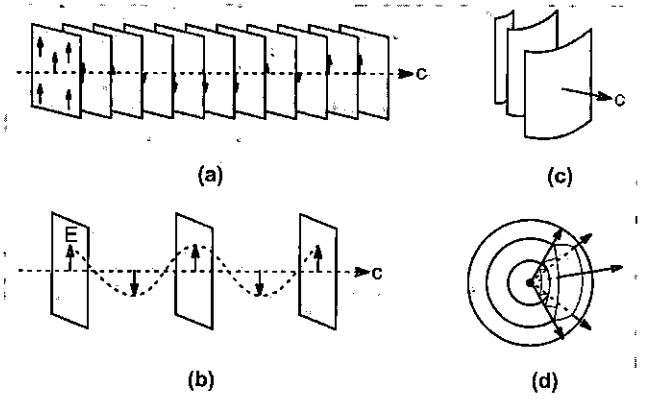


Fig. 1.1

When the light contains a single wavelength it is called **monochromatic** (literally 'one colour'). On each of the sheets the electric field has constant magnitude, it oscillates between E_{max} to $-E_{min}$ as we move from sheet to sheet. A given sheet moves to the right at the speed of light. The electric field is different in planes located at different positions along the propagation direction. For simplicity we take a particular sheet over which the electric field has a particular phase of oscillation; *e.g.*, we choose the phase where E has maximum value and is pointed upward. The rectangular sheets of Fig. 1.1 (b) represent a **wavefront**, on each of them *the wave has an equivalent phase.* When we move from sheet to sheet, the phase increases or decreases by 2π rad. Figs. 1.1 (c) and (d) show spherical and cylindrical wavefronts.

Wavefronts travel with the speed of light. The direction of energy flow is given by **Poynting vector** *A line tracing the flow of radiant energy through space is called a ray.* In an isotropic medium rays are perpendicular to wavefronts. We assume that the direction of motion of light is represented by rays. Rays do not travel; they actually indicate paths of light. **In the ray approximation** we assume that a wave through a medium travels in a straight line in the direction of its rays.

1. Parallel rays indicate plane wavefronts.
2. Diverging rays indicate expanding wavefronts.
3. Converging rays indicate collapsing wavefronts.

When a wave meets an obstacle with circular opening whose diameter is much larger than the wavelength, the wave emerging from the opening continues to move in a straight line, hence the ray approximation is valid. If the diameter of the opening is of the order of the wavelength [Fig. 1.2 (b)] the waves spread out from the opening in all directions.

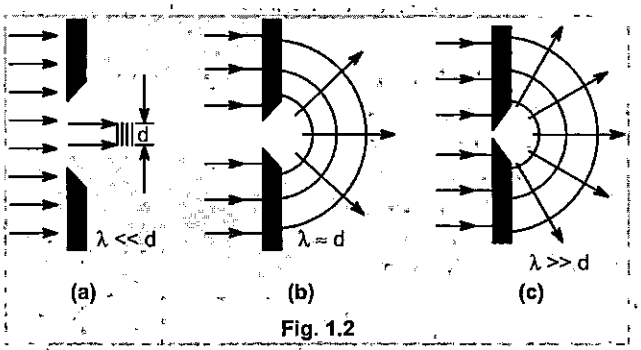


Fig. 1.2

Finally, if the opening is much smaller than the wavelength, the opening can be approximated as a point source of waves, each incident light ray subsequently travels in a unique direction. Well defined shadows exist in regions where there are no rays. When a wave encounters an object of dimensions d and if $\lambda << d$, the object casts a sharp shadow. In this chapter the ray approximation and the assumption that $\lambda << d$ have been used.

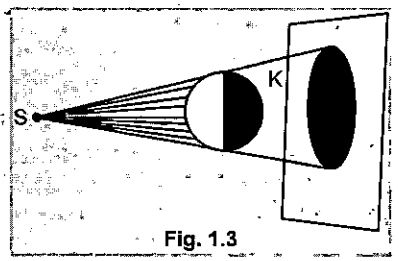


Fig. 1.3

A medium is defined as optically homogeneous if its refractive index is everywhere the same. In an optically homogeneous medium, the rays are straight: in such a medium light propagates rectilinearly. This is confirmed by the formation of shadows. If S is a point source of light, and K is a body in the path of the light from the source, then a cone of a shadow is formed beyond the body K (Fig. 1.3). No point inside this cone receives light from the source. A well-defined shadow of the body K is obtained on a screen placed at right angles to the axis of the cone.

Beams of light rays upon intersecting do not interfere and propagate independently of each other after intersection.

If a piece of wood, tissue paper and glass, etc., are held in succession to a source of light, it is seen that some substances allow no light to pass through them, whilst other substances do so to varying degrees. Hence we apply the terms —

***opaque*, to substances which allow no light to pass through them,**

***transparent*, to substances which allow much light to pass through them,**

***translucent*, to substances which allow a smaller portion of light, falling on them (or as we say, incident upon them), to pass through.**

The light which passes through is called *transmitted* light. But what happens to the light falling on an opaque body ? **Since the body is visible it must throw off, or *reflect*, light just as a wall or floor throws off, or reflects, a ball thrown on it.** But there is a difference, for whereas a ball is only reflected in the one direction, light must be reflected in all directions. **An object can be seen from all parts of a room, despite the fact that only a single source of light is illuminating it. This process by which a body *reflects incident light in all directions* is called *irregular or diffuse reflection*, or *scattering*, of light, and it is due to this process that we ordinarily see objects.**

RECTILINEAR PROPAGATION OF LIGHT

Light Rays

Observations show that in homogeneous media, light also propagates along straight lines. When an object is

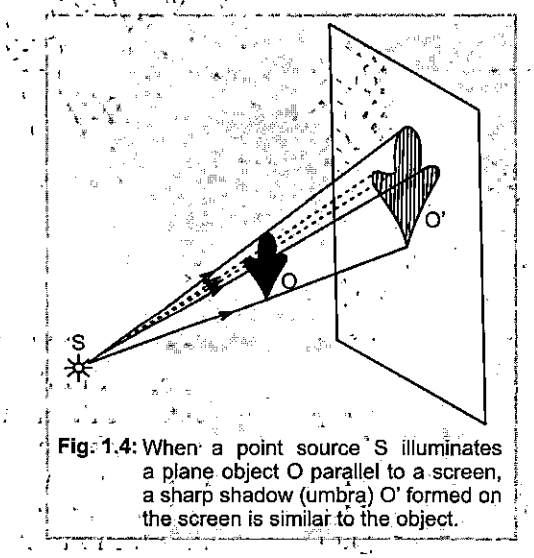


Fig. 1.4: When a point source S illuminates a plane object O parallel to a screen, a sharp shadow (umbra) O' formed on the screen is similar to the object.

illuminated by a **point source,** a sharp shadow **(umbra)** is formed (Fig.1.4), whose shape is similar to the shape of a

cross-section of the object, which is parallel to the screen. The size of the umbra is determined by the mutual arrangement of the source, the object and the screen in conformity with the laws of projecting by straight lines. Partial shadows (penumbras) observed sometimes are due to the finite size of the light source rather than due to a deviation of the direction of propagation from a straight line (Fig.1.5). Joiners employ a well-known method of verifying

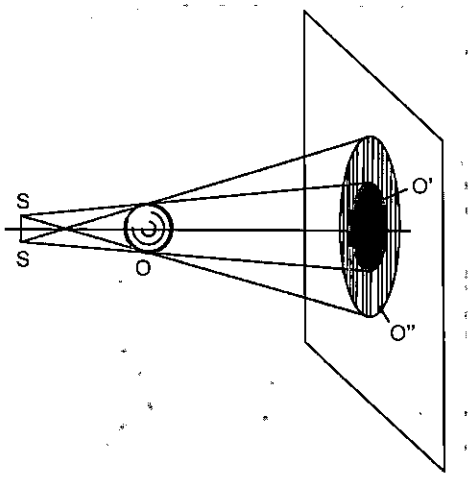


Fig. 1.5: When ball O is illuminated by an extended source SS, the umbra O' on the screen is surrounded by penumbra O".

the straightness of a planed board " by ray" (Fig.1.6). Phenomena associated with the rectilinear propagation of light are similar to those described in the previous section. If

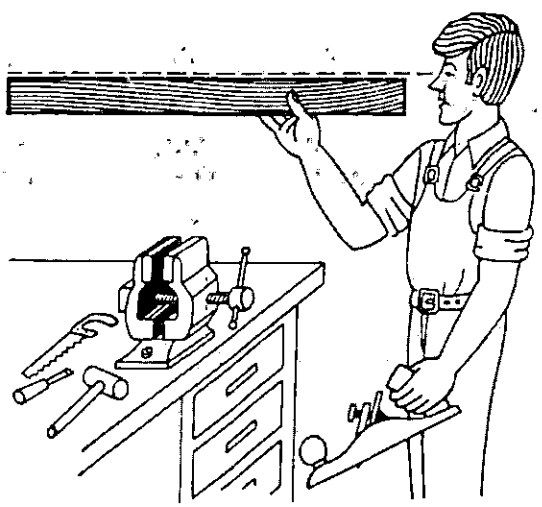


Fig. 1.6: Verification of the straightness of the edge by the "lens of sight".

we make the paths of solar rays "visible" with the help of cigarette smoke, the experiment with partitions will be repeated for light. If we interpose a light ray with a cardboard screen having one or several small holes (which are naturally much larger than the optical wavelength), we will see the path of light in the room in the form of narrow beams emanating from the holes (Fig.1.7). With any position of the screen, these beams have the same direction pointing to the Sun.

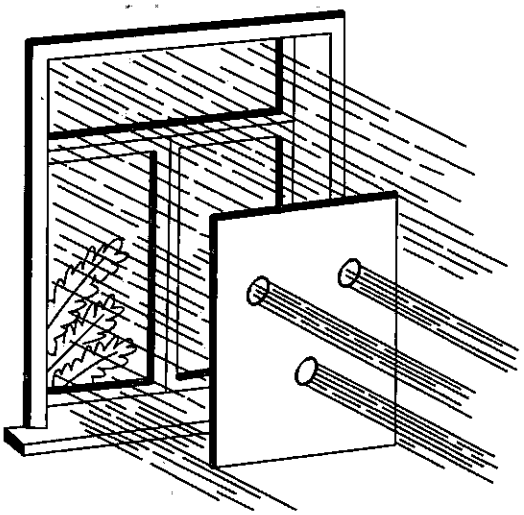


Fig. 1.7: Beams isolated from a plane light wave (the light source is the Sun) by a screen with small holes. The diameters of the holes are large in comparison with the light wavelength.

If in a dark room we place a bright bulb with a small filament into a dark box with several holes, the path of emanating rays will be seen in the dusty air of the room as a narrow beam diverging in all directions (Fig.1.8). Having marked the position of the bulb on the box wall, we will easily see that these beams intersect at the point where the bulb filament is located.

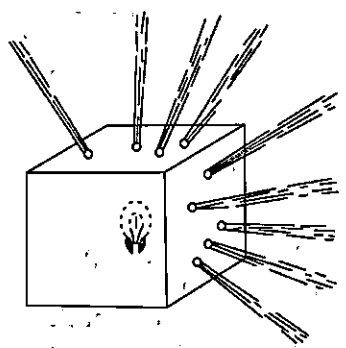


Fig. 1.8: Beams of rays isolated from a spherical light wave.

Proceeding in the same way as in the case of the waves on water surface, we draw the lines indicating the direction of light propagation. If the isolated beams are narrow, the construction of such lines presents no difficulty. These geometrical lines are **light rays.** In the cases considered above, they are nearly parallel line directed to the Sun, or the radii perpendicular to the surface of the sphere with the centre at the point where the light source (bulb filament) is located. A light wave propagates along these straight rays. So that they exceed the details of the pattern, the blurring becoming stronger with a further reduction of the aperture size. But since these spots are the traces of light beams cut by the aperture, the experiment demonstrates the divergence of a light beam upon a considerable reduction of the aperture size. Therefore, it is physically impossible to isolate an infinitely narrow beam. We have to confine ourselves to the isolation of finite width beams and then

replace them by the lines representing the axes of these beams. Thus, **a light ray is a geometrical concept.**

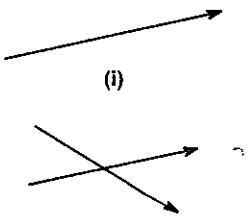
The merit of this concept consists in the possibility of establishing the direction of propagation of luminous energy. The laws governing the change in the direction of rays allow us to solve very. important optical problems concerning the change in the direction of propagation of luminous energy. For solving most of such problems, it is sufficient to replace a physical concept of light wave by a geometrical concept of light ray and carry out analysis in terms of rays.

However, the concept of light rays cannot always be used for determining the nature of propagation of light waves. There is a large number of optical phenomena which can be grasped only by analyzing light waves. Naturally, optical phenomena can be considered in the framework of the wave theory for simple cases when the ray method also provides quite satisfactory results. But since the ray method is much simpler, it is normally used for solving all problems where it is applicable. Therefore,we must outline the range of problems and the degree of accuracy with which geometrical rays can be used to distinguish these problems from those where the ray method leads to considerable errors and hence is inadmissible.

Thus, the method of **ray optics, or geometrical optics, is an approximate technique for solving problems,** which is adequate for analyzing a certain scope of questions. For this reason, it is important for those studying optics to learn how to apply the ray method correctly and to establish the limits of its applicability.

Concepts: ***(1) Light Rays Travel in Straight Line:*** *Light travels through a transparent material in straight lines called light rays. The speed of light is $v = c/n$. where n is the index of refraction of the material.*

(2) Light rays can cross: *Light rays do not interact with each other. Two rays can cross without either being affected in any way.*



(i)

(ii)

Fig. 1.9

(3) A Light Ray Travels Forever Unless it Interacts with Matter: *A light ray continues forever unless it has an interaction with matter that causes the ray to change direction or to be absorbed. Light interacts with matter in different ways:*

At an interface between two materials, light can be either reflected or ***refracted.***

Within a material, light can be either scattered or absorbed.

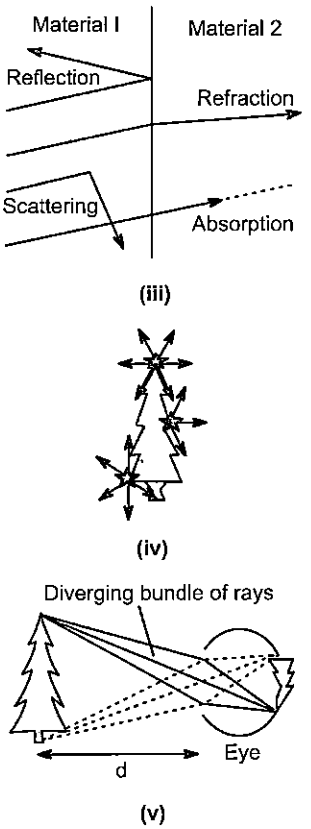


Fig. 1.9

(4) An Object is a Source of Light Rays: *An object is a source of light rays. Rays originate from* ***every*** *point on the object, and each point sends rays in* ***all*** *directions. We make no distinction between self-luminous objects and reflective objects.*

(5) The Eye Sees by Focasing a Diverging Bundle of Rays: *The eye "sees" an object when* ***diverging*** *rays from each point on the object enter the pupil and are focused to an image on the retina. From the movements the eye's lens has to make to focus the image, brain "computes" the distance d at which the rays originated, and you perceive the object as being at that point.*

(6) Objects: *Fig. 1.10 illustrates the idea that objects can be either* ***self-luminous,*** *such as the sun, flames, and light bulbs, or reflective. Most objects are reflective. A tree, unless it is on fire, is seen or photographed by virtue of reflected sunlight or reflected skylight. People, houses, and this page in the book reflect light from self-luminous sources.*

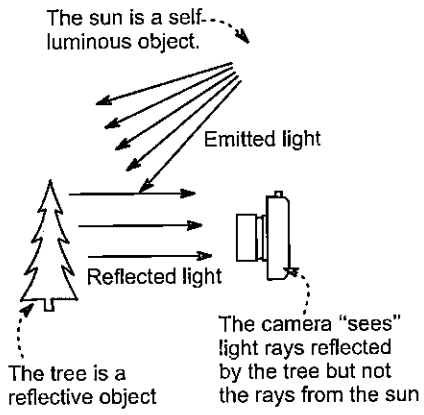

The sun is a self-luminous object.
Emitted light
Reflected light
The camera "sees" light rays reflected by the tree but not the rays from the sun
The tree is a reflective object

Fig. 1.10 Self-luminous and reflective objects.

(7) *Light rays from an object are emitted in all directions, but we do not feel it unless they enter the pupil of your eye. Consequently, most light rays go completely unnoticed. For example, light rays travel from the sun to the tree in Fig. 1.10, but you're not aware of these unless the tree reflects some of them into your eye. Or consider a laser beam. It's almost impossible to see a laser beam from the side unless there's dust in the air. The dust scatters a few of the light rays toward your eye, but in the absence of dust you would be completely unaware of a very powerful light beam traveling past you.* ***Light rays exist independently of whether you are seeing them.***

(8) *Fig. 1.11 shows two idealized sets of light rays. The diverging rays from a* ***point source*** *are emitted in all directions. It is useful to think of each point on an object as a point source of light rays. A* ***parallel bundle*** *of rays could be a laser beam. Alternatively it could represent a* ***distant object,*** *such as a star so far away that the rays arriving at the observer are essentially parallel to each other.*

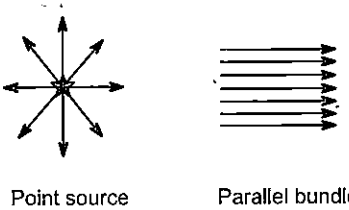


Figure 1.11: Point sources and parallel bundles represent idealized objects.

(9) Ray Diagrams: *Rays originate from every point on an object and travel outward in all directions, but a diagram trying to show all these rays would be hopelessly messy and confusing. To simplify the picture, we usually use a* ***ray diagram*** *showing only a few rays. For example, Fig. 1.12 is a ray diagram showing only a few rays leaving the top and bot tom points of the object and traveling to the right. These rays will be sufficient to show us how the object is imaged by lenses or mirrors.*

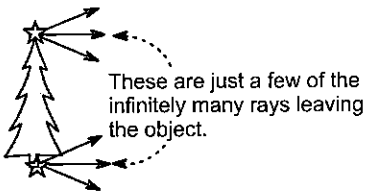


Figure 1.12: A ray diagram simplifies the situation by showing only a few rays.

Note: Ray diagrams are the basis for a *pictorial representation* a ray diagram does not shows all of the rays. They rays shown on the diagram are just a subset of the infinitely many rays leaving the object.

REGULAR REFLECTION

A special form of reflection of light—reflection from a polished surface, *i.e.*, a surface so smooth that the surface particles are all in the same plane, and so all the rays in a parallel beam falling on them are reflected in parallel paths. In this case there are definite laws regarding the reflection of the light, and so we call it **regular reflection.** The reception of a regularly reflected beam of light from a surface gives the impression of a brightness at the surface, which is thus **said to be polished.** Surfaces specially made to reflect light regularly are mirrors. Ordinary mirrors are made by pressing **mercury between glass and tinfoil.** A mercury-tin amalgam adheses to the glass and, in time, hardens. It is coated with shellac varnish and backed with red oxide of iron paint, to preserve it. For very good mirrors a fine layer of silver is deposited on specially smooth glass by a chemical or electrolytic process.

A mirror is not a perfect regular reflector, *i.e.*, **there is a loss of intensity on reflection.** If a beam of light be directed on to a mirror in an otherwise dark room, the mirror is visible from all parts of the room, *i.e.*, it does scatter some of the light.

Diffuse Reflection

Most objects are seen by virtue of their reflected light. For a "rough" surface, the law of reflection $\theta_r = \theta_i$ is obeyed at each point but the irregularities of the surface cause the reflected rays to leave in many random directions. This situation, shown in Fig.1.13, is called **diffuse reflection.** It is how you see this page, the wall, your hand, your friend, and so on. Diffuse reflection is far more prevalent than the mirror like specular reflection.

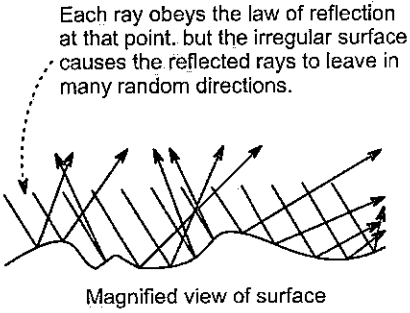

Each ray obeys the law of reflection at that point. but the irregular surface causes the reflected rays to leave in many random directions.
Magnified view of surface

Fig. 1.13

"Rough" surface, means a surface that is rough or irregular in comparison to the wavelength of light. Because visible-light wavelengths are $\approx 0.5\mu m$, any surface with irregularities larger than $1\mu m$ will cause diffuse reflection rather than specular reflection. A piece of paper may feel quite smooth to your hand, but a microscope would show that the surface consists of distinct fibers much larger than $1\mu m$. The irregularities on a mirror or a piece of polished metal are much smaller than $1\mu m$. The law of reflection is equally valid for both specular and diffuse reflection, but the nature of the surface causes the outcomes to be different.

When light is reflected from a rough surface, rays are scattered more or less randomly in different directions. This type of reflection is called **diffused reflection.** When light rays strike an optically smooth surface (a surface with irregularities of a size much less than the wavelength of light) the reflection of the wavefront is quite regular called **specular reflection.**

A line perpendicular to the surface at the point where the incident ray strikes the surface is called a normal line or simply the **normal** to the surface. The angle i that the incident ray makes with the normal is the **angle of incidence.** The angle r that the reflected ray makes with the normal is the angle of reflection.

Let $\hat{\mathbf{S}}$ represent a unit vector pointed in the direction of energy propagation. Then $\hat{\mathbf{S}}_i$ is in the direction of incident ray that makes an angle i with a normal $\hat{\mathbf{N}}$ drawn to the reflecting surface. The plane formed by $\hat{\mathbf{S}}_i$ and $\hat{\mathbf{N}}$ is called the **plane of incidence.** According to law of reflection $\hat{\mathbf{S}}_r$ lies in the plane of incidence and makes an angle $i = r$, shown in Fig. 1.14.

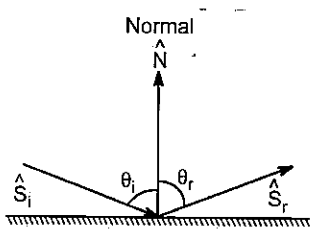


Fig. 1.14: In specular reflection the incoming ray, reflected ray and a normal to the surface all lie in the same plane. The angle of incidence (θ_i) equals the angle of reflection (θ_r).

When light reaches the interface between two transparent media, a part of it passes to the second medium (is refracted), while the other is reflected to the first medium.

The **law of reflection of light** states that *the reflected ray lies in the same plane with the incident ray and with the normal to the reflecting surface at the point of incidence, the angle of reflection being equal to the angel of incidence* (Fig.1.15).

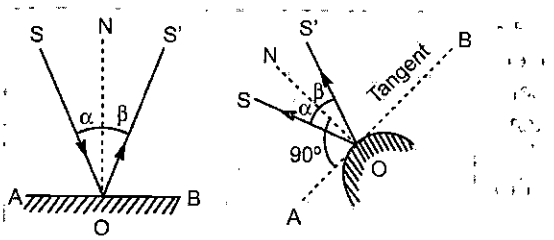


Fig. 1.15

The *angle of incidence* is the angle α between the normal and the incident ray, while the *angle of reflection* is the angle β between the normal and the reflected ray.

Light rays are reversible. This means that a ray passed against another ray that has covered the distance between points 1 and 2 propagates along the same path but in the opposite direction. In particular, if the ray propagating along SO is reflected in the direction OS', the ray incident along $S'O$ is reflected along OS.

The image of a point source formed by a mirror (or lens) is found at the intersection or continuation of light rays emerging form this point and reaching the eye of an observer.

If the rays emerging from a point source and getting to the eye do not intersect but their continuations do, the image is called *virtual.*

Fig. 1.16 illustrates the construction of the image of a point source S formed by a *plane mirror AB.* For this purpose, it is sufficient to take (see Fig.1.16 two rays SO_1 and SO_2 incident at points O_1 and O_2 of the mirror. The reflected rays o not intersect, but their continuations intersect at point S'. The triangles SO_1O_2 and $S'O_1O_2$ are equal since O_1O_2 is their common side,

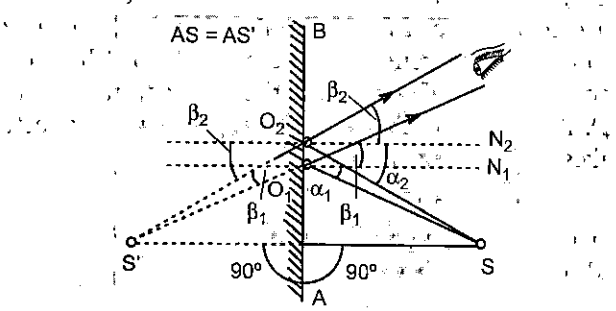


Fig. 1.16

$< SO_1O_2 = (90°+\alpha_1) =< S'O_1O_2 = (90°+\beta_1)$ and

$< SO_2O_1 = (90°-\alpha_2) =< S'O_2O_1 = (90°-\beta_2)$.

Consequently, the heights of the triangles are also equal, and the image of a point source (object) fromed by a plane mirror is virtual and is *symmetric* to the point source (object).

If a surface is not a mirror for given rays, diffuse reflection is observed since rays incident on such a surface are reflected in all directions from the nonuniformities Fig. 1.17.

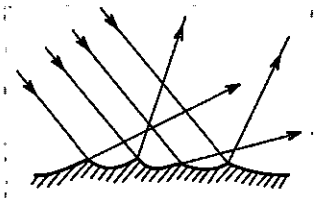


Fig. 1.17

Example 1

A light ray is incident at angle θ on the reflecting surface shown in Fig. 1E.1. The reflecting surface is now rotated through an angle ϕ as shown in Fig. 1E.1. Find the angle between the incident and reflected ray.

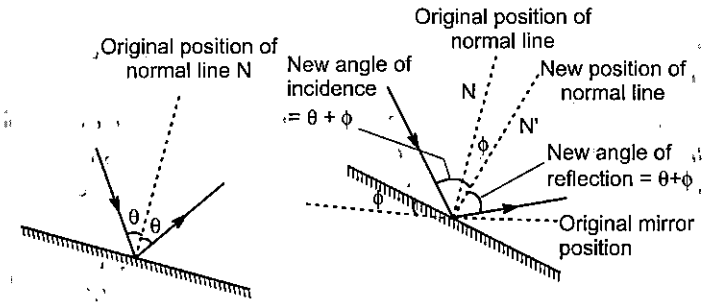


Fig. 1E.1

Solution: In the initial position of the mirror, the angle between the reflected ray and the incident ray is 2θ, according to the law of reflection.

As the mirror rotates through an angle ϕ, the normal line also rotates through the same angle ϕ. The angle of incidence now is $\theta + \phi$, as shown in Fig. 1E.1(b); from law of reflection angle of reflection must be $\theta + \phi$. The angle between the new reflected and the incident ray then is $2(\theta + \phi)$. Therefore the change in the angle that the reflected ray makes with the incident ray is

$$2(\theta + \phi) - 2\theta = 2\phi$$

which is independent of angle of incidence θ. This device is called an **optical lever.**

THE FORMATION OF IMAGES BY A PLANE MIRROR

When a given **object** is placed infront of an optical device, we imagine each point on the object as an **object point.** Each object point is a source of many diverging rays in different directions. An **extended object** consists of many different object points. If the light rays intersect or appear to intersect at some point, that point is the **image point** of the object point.

The drawing of reflected rays in Fig. 1.18 indicates an image point directly behind the mirror and at the same distance from the mirror as the source point; this result is shown in figure. The image at point P' is called a virtual image point. In general, images are referred to as either real or virtual.

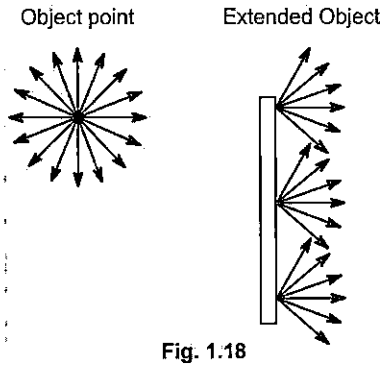


Fig. 1.18

A **real image point** is defined as a point at which converging light rays intersect, **images from which all the light rays actually do emanate.** A **virtual image point** is defined as a point from which rays of light appear to be diverging, **all the rays of light do not actually emanate from the image.**

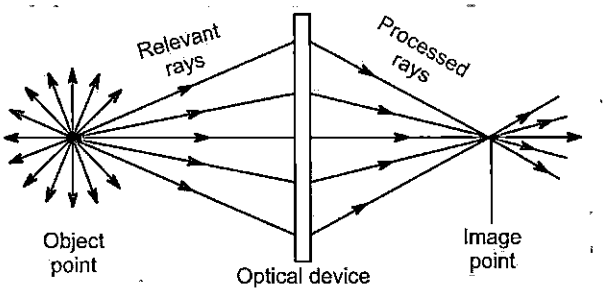


Fig. 1.19

OBJECT

Point of intersection of incident rays is called object.

Real Object

An object is real if two or more incident rays actually emanate or seem to emanate from a point.

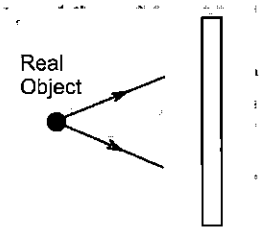
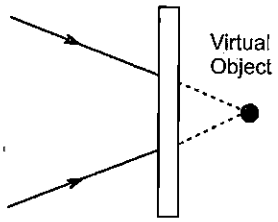


Fig. 1.20

Virtual Object

Now consider a converging set of rays as shown in Fig. 1.21. If not intercepted, the rays will meet at a point. However, if the rays are intercepted by an optical element placed as shown in the figure, then the point of convergence is a virtual point behind the optical element. This point is called the virtual object for the optical element.



Optical Element
A virtual object that appears to be behind the optical element

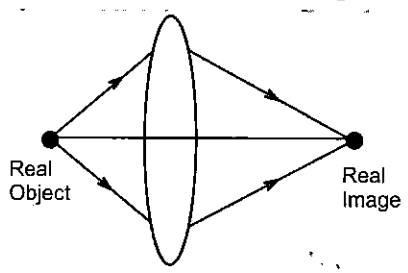
Fig. 1.21

IMAGE

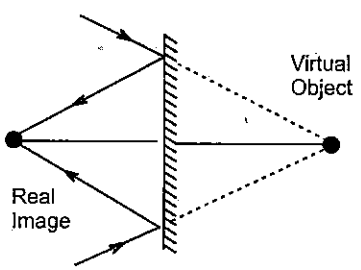
An image is the point of convergence or apparent point of divergence of rays after they interact with a given optical element. An object provides rays that will be incident on an optical element. The optical element reflects or refracts the incident light rays which then meet at a point to form an image.

Real Image

Real images are formed when the reflected or refracted rays actually meet or converge to a point. If a screen is placed at the point, a bright spot will be visible on the screen. Thus a real image can be captured on a screen. Examples of a real images are shown in Fig. 1.22(a) and (b).



(a) A real image formed by a real object placed in front of the convex lens

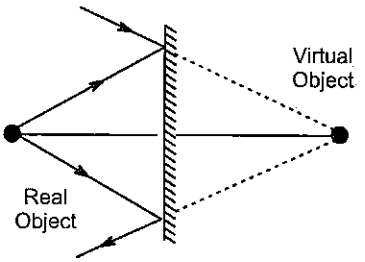


(b) A Real image formed by a virtual object that appears to be behind the mirror

Fig. 1.22

Virtual Image

When light rays, after interacting with the optical element, actually meet at a point the image formed is a real image. However, if the rays do not meet at a point but a appear to emanate form a point then a virtual image is formed.



A virtual image formed by a real object in front of the mirror

Fig. 1.23

Object/Image	Criteria
Real object	When incident rays are of diverging nature.
Virtual object	When incident rays are of converging nature. Incident rays converge to a point behind the optical element.
Real image	When rays after interacting with the optical element are of converging nature.
Virtual image	When rays after interacting with the optical element. are of diverging nature.

A real image can be captured on a screen. Preferably the surface should be white in color so that we can see the image clearly. This image on the screen can be perceived by the eye.

How about virtual images ? Since they cannot be captured on a screen is it possible to see them. From practical experience we know that we can see ourselves in a plane mirror which is indeed a virtual image. So it is possible to see a virtual image. But how is it done? There is a lens in our eye that focuses the diverging set of rays on to the retina forming a real image inside our eyes.

In a similar manner, a camera can also convert a virtual image into a real image which is captured on the film (Fig. 1.24).

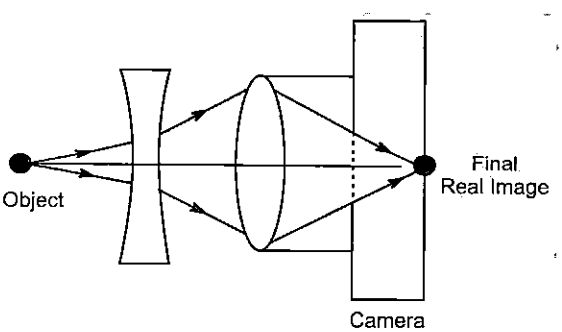


Fig. 1.24

IMAGE FORMATION FROM A PLANE SURFACE

Point Object

(i) Distance of object from mirror = Distance of image from the mirror.

All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called image.

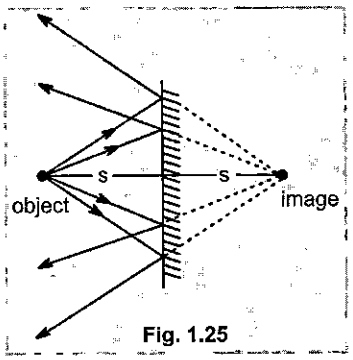


Fig. 1.25

(ii) The line joining a point object and its image is normal to the reflecting surface.

(iii) The size of the image is the same as that of the object.

(iv) For a real object the image is virtual and for a virtual object the image is real.

(v) The laws of reflection are valid for any smooth reflecting surface irrespective of geometry.

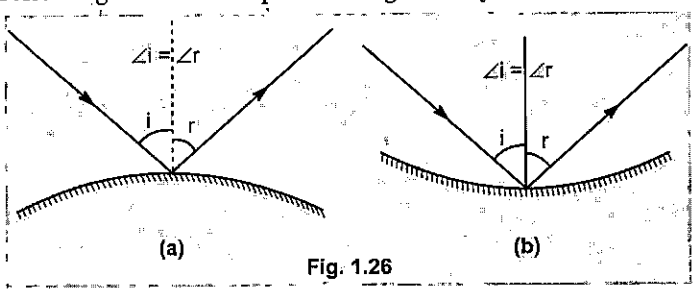


Fig. 1.26

(vi) Whenever reflection takes place, the component of incident ray parallel to reflecting surface remains unchanged, while component perpendicular to reflecting surface *(i.e., along normal)* reverses in direction

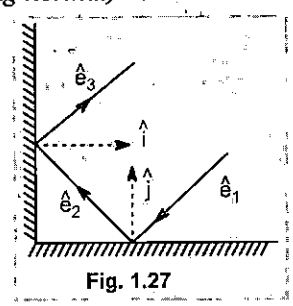


Fig. 1.27

Consider incident ray along unit vector $\hat{\mathbf{e}}_1$ given $\hat{\mathbf{e}}_1 = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ similarly $\hat{\mathbf{e}}_3 = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ unit vector along reflected ray will be given by $\hat{\mathbf{e}}_2 = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ diverge.

Similarly consider on incident ray $\vec{\mathbf{e}}_1 = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ on a mirror.

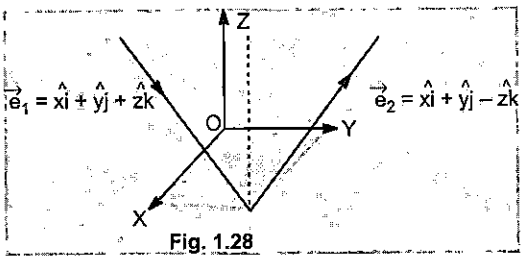


Fig. 1.28

VECTOR FORM OF LAWS OF REFLECTION

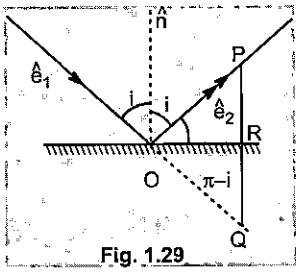


Fig. 1.29

$$\hat{\mathbf{e}}_2 = \hat{\mathbf{e}}_1 - 2(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}$$

where $\hat{\mathbf{e}}_1$ = unit vector along incident ray

$\hat{\mathbf{n}}$ = unit vector along normal

$\hat{\mathbf{e}}_2$ = unit vector along reflected ray

Form triangle law of addition of vectors

Consider two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ as shown in Fig. 1.30.

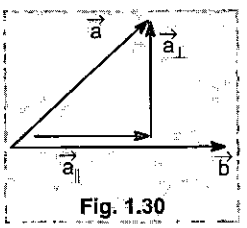


Fig. 1.30

Here we can express $\vec{\mathbf{a}}$ as its components $\vec{\mathbf{a}}_{\parallel}$ and $\vec{\mathbf{a}}_{\perp}$

Such that $\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\parallel} + \vec{\mathbf{a}}_{\perp}$

$$\vec{\mathbf{a}}_{\parallel} = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}\hat{\mathbf{b}}$$

also $\vec{\mathbf{a}}_{\perp} = \vec{\mathbf{a}} - \vec{\mathbf{a}}_{\parallel}$

$$\vec{\mathbf{e}}_1 = \vec{\mathbf{e}}_{\parallel} + \vec{\mathbf{e}}_{\perp}$$

where, $\vec{\mathbf{e}}_{\parallel}$ = component of $\hat{\mathbf{e}}$ parallel to mirror

$$= \hat{\mathbf{e}}_1 - (\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}$$

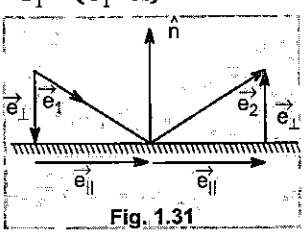


Fig. 1.31

$\vec{\mathbf{e}}_{\perp}$ = component of $\hat{\mathbf{e}}_1$ perpendicular to mirror

$$= (\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}$$

Using remark we get ;

$$\hat{e}_2 = \vec{e}_{\parallel} - \vec{e}_{\perp} = \hat{e}_1 - 2\hat{n}\,(\hat{e}_1 \cdot \hat{n})$$

CORNER REFLECTOR

When incident ray suffers two successive reflection.

From vector form of laws of reflection

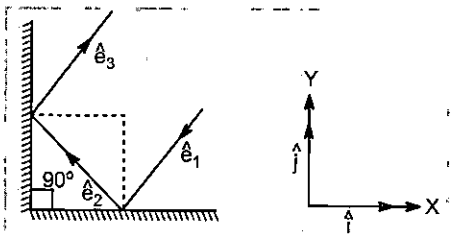


Fig. 1.32

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{j})\hat{j} \qquad ...(1)$$

Similarly, $\hat{e}_3 = \hat{e}_2 - 2(\hat{e}_2 \cdot \hat{i})\hat{i}$ ...(2)

From eqns. (1) and (2), we get ;

$$\hat{e}_3 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{j})\hat{j} - 2[\{\hat{e}_2 \cdot \hat{i})\} \cdot \hat{i}]\hat{i}$$

$$= \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{j}) - 2(\hat{e}_1 \cdot \hat{i})\hat{i} + 4[(\hat{e}_1 \cdot \hat{j})\hat{j} \cdot \hat{i}]\hat{i}$$

$$= \hat{e}_1 - 2[\hat{e}_1 \cdot \hat{i})\hat{i} + (\hat{e}_1 \cdot \hat{j})\hat{j}] \qquad (\because \hat{i} \cdot \hat{j} = 0)$$

or $\hat{e}_3 = \hat{e}_1 - 2\hat{e}_1$

or $\hat{e}_3 = -\hat{e}_1$ ...(3)

Hence, $\hat{e}_3$ is antiparallel to $\hat{e}_1$.

Alternatively:

Let, $\hat{e}_1 = x\hat{i} + y\hat{j}$

then, $\hat{e}_2 = x\hat{i} - y\hat{j}$

and $\hat{e}_3 = -x\hat{i} - y\hat{j}$

Clearly, $\hat{e}_3 = -\hat{e}_1$

Such a combination of mirrors is used in periscope.

The result can be extended to three plane mirrors arranged mutually perpendicular to each other. This arrangement of three mutually perpendicular mirrors is also known as corner reflector. If the incident ray is represented by $x\hat{i} + y\hat{j} + z\hat{k}$ then after three reflections final reflected ray is given by $-x\hat{i} - y\hat{j} - x\hat{k}$. Thus we always see our reflected image (put only upside down), independent of our position.

Concepts: ***(1)*** *Plane mirror changes right handed co-ordinate system to left handed co-ordinate system and vice-versa.*

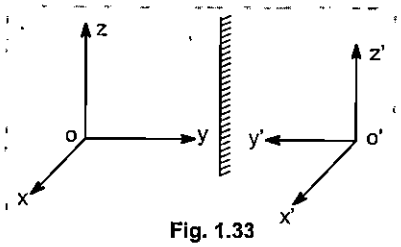


Fig. 1.33

(2) *The image of a horizontal object becomes vertical and the image of a vertical object becomes horizontal if plane mirror is inclined at an angle 45° with horizontal.*

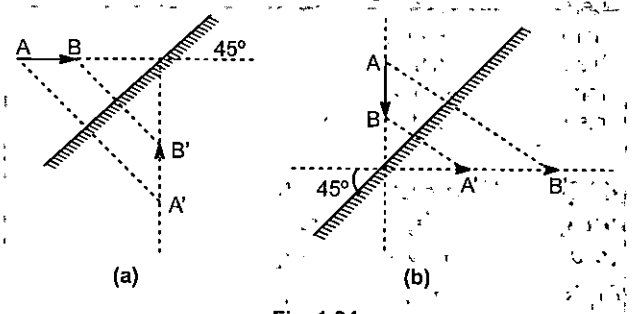


Fig. 1.34

(3) The minimum height (or length) of a mirror required for a person to see his complete image in a plane mirror:

AB is the person with E as his eyes. M_1M_2 is a plane mirror in front of him. (Fig.1.35) .

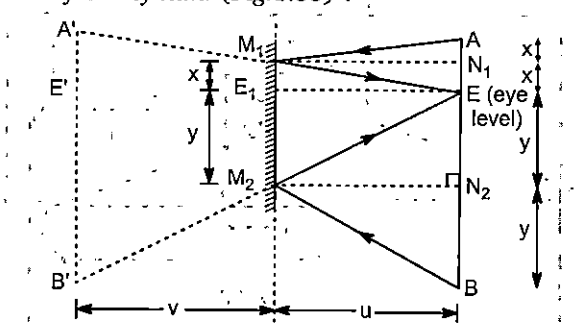


Fig. 1.35

The rays coming from the extreme and portions of his body, i.e., A and B, should enter his eyes after reflection.

The light ray AM_1 is incident ray and M_1E the reflected ray

$$M_1E_1 = x(\text{say}) = N_1E = \frac{1}{2}(AE) \qquad ...(1)$$

Similarly the light rays BM_2 and M_2E are incident and reflected rays respectively

$$M_2E_1 = y(\text{say}) = N_2E = \frac{1}{2}(BE). \quad ...(2)$$

Adding eqns. (1) and (2)

$$x+y=\text{length of mirror} = \frac{1}{2}(AE+BE) = \frac{1}{2}(AB)$$

$$= \frac{1}{2}\text{ (height of the person).}$$

Note: For a person of height H, with a plane mirror of height $\frac{H}{2}$, to view his complete image, the distance between mirror and person does not matter. However, the upper edge of the mirror should be at a level exactly midway between that of his eye level and the top most level or the lower edge of the mirror should be at a level exactly midway between that of his eye level and the bottom most level.

(4) If a plane mirror be fitted on a wall and a person gazing at it, intends to view the complete image of the rear wall of height *H.*

If η is the fraction of the distance of the person from the mirror, as compared to the length of the room. (Fig. 1.36).

From similar triangles AM_1N_1 and EM_1P_1

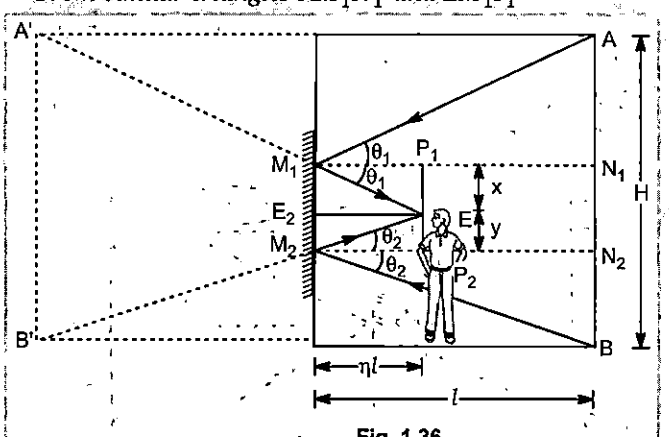


Fig. 1.36

$$\frac{x}{\eta l} = \frac{AN_1}{l} \text{ or } AN_1 = \frac{x}{\eta} \quad ...(1)$$

Similarly, from similar triangle BM_2N_2 and EM_2P_2

$$\frac{y}{\eta l} = \frac{BN_2}{l} \text{ or, } BN_2 = \frac{y}{\eta} \quad ...(2)$$

Adding eqns. (1) and (2), we get

$$AN_1 + BN_2 = (x+y)/\eta. \quad ...(3)$$

Now, total height of the room = AB = H

or $AN_1 + N_1N_2 + BN_2 = H$

or $AN_1 + (M_1E_2 + E_2M_2) + BN_2 = H$

or $\frac{x}{\eta} + (x+y) + \frac{y}{\eta} = H$

$$x + y = \text{height of the mirror}$$

$$(x+y) = \frac{\eta H}{(\eta+1)}$$

$$\text{Height of mirror} = \frac{\eta H}{(\eta+1)}$$

Note that the height of mirror obtained above is minimum since, light coming from the extreme edges of the room A and B is just able to enter the person's eye after reflection from the mirror.

Note : (1) Result is independent of the height or the eye level of the person.

(2) $E_2M_2 = EP_2 = y$.

But $y + \frac{y}{\eta} = h$, height of eye level above the floor of the room

$$y = \frac{\eta h}{(\eta+1)}$$

For a person to see the complete image of the rear wall the lower edge of the mirror, *i.e.*, M_2 should be at a level $\frac{\eta h}{\eta+1}$ lower than his eye level.

(3) If the person stands at the middle of the room, then $\eta = \frac{1}{2}$.

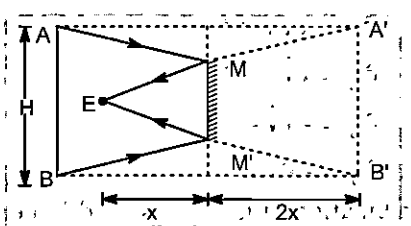


Fig. 1.37

∴ Height (minimum) of mirror required is

$$= \frac{\left(\frac{1}{2}\right)H}{\left(\frac{1}{2}+1\right)} = \frac{H}{3}$$

The Minimum Width of a Plane Mirror Required for a Person to See the Complete Width of his Face

The minimum width of a plane mirror required for a person to see the complete width of his face is $(D-d)/2$ where, D is the width of his face and d is the distance between his two eyes. (Fig. 1.38)

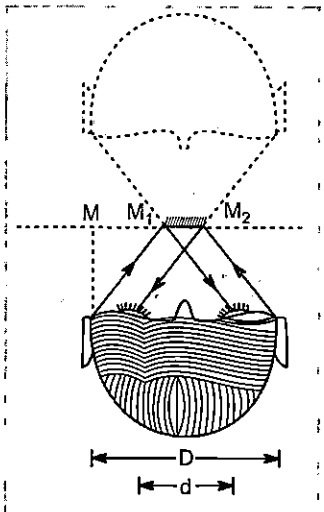


Fig. 1.38

$$MM_1 = \frac{1}{2}\left[D - \frac{1}{2}(D-d)\right]$$

$$= \frac{(D+d)}{4} \quad ...(1)$$

and $$MM_2 = D - \frac{(D+d)}{4}$$

$$= \frac{(3D-d)}{4} \quad ...(2)$$

∴ Width of the mirror $= M_1M_2$

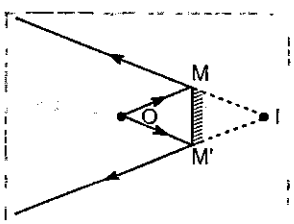
$= MM_2 - MM_1$

$$= \frac{2D-2d}{4} \quad \text{[From eqns. (1) and (2)]}$$

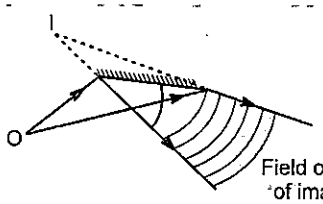
$$= \frac{(D-d)}{2}$$

Field of View

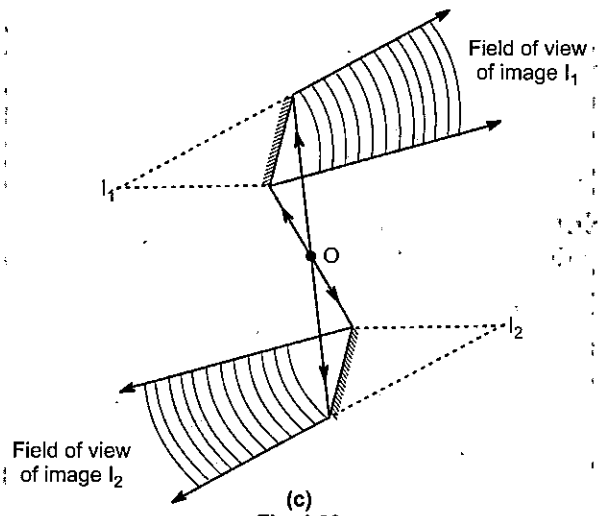
This is the region where diverging rays from object or image are present. If our eyes are present in field of view and receive diverging rays then only we can see image. Field of view of image is decided by drawing extreme reflected or refracted rays.



Field of view of image
Fig. 1.39 (a)



(b)



(c)
Fig. 1.39

Virtual object has no field of view *i.e.,* it cannot be seen.

Illustration 1: A point source of light S, placed at a distance L in front of the centre of a mirror of width d. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in Fig. 1.40(a) Find the distance over which he can see the image of the light source in the mirror.

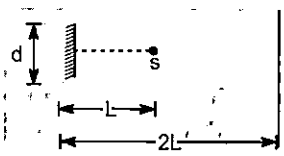


Fig.1.40 (a)

Solution: Man will see image as long as he is in the field of view of image.

From similar $\Delta MM'I$ and ΔPQI

$$\frac{d}{L} = \frac{PQ}{3L}$$

or $$PQ = 3d$$

∴ Required distance = $3d$

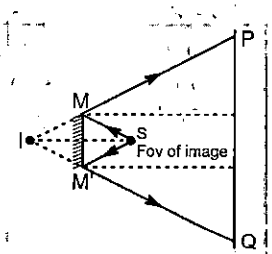


Fig. 1.40 (b)

Example 2

Fig. 1E.2(a) shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (The region is called field of view.)

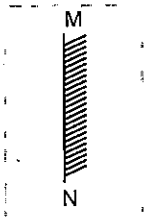


Fig. 1E.2 (a)

Solution: See Fig. 1E.2 (b), consider any two rays emanating from the object N_1 and N_2 are normals ;

$$i_1 = r_1 \text{ and } i_2 = r_2$$

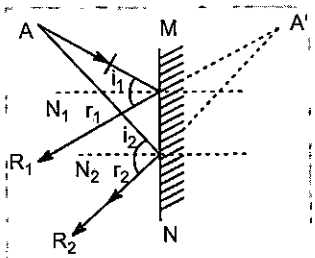


Fig. 1E.2 (b)

The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror such that their meeting point is A'. To obtain the region in which reflected rays are present, join A' with the ends of mirror and extend. The following figure shows this region as shaded. In Fig. 1E.2 (c) there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q cannot see the image because they do not receive any reflected ray.

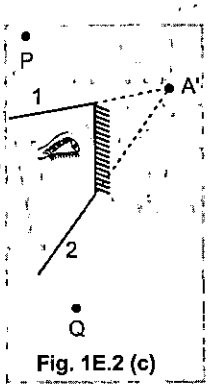


Fig. 1E.2 (c)

Example 3

Find the region on Y-axis in which reflected rays are present. Object is at A (2,0) and MN is a plane mirror, as shown in Fig. 1E.3(a).

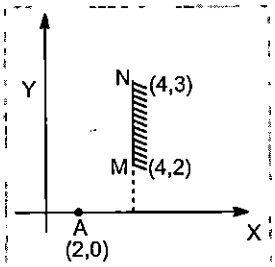


Fig. 1E.3 (a)

Solution: The image of point A, in the mirror is at A' (6,0). Join $A'M$ and extend to cut Y-axis at M' (Ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join $A'N$ and extend to cut Y-axis at N' (Ray originating from A which strikes the mirror at N get reflected as the ray NN' which appears to come from A').

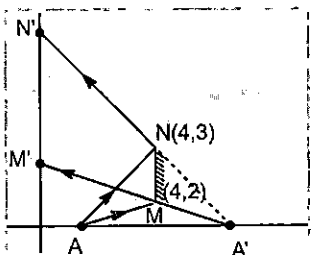


Fig. 1E.3 (b)

From geometry.

$M' \equiv (0,6)$

$N' \equiv (0,9)$. $M'N'$ is the region on Y axis in which reflected rays are present.

Illustration 2: A ray of light is incident on a plane mirror at a certain point. The mirror is capable of rotation about an axis passing through X as shown in Fig. 1.34. Prove that if the mirror turns through a certain angle, the reflected ray turns through double the angle.

Solution: Let XY be the position of mirror initially and mirror be rotated to a new position XY' through an angle θ about an axis passing through X.

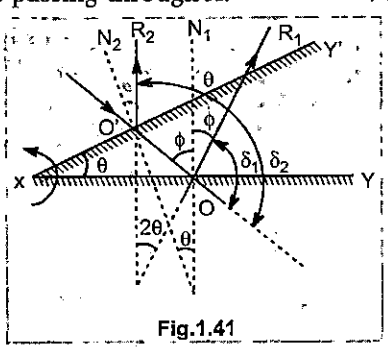


Fig.1.41

δ_1 = Deviation in first position of mirror

$= \pi - 2\phi$

δ_2 = Deviation in second position of mirror

$= \pi - 2(\phi - \theta)$ [$\because$ Angle of incidence in second position $= \phi - \theta$]

$\therefore \delta_1 - \delta_2 = \pi - 2(\phi - 2\theta) - (\pi - 2\phi) = 2\theta$

$\pi - 2\phi - \pi + 2\phi - 2\theta$

Hence, reflected ray rotates twice the angle turned by the mirror.

Remarks

(1) The angle turned by the reflected ray is twice the angle turned by the mirror. So, if a mirror turns through a certain angle, keeping incident ray fixed the reflected turns through double that angle in same sense i.e., if mirror rotates clockwise then reflected ray will rotate clockwise.

(2) The above statement is valid for rotation of mirror about any point. (See illustration 2).

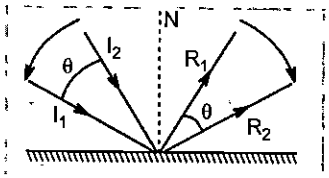


Fig.1.42: Rotation of incident ray

(3) If mirror is kept fixed and incident ray is rotated then reflected ray will rotate in opposite sense by same angle.

(4) If mirror and incident ray both are rotated then net rotation suffered by reflected ray will be algebraic sum of rotation suffered by reflected ray due to mirror rotation and incident ray rotation separately keeping sense of rotation in mind.

ANGLE OF DEVIATION

When a light ray strikes a plane mirror at an angle of incidence i, the deviation of the reflected ray is $(\pi - 2i)$. The angle of deviation is the angle made by the reflected ray with the direction of incident ray.

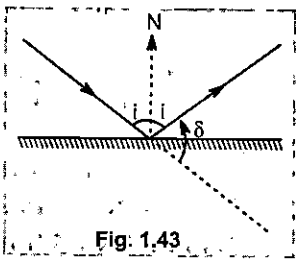


Fig. 1.43

If several plane mirrors are placed on the path of a light ray, each mirror deviation ray given by the deviation can be clockwise or anticlockwise. In the above case, the deviation is **anticlockwise.** In case of multiple reflections, clockwise and anticlockwise deviations are given opposite signs, net deviation is the algebraic sum of the deviations due to the individual mirrors.

Illustration 3: Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror and parallel to the second is reflected from the second mirror parallel to the first mirror. What is the angle between the two mirror and find total deviation produced in the incident ray due to the two reflections.

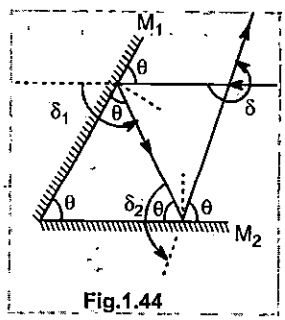


Fig.1.44

Solution: From Fig. 1.44

$$3\theta = 180^\circ$$

$$\therefore \quad \theta = 60^\circ$$

$$\delta_1 = 180^\circ - 2\times 30^\circ$$

$$= 120^\circ \text{ anticlockwise}$$

$$\delta_2 = 180^\circ - 2\times 30^\circ$$

$$= 120^\circ$$

$\therefore$ Total deviation $= \delta_1 + \delta_2$

$= 240^\circ$ anticlockwise or 120° clockwise

Illustration 4 : Two plane mirrors are inclined to each other at an angle θ. A ray of light is reflected first at one mirror and then at the other. Find the total deviation of the ray.

Solution: Let

α = Angle of incidence for M_1

β = Angle of incidence for M_2

δ_1 = Deviation due to M_1

δ_2 = Deviation due to M_2

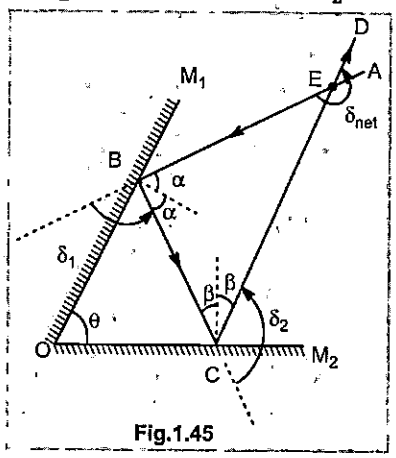


Fig.1.45

From Fig. 1.45

$$\delta_1 = \pi - 2\alpha, \quad \delta_2 = \pi - 2\beta$$

Both deviation are in same sense *i.e.,* anticlockwise

$\therefore$ δ_{net} = Total deviation $= \delta_1 + \delta_2$

$$= 2\pi - 2(\alpha + \beta)$$

Now in ΔOBC,

or $(90^\circ - \alpha) + (90^\circ - \beta) + \theta = 180^\circ$

or $\alpha + \beta = \theta$

Hence, $\delta_{net} = 2\pi - 2\theta$

Illustration 5: Calculate deviation suffered by incident ray in situation as shown in Fig. 1.46 (a) after three successive reflections.

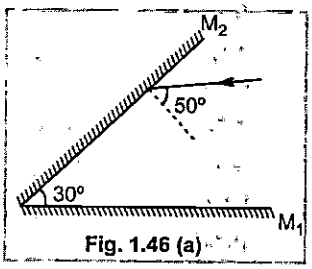


Fig. 1.46 (a)

Solution:

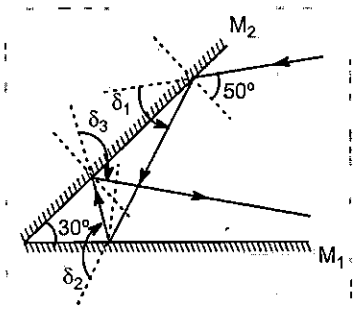


Fig. 1.46 (b)

From Fig. 1.46 (b)

$\delta_1 = 180°-2\times 40° = 100°$ anticlockwise

$\delta_2 = 180°-2\times 20° = 140°$ clockwise

$\delta_3 = 180°-2\times 10° = 160°$ clockwise

δ = Total deviation $100°+140°+160°$

$= 100°$ clockwise or $260°$ anticlockwise

Example 4

Show that the angle of deviation of a light ray striking two plane mirrors, successively, kept inclined to each other, depends only upon the angle between the mirrors.

Solution: Let the two plane mirrors M_1 and M_2 be kept inclined at an angle α to each other. Let the incident ray strike the plane mirror at an angle of incidence θ_1. (Fig.1E.4)

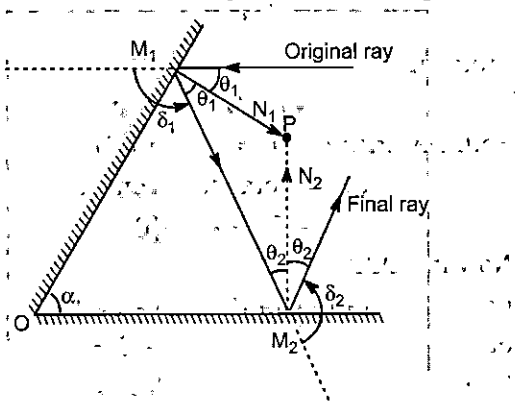


Fig. 1E.4

Deviation produced at first reflection is

$\delta_1 = (\pi - 2\theta_1)$. (anticlockwise sense) ...(1)

When the light ray strikes the mirror M_2 at an angle of incidence θ_2 given by

$\theta_2 = \pi - [\angle M_1PM_2 + \theta_1]$

$= [(\pi - \angle M_1PM_2) - \theta_1] = \alpha - \theta.$

[$\because M_1OM_2P$ is a cyclic quadrilateral so $M_1PM_2 = \pi - \alpha$]

The deviation produced due to the reflection on the mirror M_2 is

$\delta_2 = \pi - 2(\alpha - \theta_1)$. (anticlockwise sense) ...(2)

Total deviation produced is

$\delta_{Net} = \delta_1 + \delta_2 = 2(\pi - \alpha)$,

which evidently depends only on α, the angle between the two mirrors.

Some Applications:

(1) If the mirrors be mutually perpendicular, then net deviation is π. The light ray reverses its direction, irrespective of its angle of incidence.

(2) If the incident light ray and the emergent light ray are parallel to the two mirrors, then net deviation = $(\pi + \alpha)$.

$\therefore \quad \pi + \alpha = 2(\pi - \alpha)$

or $\quad 3\alpha = \pi$ or $\alpha = \frac{\pi}{3}$.

The two mirrors are inclined at an angle $\frac{\pi}{3}$ (or 60°) to each other.

EXTENDED OBJECT

An extended object like AB shown in Fig. 1.47 is combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite image will be formed A' will be images of A,C' will be image of C,B' will be image of B etc. All point images together form extended image. Thus extended image is formed of an extended object.

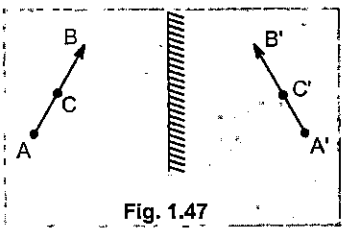


Fig. 1.47

Properties of image of an extended object, formed by a plane mirror:

(1) Size of extended object = Size of extended image.

(2) The image of erect, if the extended object is placed parallel to the mirror.

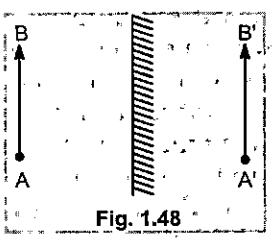


Fig. 1.48

(3) The image is inverted if the extended object lies perpendicular the plane mirror.

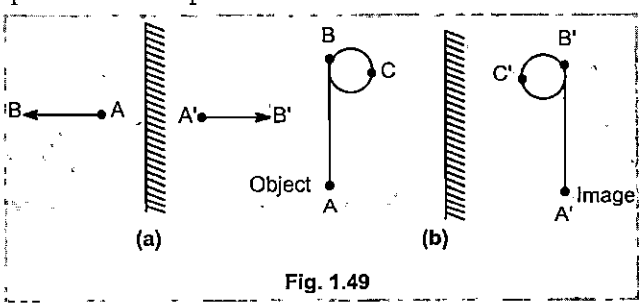


Fig. 1.49

Note: It an extended horizontal object is placed infornt of a mirror inclined 45° with the horizontal, the image formed will be vertical. See Fig. 1.50.

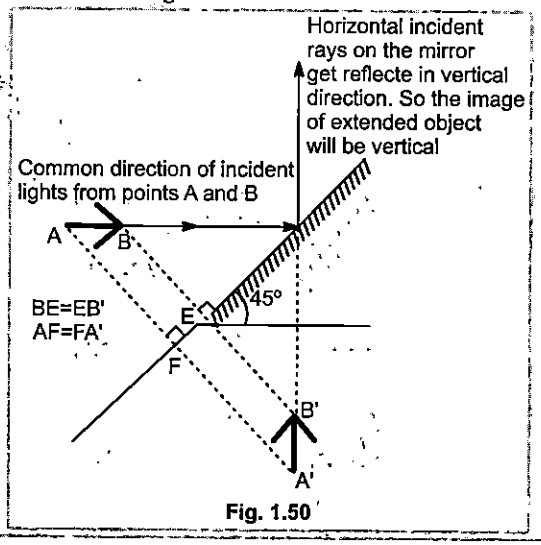


Fig. 1.50

Example 5

Fig. 1E.5 shows an object AB and a plane mirror MN placed parallel to object. Indicate the mirror length required to see the image of object if observer's eye is at E.

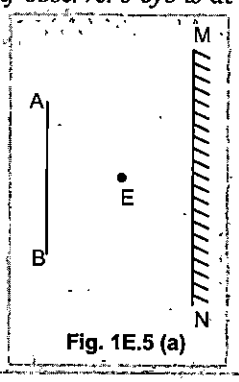


Fig. 1E.5 (a)

Solution:

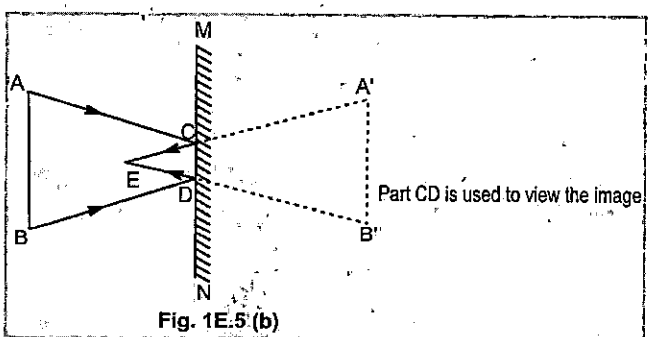


Fig. 1E.5 (b)

Velocity of Image :

Let, $X_{O/M}$ = X co-ordinate of object w.r.t. mirror

$X_{I/M}$ = X co-ordinate of image w.r.t mirror

$Y_{O/M}$ = Y co-ordinate of object w.r.t. mirror

$Y_{I/M}$ = Y co-ordinate of image w.r.t. mirror

For plane mirror

$X_{O/M} = -X_{I/M}$

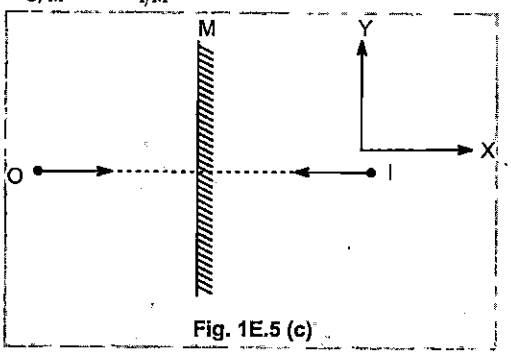


Fig. 1E.5 (c)

Differentiating both sides w.r.t. 't'

$$\frac{d}{dt}(X_{O/M}) = \frac{-d}{dt}(X_{I/M})$$

$$[\vec{v}_{O/M}]_X = -[\vec{v}_{IX} - \vec{v}_{MX}]$$

$$\therefore \quad \vec{v}_{IX} = 2\vec{v}_{MX} - \vec{v}_{OX}$$

Similarly, $Y_{I/M} = Y_{O/M}$

Differentiating both side w.r.t. 't' we get

$$(\vec{v}_{I/M})_Y = (\vec{v}_{O/M})_Y$$

In nutshell, for sloving numerical problems involving calculation of velocity of image of object with respect to any observer, always calculate velocity of image first with respect to mirror using following points.

$$(\vec{v}_{I/M})_{\parallel} = (\vec{v}_{O/M})_{\parallel}$$

$$(\vec{v}_{I/M})_{\perp} = (\vec{v}_{O/M})_{\perp}$$

$$\vec{v}_{I/M} = (\vec{v}_{I/M})_{\parallel} + (\vec{v}_{I/M})_{\perp}$$

Velocity of image with respect to required observer is then calculated using basic equation for relative motion.

$$\vec{v}_{A/B} = \text{Velocity of } A \text{ with respect } B$$

$$= \vec{v}_A - \vec{v}_B$$

(i) If an object moves towards (or away from) a plane mirror at speed v, the image will also approach (or recede) at the same speed v, and the relative velocity of image with respect to object will be $2v$, as shown in Fig. 1.51.

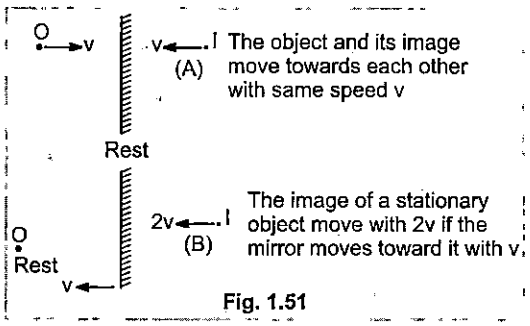


Fig. 1.51

(ii) If the mirror is moved toward (or away from) the object with speed v, the image will also move toward(or away form) the object with a speed $2v$, as shown in figure.

Case (i)

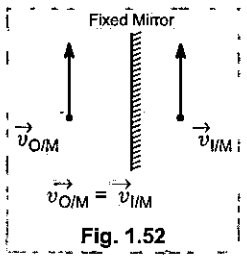


Fig. 1.52

If mirror is fixed and object moves in plane parallel to mirror, velocity of object is equal to velocity of image.

Case (ii)

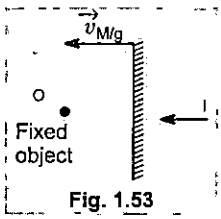


Fig. 1.53

$$\vec{\mathbf{v}}_{I/g} = -\vec{\mathbf{v}}_{O/g} + 2\vec{\mathbf{v}}_{M/g}$$

as $\vec{\mathbf{v}}_{O/g} = 0$

$$\vec{\mathbf{v}}_{I/g} = +2\vec{\mathbf{v}}_{M/g}$$

Case (iii)

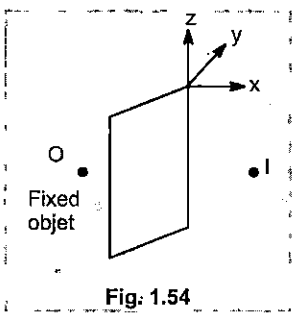


Fig. 1.54

If mirror has velocity

$\vec{\mathbf{v}}_{M/g} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$

$v_x\hat{\mathbf{i}}$ = velocity component of mirror perpendicular to mirror.

$\vec{\mathbf{v}}_{\parallel} = v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$ = velocity component of mirror in the plane of mirror.

$$\vec{\mathbf{v}}_{M/g} = \vec{\mathbf{v}}_{\perp} + \vec{\mathbf{v}}_{\parallel}$$

Velocity if image is not effected by motion of mirror in its own plane *i.e.*, $\vec{\mathbf{v}}_{\parallel} = v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$ does not change the velocity of image.

Only the $\vec{\mathbf{v}}_{\perp} = \vec{\mathbf{v}}_x\hat{\mathbf{i}}$ has effect an velocity of image. Thus in the above situation

$$\vec{\mathbf{v}}_{I/g} = -\vec{\mathbf{v}}_{O/g} + 2\vec{\mathbf{v}}_{M/g} = -2v_x\hat{\mathbf{i}}$$

Case (iv) General case

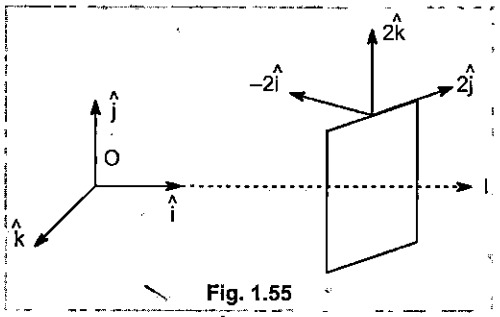


Fig. 1.55

Given $\vec{\mathbf{v}}_{M/g} = \underbrace{-2\hat{\mathbf{i}}}_{\vec{\mathbf{v}}_{M\perp}} + \underbrace{2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}_{\vec{\mathbf{v}}_{M\parallel}}$

Velocity component of mirror ⊥ to mirror

Velocity component of mirror in its plane

Given $\vec{\mathbf{v}}_{O/g} = \underbrace{\hat{\mathbf{i}}}_{\vec{\mathbf{v}}_{O\perp}} + \underbrace{\hat{\mathbf{j}} + \hat{\mathbf{k}}}_{\vec{\mathbf{v}}_{O\parallel}}$

velocity of object perpendicular to mirror

Velocity component of object parallel to mirror

Note that $\vec{\mathbf{v}}_{M/I}$ has no effect on velocity of image.

$$[\vec{\mathbf{v}}_{I/g}]_{\perp} = -[\vec{\mathbf{v}}_{O/g}]_{\perp} + [2\vec{\mathbf{v}}_{M/g}]_{\perp}$$

Component ⊥ to mirror

$$= -\hat{\mathbf{i}} + 2[-2\hat{\mathbf{i}}]$$

$$= -5\hat{\mathbf{i}}$$

$$[\vec{\mathbf{v}}_{I/g}]_{\parallel} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Component || to mirror,

Thus $\vec{\mathbf{v}}_{P/g} = -5\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Illustration 6 : A point object is moving with a speed v before an arrangement of two mirrors as shown in Fig. 1.56(a). Find the velocity of image in mirror M_1 with respect to image in mirror M_2.

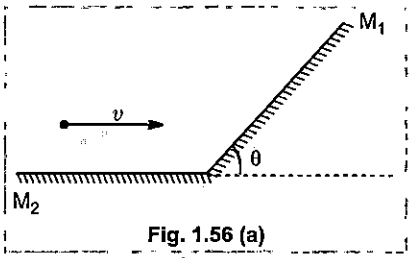


Fig. 1.56 (a)

Solution. Velocity of image in mirror M_1 and M_2 is as shown in Fig. 1.56(b).

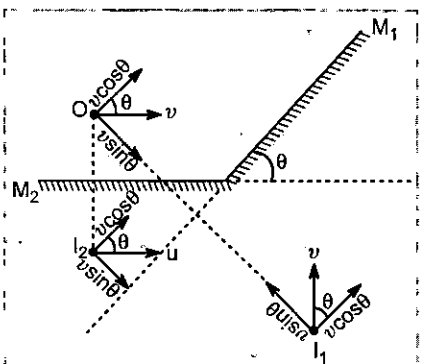


Fig. 1.56 (b)

$\vec{v}_{1/2}$ = Velocity of I_1 w. r. t. I_2

$= \vec{v}_1 - \vec{v}_2$

or $v_{1/2} = 2v\sin\theta$

Example 6

Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the Fig. 1E.6. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB and CD of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces w.r.t. each other is:

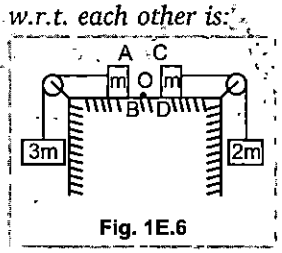


Fig. 1E.6

(a) 5g/6 *(b) 5g/3*

(c) g/3 *(d) 17g/6*

Solution: We know that

$V_I = 2V_M + V_O$

differentiating $a_I = 2a_M + a_O$

$a_o = 0$; $a_I = 2a_M$

$$a_A = \frac{3}{4}g \; ; \qquad a_C = \frac{2g}{3}$$

Accelerate of image in $AB = 2a_A = \frac{3g}{2}$

Acceleration of image in AB w.r.t. that

$$CD = \frac{3g}{2} + \frac{4g}{3} = \frac{17g}{6}\ \text{m/s}^2$$

Image formed by two Plane Mirrors

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

Example 7

Fig. 1E.7(a) shows a point object placed between two parallel mirrors. Its distance from M_1 is 2cm and that from M_2 is 8 cm. Find the distance of image from the two mirrors considering reflection on mirror M_1 first.

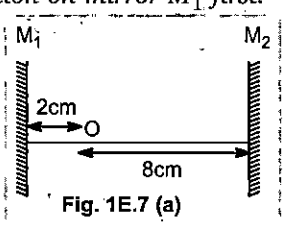


Fig. 1E.7 (a)

Solution: To understand how images are formed see the following figure and table. You will required to know that symbols like I_{121} stands for. See the Fig. 1E.7 (b)

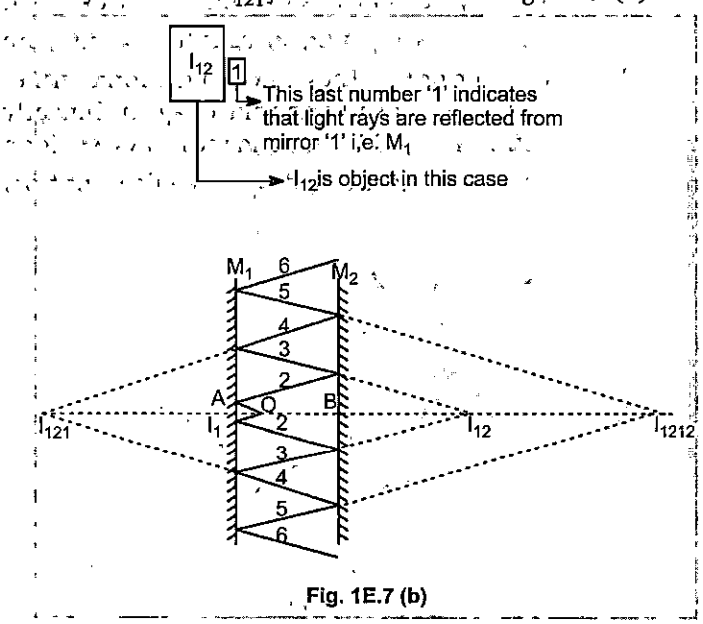


Fig. 1E.7 (b)

Incident rays	Reflected by	Reflected rays	Object	Image	Object distance	Image distance
Rays 1	M_1	Rays 2	O	I_1	$AO = 2cm$	$AI_1 = 2cm$
Rays 2	M_2	Rays 3	I_1	I_{12}	$BI_1 = 12cm$	$BI_{12} = 12cm$
Rays 3	M_1	Rays 4	I_{12}	I_{121}	$AI_{12} = 22cm$	$AI_{121} = 22cm$
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	$BI_{12} = 32cm$	$BI_{1212} = 32cm$

Ans so on .....

Similarly images will be formed by the rays striking mirror M_2 first. Total number of image $=\infty$.

NUMBER OF IMAGES FORMED BY TWO INCLINED PLANE MIRRORS

When mirrors are parallel : In this case, infinite images are formed due to multiple reflections. Image I_1 formed after reflection at M_1 is object for reflection at M_2. Thus each image acts as object for second mirror.

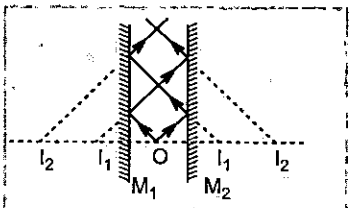


Fig. 1.57. when mirror are parallel

Example 8

Consider two perpendicular mirrors. M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinate of object as (x,y), find the position and number of images.

Solution: Rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at $(x,-y)$, such that O and I_1 are equidistant from mirror M_1. These rays do not form further image because they do not strike any mirror again. Similarly rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at $(-x,y)$; such that O and I_2 are equidistant from mirror M_2.

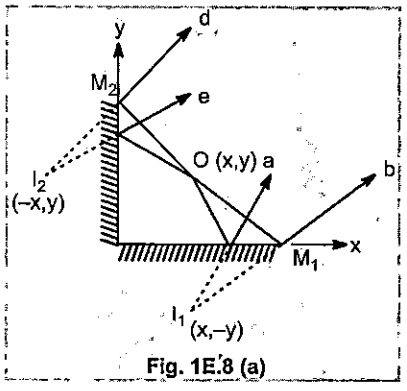


Fig. 1E.8 (a)

Now consider those rays which strike mirror M_2 first and then the mirror M_1.

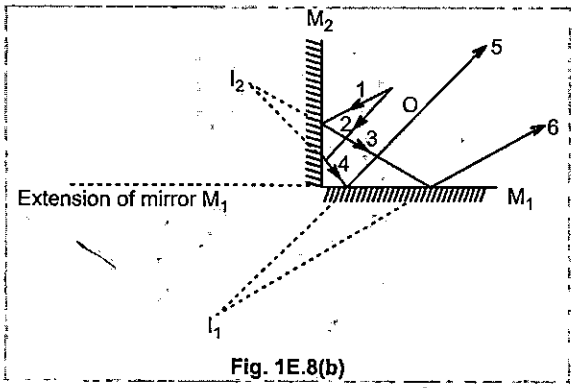


Fig. 1E.8(b)

For incident rays 1,2 object is O, and reflected rays 3,4 form image I_2.

Now rays 3, 4 incident on M_1 (Object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

I_2 and I_{21}, are equidistant from M_1. To summarize see the Fig. 1E.8 (c).

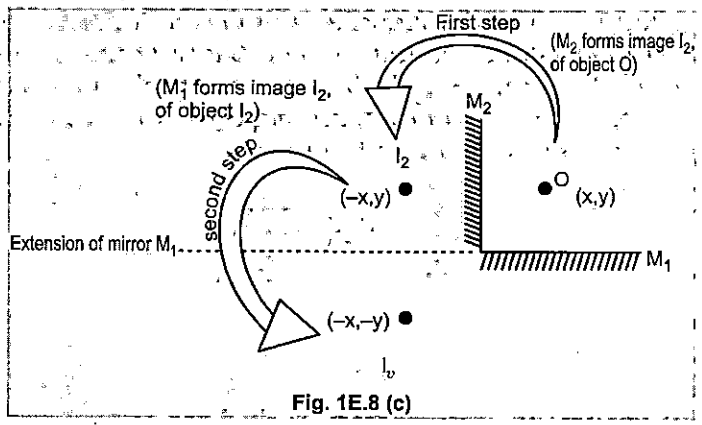


Fig. 1E.8 (c)

For rays reflecting first from M_1 and then from M_2, first image I_1 (at $(x,-y)$) will be formed and this will function as object for mirror M_2 and then its image I_{12} [at $(-x,-y)$] will be formed.

I_{12} and I_{21} coincide.

$\therefore$ Three images are formed.

Concept: ***(i)*** *Note that the third image is formed due to rays undergoing two successive reflection. Also, object and its images lie on a circle whose equation is given by* $x^2 + y^2 = a^2 + b^2$.

(ii) *When an object is placed infront of arrangement of three mutually perpendicular mirrors, then total seven images are formed. Further, object and its images lie on a sphere whose equation is given by* $x^2 + y^2 + z^2 = a^2 + b^2 + c^2$ *where a, b and c are co-ordinates of object.*

Locating All the Images Formed by Two Plane Mirrors :

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in Fig. 1.58.

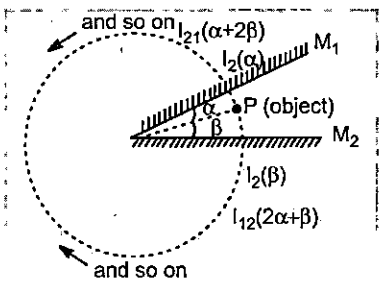


Fig. 1.58

Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I Similarly image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I

Now I_2 will act as an object for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when hsi will process stop.

Hint: The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

Number of Images formed by Two Inclined Mirrors:

(i) If $\frac{360^\circ}{\theta}$ = even number ; number of image $= \frac{360^\circ}{\theta} - 1$

(ii) If $\frac{360^\circ}{\theta}$ = odd number; number of image $= \frac{360^\circ}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\frac{360^\circ}{\theta}$ = odd number ; number of image $= \frac{360^\circ}{\theta}$, if the object is not placed on the angle bisector.

(iv) If $\frac{360^\circ}{\theta} \neq$ integer, then count the number of images as explained above.

Example 9

Two mirrors are inclined by an angel 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror and the total number of images using (i) direct formula and (ii) counting the images.

Solution: Figure is self explanatory

Number of image

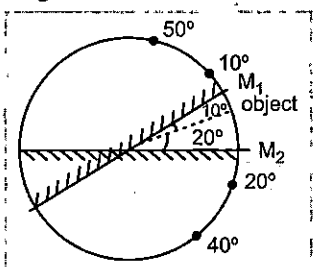


Fig. 1E.9

(i) Using direct formula ; $\frac{360^\circ}{30^\circ} = 12$ (even number)

$\therefore$ number of images $= 12 - 1 = 11$

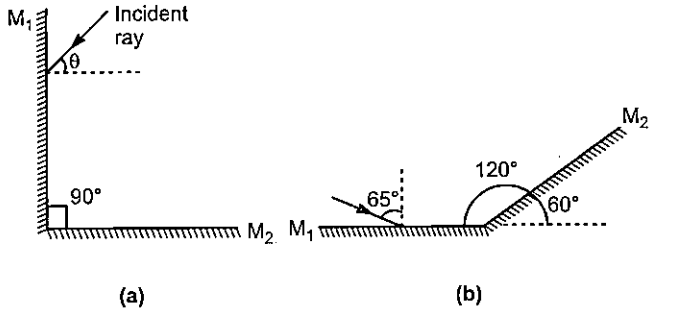
(ii) By counting. See the following table

Image formed by Mirror M_1 (angles are measured from the mirror M_1.)	Images formed by Mirror M_2 (angles are measured from the mirror M_2.)
10°	20°
+30°	
50°	40°
+30°	
70°	80°
+30°	
110°	100°
+30°	
130°	140°
+30°	
170°	160°
Stop because next angle will be more than 180°.	Stop because next angle will be more than 180°.

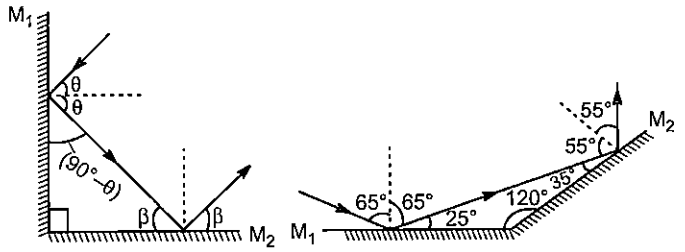
To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors = 160°+170°+30° = 360°. Therefore in this case the last images coincide. Therefore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 − 1 (as the last images coincide) = 6 + 6 − 1 = 11

Example 10

In Figs. 1E.10(a) and (b) two mirrors are inclined at 90° and 120° respectively. Trace the rays as it is reflected from the two mirrors, M_1, M_2.



(a) (b)



(c) (d)

Fig. 1E.10

Solution: The law of reflection is applicable at each surface. From the figure.

$$\beta + 90° + 90° - \theta = 180°$$

$$\beta = \theta$$

Therefore the reflected ray is antiparallel to the incident ray, whatever be the angle of incidence.

Example 11

An object is placed between two perpendicular plane mirrors. How many images are seen ?

Solution: In Fig. 1E.11 we use the tail of an arrow as the object. Its images in mirrors M_1 and M_2 are I_1 and I_2 as shown. In these cases, light reaches the eye after one reflection. In addition, light can enter the eye after being reflected by both mirrors.

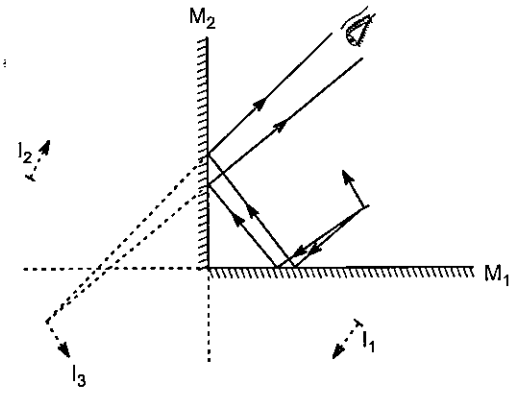


Fig. 1E.11

The rays leading to the formation of image I_3 are shown. Notice that I_3 would be an image of I_1 if M_2 were extended downward. Similarly, I_3 is an image of I_2 in the 'virtual mirror' formed by extending M_1 to the left.

Example 12

(a) What is the minimum length of mirror required for a person to see his or her full height? Assume that the eyes are a distance a below the top of the head and a distance b above the feet. (b) Does the person's horizontal distance from the mirror matter ? (c) Does the vertical position of the mirror matter ?

Solution: (a) Rays enter the eye from the feet and the top of the head after reflection at the mirror, as shown in Fig. 1E.12. We know that the angle of incidence is equal to the angle of reflection. Light from the feet reaches the eye

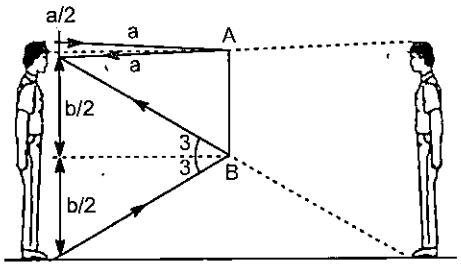


Fig. 1E.12

after reflection at point B, which is $b/2$ above the floor. Light from the top of the head reaches the eye after reflection at point A, located $a/2$ below the top of the head. The person's total height is $a + b$, and the required length of mirror is $a/2 + b/2$, which is 50% of the full height of the person.

(b) No.

(c) Yes, the bottom of the mirror must be at a height $b/2$

Example 13

Rearview mirrors in cars have a day setting and a night setting. The night setting greatly diminishes the intensity of the image in order that lights from trailing vehicles do not blind the driver. How does such a mirror work ?

Solution: Fig. 1E.13 shows a cross-sectional view of a rearview mirror for each setting. The unit consists of a reflective coating on the back of a wedge of glass. In the day setting [Fig. 1E.13(a)], the light from an object behind the car strikes the glass wedge at point 1. Most of the light enters the wedge, refracting as it crosses the front surface, and reflects from the back surface to return to the front surface, where it is refracted again as it reenters the air as ray *B* (for bright). In addition, a small portion of the light is reflected at the front surface of the glass, as indicated by ray *D* (for *dim*).

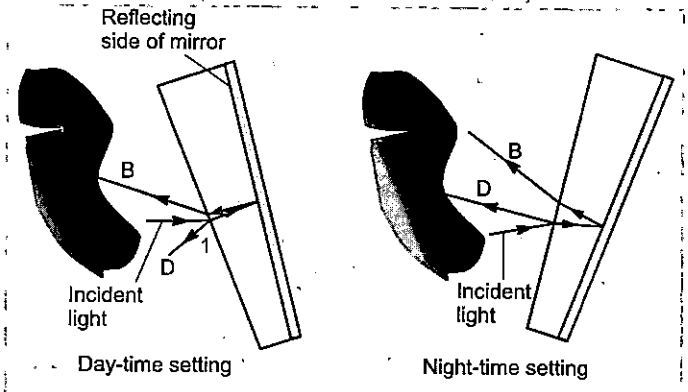


Fig. 1E.13

This dim reflected light is responsible for the image that is observed when the mirror is in the night setting [Fig. 1E.13 (b)]. In this case, the wedge is rotated so that the path followed by the bright light (ray *B*) does not lead to the eye. Instead, the dim light reflected from the front surface of the wedge travels to the eye, and the brightness of trailing headlights does not become a risk.

Example 14

A plane mirror is inclined at an angle θ with the horizontal surface. A particle is projected from point P (see Fig. 1E.14) at t = 0 with a velocity v at an angle α with the horizontal. The image of the particle is observed from the frame of the particle projected. Assuming the particle does not collide the mirror, find the

(a) time when the image will come momentarily at rest w.r.t. the particle, (b) path of the image as seen by the particle.

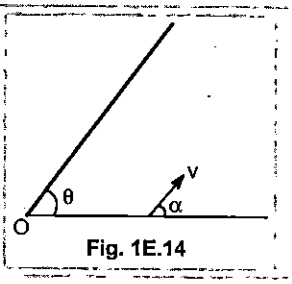


Fig. 1E.14

Solution: (a) The image will appear to be at rest w.r.t. the particle, at the instant the velocity of the particle is parallel to the mirror,

$$i.e., \quad \frac{v_y}{v_x} = \tan\theta \quad \text{or} \quad \frac{v\sin\alpha - gt}{v\cos\alpha} = \tan\theta$$

$$\text{or} \qquad t = \frac{v\cos\alpha(\tan\alpha - \tan\theta)}{g}$$

(b) Path will be a straight line. Because at any instant velocity of the particle can be resolved into two components, one parallel to the mirror and the other perpendicular to it, parallel components of the particle and its image are identical. Hence the path will be a straight line perpendicular to the mirror.

Example 15

An oblong object PQ of height h stands erect on a flat horizontal mirror. Sunrays fall on the object at a certain angle. Find the length of the shadow on a screen placed beyond the shadow on the mirror.

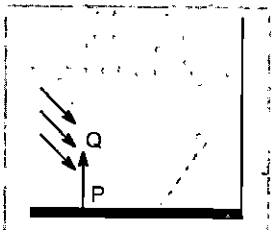


Fig. 1E.15 (a)

Solution: The formation of shadow on the mirror and on the screen is shown in Fig. 1E.15 (b) in compliance with the principle of rectilinear propagation of light and law of reflection.

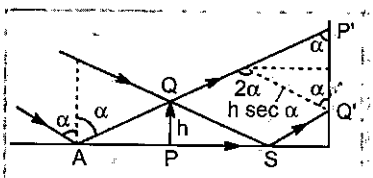


Fig. 1E.15 (b)

PS is the shadow on the mirror and *P'Q'* is the inverted shadow of *PQ* on the screen. Let α = angle of incidence. Then *PS* = *h* tan α and *QS* = *h* sec α. From the property of triangle,

$$P'Q' = 2(h\sec\alpha)\cos\alpha = 2h$$

Example 16

Determine graphically the positions of the eye when an observer can simultaneously see in a flat mirror of finite dimensions the image of a point and a section of a straight line placed w.r.t. the mirror as shown in Fig. 1E.16(a).

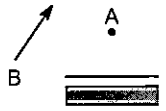


Fig. 1E.16 (a)

Solution: Observer can see only when the eye is placed inside the triangle *DEH* limited by the rays *DG* and *EF* [Fig.1E.16(b)]

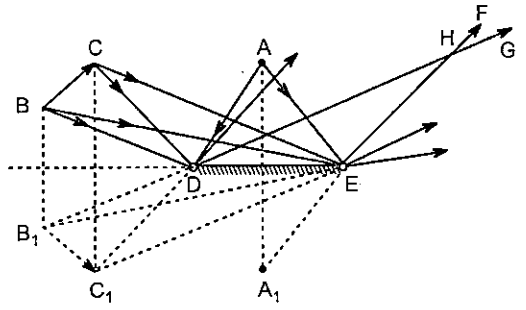


Fig. 1E.16 (b)

Example 17

An object O'O and a mirror AC are placed as shown in Fig. 1E.17(a). Construct the image of this object in the mirror. Where should the eye be placed to observe the image of the entire object ?

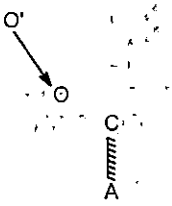


Fig. 1E.17 (a)

Solution: See Fig. 1E.17 (a). The rays coming from the point *O'* will be propagated inside the band restricted by the straight lines *AB* and *CD* after reflection from the mirror. The rays coming from *O* will be inside the bands *AE* and *CF*.

The rays coming from all the points on the object will only arrive at each point in space between the straight lines *AB* and *CF*. The eye can only see the entire image of the object if it is at one of the points enclosed between the rays *AB* and *CF*.

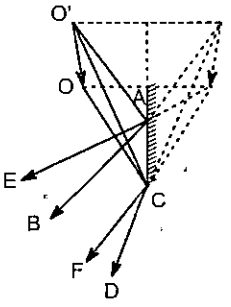


Fig. 1E.17 (b)

Example 18

A mirror 1 m high hangs on a wall. A man stands a distance of 2 m away from the mirror. What is the height of the portion of the opposite wall in the room that can be seen by the man in the mirror without changing the position of his head? The wall is 4 m from the mirror.

Solution: The image of the wall will be behind the mirror at a distance $l_2 = 4$m. If the eye is placed at the point A (Fig. 1E.18) it will see only the rays coming from all the points in the section of the wall image *DE* after the reflection in the mirror *BC*. Thus, the section of the wall visible in the mirror will have dimensions

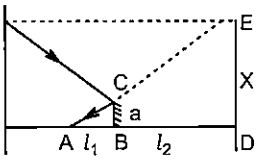


Fig. 1E.18

$$X = \frac{l_1 + l_2}{l_1} a = 3\text{m}$$

Example 19

A narrow beam of light S is incident on a dihedral angle $\alpha = 60°$ formed by identical flat mirrors OM and ON secured on axis O [Fig.1E.19(a)]. After being reflected from the mirrors the light is focussed by lens L and gets into stationary receiver R. The mirrors rotate with a constant angular velocity.

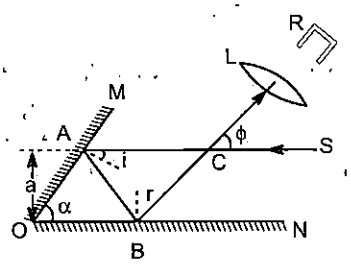


Fig. 1E.19 (a)

What part of the light energy of the beam will reach the receiver during a time that greatly exceeds the period of rotation if the beam passes at a distance a from an axis equal to half the length of mirror OM ?

Solution: The beam reflected from mirror *ON* forms with the incident beam an angle ϕ [see Fig. 1E.19(a)] that does not depend on the angle of incidence *i*. From triangle *ABC*, we have $\phi = 180°-2(i + r)$ and in triangle *OAB*, we have $\alpha + (90°-i) + (90°-r) = 180°$. Hence, $\phi = 180°-2\alpha = 60°$. When the mirror rotates, the direction of the reflected beam does not change.

Thus, if the beam that fell on mirror *OM* is reflected from mirror *ON*, it will always get into the receiver. As can easily be seen in Fig. 1E.19 (b) showing two extreme positions of the mirror at which the beam gets into the

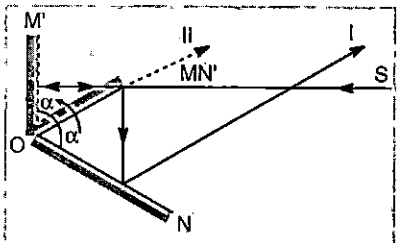


Fig. 1E.19 (b)

receiver (*OM, ON* and *OM', ON'*), this occurs during one-sixth of a revolution. For this reason one-sixth of all the energy of the beam will get into the receiver during one revolution which is a sufficiently large interval of time.

Example 20

In what direction should a beam of light be sent from point A [Fig. 1E.20(a)] contained in a mirror box for it to fall onto point B after being reflected once from all four walls ?

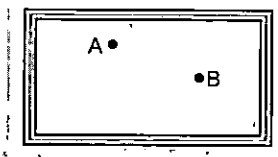


Fig. 1E.20 (a)

Points A and B are in one plane perpendicular to the walls of the box (i.e. in the plane of the drawing).

Solution: Let us plot the image of point *B* in mirror *bd* [Fig. 1E.20(b). Let us then construct image B_1 in mirror *cd*. Also, B_3 is the image of B_2 in mirror *ac* and B_4 is the image of B_3 in mirror *ab*.

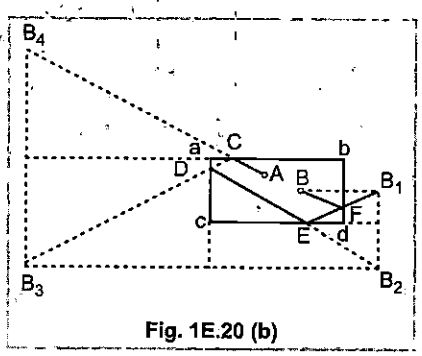


Fig. 1E.20 (b)

Let us connect points *A* and B_4. Point *C* is the point of intersection of *ab* with line AB_4. Let us now draw line B_3C from B_3 and connect point *D* at which this line intersects *ac* with B_2, *E* with B_1 and *F* with *B*.

It can be stated that broken line *ACDEFB* is the sought path of the beam. Indeed, since B_3CB_4 is an isosceles triangle, *CD* is the reflection of beam *AC*.

Similarly, it is easy to show that *DE* is the reflection of *CD*, etc.

This solution of the problem is not unique, since the beam should not necessarily be sent initially to mirror *ab*.

Example 21

(a) In an experiment light is repeatedly reflected from moving mirrors (Fig. 1E.21) The mirrors were placed on discs revolving in different directions.

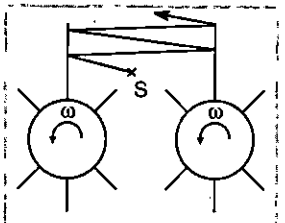


Fig. 1E.21

(i) The angular velocity ω of rotation of the discs being known, find the angular velocity Ω of rotation of a beam that is consecutively reflected n times from the mirrors.

(ii) Determine the linear velocity of the n^{th} image at the moment when the mirrors are parallel to each other and their reflecting portions move with a velocity v in different directions.

(b) Solve the problem if the discs rotate in the same direction.

Solution: (a) (i) The beam reflected from the first mirror forms an angle 2α with the incident beam (α is the angle of incidence). During the time *t* the mirror will turn through an angle ωt and the new angle of incidence will become equal to $\alpha + \omega t$, as will the angle of reflection. Therefore, the angle between the incident and reflected beams will increase by $2\omega t$, *i.e.*, the reflected beam will turn through an angle $2\omega t$.

In view of this, the angle of incidence on the second mirror, provided it does not rotate, would be $\beta + 2\omega t$, where β is the angle of incidence with immobile discs. But the mirror also revolves through the angle ωt during the time *t*, and therefore the angle of incidence becomes $\beta + 3\omega t$. The angle of reflection will be the same. Thus, after two reflections the beam will turn through the angle $3\omega t$ from its direction with immobile mirrors. After there reflections the beam will turn through $5\omega t$ and after *n* reflections through $(2n-1)2\omega t$. In this way its angular velocity will be $\Omega = (2n-1)2\omega$.

(ii) When the mirror moves from the source with a velocity *v*, the image will move away from the source with a velocity $2v$ and from the second mirror with a velocity $3v$. Therefore, the second image moves with a velocity $3v$ w.r.t. the second mirror and with a velocity $4v$ w.r.t. the source. The velocity of the third image w.r.t. the source will be $6v$ and the velocity of the n^{th} image $2nv$.

(b) (i) When the first mirror turns through an angle ωt the reflected beam will turn through an angle $2\omega t$. Hence, the angle of incidence on the second mirror will also increase by $2\omega t$, and if the mirror did not revolve, the angle of reflection would also increase by $2\omega t$. After two reflections the beam would turn through $2\omega t$ as compared with the case of immobile mirrors.

Since the second mirror does rotate, however, the angle of the beam incident on it decreases by ωt during the time t. The angle of reflection decreases by the same amount and for this reason the reflected beam will travel in the same direction as with immobile discs.

Since this line of reasoning may be adopted for any two consecutive reflections, the angular velocity of rotation of the beam subjected to n reflections will be $\Omega = 0$ if n is even and $\Omega = 2\omega$ if n is odd.

(ii) The first image moves away from the source with a velocity $2v$ and from the second mirror with a velocity v. Therefore, the second image moves w.r.t. the second mirror with a velocity v, *i.e.*, it is immobile w.r.t. the source.

Reasoning similarly, we find that the sought linear velocity of the n^{th} image is zero if n is even, and $2v$ if n is odd.

Example 22

When sunrays pass through a small opening in the foliage at the top of a high tree, they produce an elliptical spot on the ground. The major and minor axes of the ellipse are $a = 12$ cm and $b = 10$ cm respectively. What is the height of the tree ? The angular dimensions of the sun's disc are $\beta = \frac{1}{108}$ rad.

Solution: The cone of rays through the foliage will have elliptical section by the ground plane. The section of the cone perpendicular its axis is a circle of diameter b. While the length of the section by the plane through the axis of the cone and the foot of the tree is elongated to a, the side perpendicular to it remains unchanged.

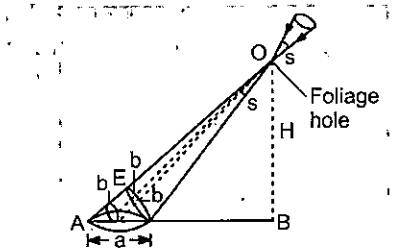


Fig. 1E.22

$$\therefore \quad \sin\alpha = \frac{b}{a} \text{ where } \alpha = \angle OAD$$

Now $DE = OD \times \beta = \dfrac{H}{\sin\alpha}\beta$

Also $DE = a\sin\alpha$

$$\therefore \quad a\sin\alpha = \frac{H}{\sin\alpha}\times\beta \Rightarrow a\sin^2\alpha = H\beta$$

$$\Rightarrow \quad H = \frac{a\sin^2\alpha}{\beta} = \frac{a.b^2/a^2}{\beta} = \frac{b^2}{a\beta}$$

Here $$H = \frac{10^2}{12\times\frac{1}{108}} = \frac{100\times108}{12} = 900\,\text{cm} = 9\,\text{m}$$

Example 23

A plane mirror is placed along the xz-plane and an object P is placed at point $(0, a)$ the mirror rotates about z-axis with constant angular velocity ω. Find position and velocity of image as a function of time $t\left(<\frac{\pi}{2\omega}\right)$.

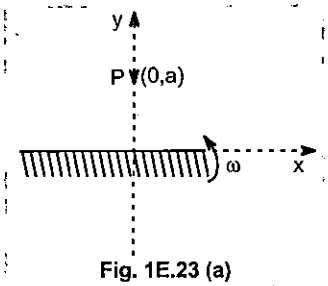


Fig. 1E.23 (a)

Solution: $\omega = \theta/t \Rightarrow \theta = \omega t$

$$y = -(2a\cos^2\theta - a)$$

$$y = -a(2\cos^2\theta - 1) \Rightarrow y = -a[2\cos^2\omega t - 1]$$

$$\Rightarrow \quad y = -a\cos 2\omega t$$

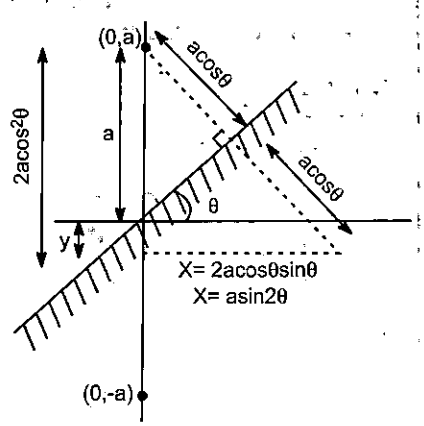


Fig. 1E.23 (b)

$$x = a\sin 2\theta = a\sin 2\omega t$$

$$\vec{\mathbf{r}}_1 = (a\sin 2\omega t\hat{\mathbf{i}} - a\cos 2\omega t\hat{\mathbf{j}})$$

$$V_x = \frac{dx}{dt} = a\cos 2\omega t \times 2\omega = 2\omega\cos 2\omega t$$

$$V_y = \frac{dy}{dt} = 2a\omega\sin 2\omega t$$

Example 24

A light ray parallel to the x-axis strikes the outer reflecting surface of a sphere at a point (2, 2, 0). Its center is at the point (0, 0, −1). The unit vector along the direction of the reflected rays is $x\hat{i}+y\hat{j}+z\hat{k}$. Find the value of $\frac{yz}{x^2}$.

Solution: $\hat{n}=\frac{2\hat{i}+2\hat{j}+k}{3}$

$\hat{e}=-\hat{i}$

Using $\hat{r}=\hat{e}-2(\hat{e}.\hat{n})\hat{n}$

$\hat{r}=\hat{i}-\frac{2(-2)}{3}\frac{(2\hat{i}+2\hat{j}+\hat{k})}{3}$

$=-\hat{i}+\frac{4}{9}(2\hat{i}+2\hat{j}+\hat{k})$

$=\frac{-\hat{i}+8\hat{j}+4\hat{k}}{9}$

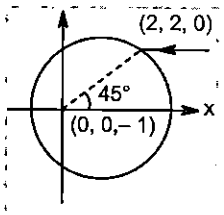


Fig. 1E.24

Example 25

The mirror of length 2l makes 10 revolutions per minute about the axis crossing its midpoint O and perpendicular to the plane of the figure. There is a light source in point A and an observer in point B of the circle of radius R drawn around centre O (AOB = 90°).

(a) Along what curve does the virtual image of the point - like light source A move ?

(b) At what speed does the virtual image of A move ?

(c) What is the ratio l/R if the observer B first sees the light source when the angle of the mirror is $\phi = 15\%$?

Solution: (a) Circle

(b) $\omega_M = 10\times\frac{2\pi}{60}=\frac{\pi}{3}$ rad/s

$\omega_1 = 2\omega_M = \frac{2\pi}{3}$ rad/s

$v_1 = \omega_1 R = \frac{2\pi}{3}R$ m/s

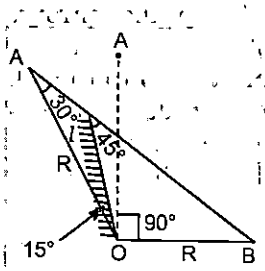


Fig. 1E.25

(c) In $\Delta OA'C$

$\frac{\sin 30°}{l}=\frac{\sin 135°}{R}$

$\Rightarrow \frac{1}{2l}=\frac{1}{R\sqrt{2}}$

$\Rightarrow \frac{l}{R}=\frac{1}{\sqrt{2}}$

Example 26

An elevator at rest which is at 10^{th} floor of a building is having a plane mirror fixed to its floor. A particle is projected with a speed $\sqrt{2}$ m/s and at 45° with the horizontal as shown in the figure. At the very instant of projection, the cable of the elevator breaks and the elevator starts falling freely. What will be the separation between the particle and its image 0.5 second after the instant of projection ?

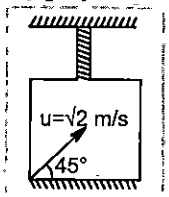


Fig. 1E.26 (a)

Solution: $\vec{v}_{E/g}=-gt\hat{j};\vec{v}_{p/g}=\hat{i}+(1-gt)\hat{j}$

$u\sin\theta=\sqrt{2}\sin 45°=1$ m/s

$u\cos\theta=\sqrt{2}\cos 45°=1$ m/s

$\vec{v}_{P/E}=\hat{i}+\hat{j};|\vec{v}_{P/E}|=\sqrt{2}$ m/s

in 0.5sec $=\sqrt{2}\times 0.5=\frac{1}{\sqrt{2}}$ m

$\sin 45°=\frac{y}{(1/\sqrt{2})};y=\frac{1}{2}=0.5$m

PP' = 1 m

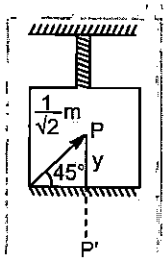


Fig. 1E.26 (b)

Example 27

Two large plane mirrors OM and ON are arranged as shown. Find the length of the part of large screen SS' in which two images of the object placed at P can be seen ?

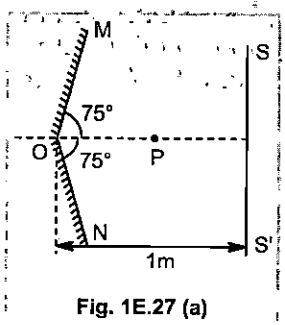


Fig. 1E.27 (a)

Solution:

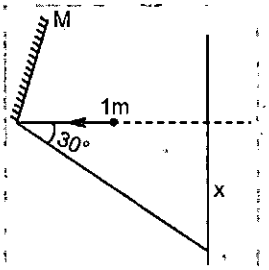


Fig. 1E.27 (b)

$$\therefore \quad x = 1\tan 30° = \frac{1}{\sqrt{3}}\text{ m}$$

Similar ray diagram for mirror N will give total part $= 2x = \frac{2}{\sqrt{3}}$

Example 28

A man 2 m tall stands 5 m in front on a large vertical mirror. Then the angle subtended at his eye by his image in the plane mirror is nearly.

(a) 0.4 rad (b) 0.2 rad (c) 0.2° (d) 0.4°

Solution: (b)

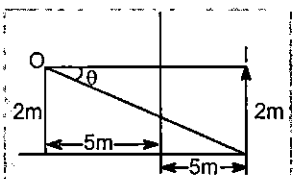


Fig. 1E.28

$$\tan\theta = \frac{2}{10}, \tan\theta = \theta$$

$$\theta = 0.2$$

Example 29

A point source of length S is iocated on a wall.A plane mirror M having length l is moving parallel to the wall with constant velocity v. The bright patch formed on the wall by reflected light will

(a) move with uniform velocity v and will have a length 2l

(b) move with uniform velocity 2v and will have a length l

(c) move with uniform velocity 2v and will have a length 2l

(d) move with uniform velocity but will have a changing length.

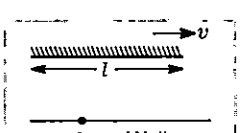


Fig. 1E.29 (a)

Solution: (c)

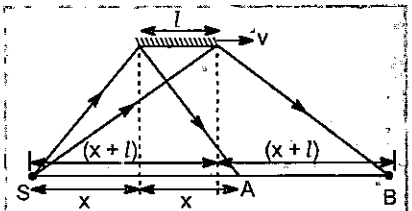


Fig. 1E.29 (b)

$$SA = 2x, SB = 2(x + l)$$

$$AB = SB - SA = 2l$$

Length of the patch will be = $2l$

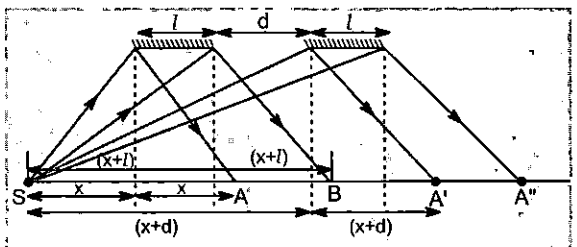


Fig. 1E.29 (c)

$$SA = 2x$$

$$SA' = 2(x + d)$$

$$SA' - SA = 2(x + d) - 2x = 2d$$

So if plane mirror is moved then patch will move $2d$ in same time so velocity of patel with be $2v$ if velocity of mirror is v.

SPHERICAL MIRRORS

Images Formed by Curved Mirrors

A **spherical mirror** is a reflecting surface whose shape is a section of a spherical surface. If the inside surface of the mirror is polished, it is a **concave mirror.** If the outside surface is polished, it is a convex mirror. The radius of the spherical surface is the **radius of curvature** R of the mirror. The centre of the sphere of which the mirror is a part is called the **centre of curvature.** A **plane mirror** can be treated as a special case of a spherical mirror: one which has an **infinite radius of curvature.**

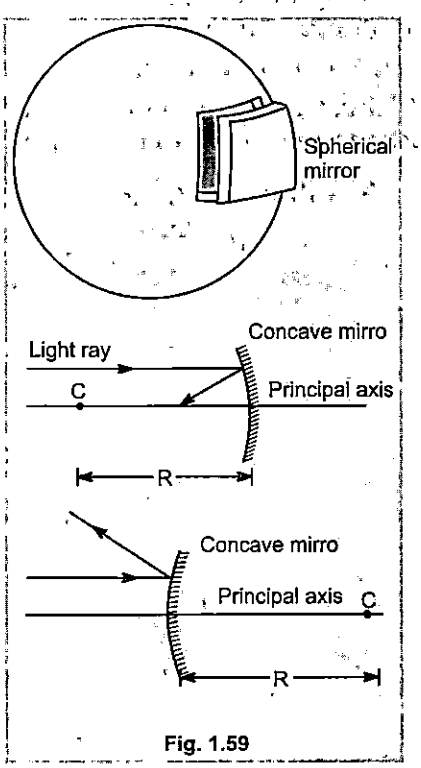


Fig. 1.59

A mirror with its reflecting surface shaped like a paraboloid revolution is called a **parabolic mirror** (mirror formed by rotating a parabola about its axis of symmetry). All rays travelling parallel to the axis of the parabola will go through the focus of the parabola. This point is known as the **focal point** of the mirror. The converging rays are said to be focussed. Spherical mirrors behave similarly to parabolic mirrors for rays that travel close to the mirror axis, **paraxial rays.** A spherical mirror is not perfect in focussing the parallel rays. The farther a ray is from the mirror axis, the greater is the error in focus. This 'defect' of spherical mirror is called **spherical aberration.**

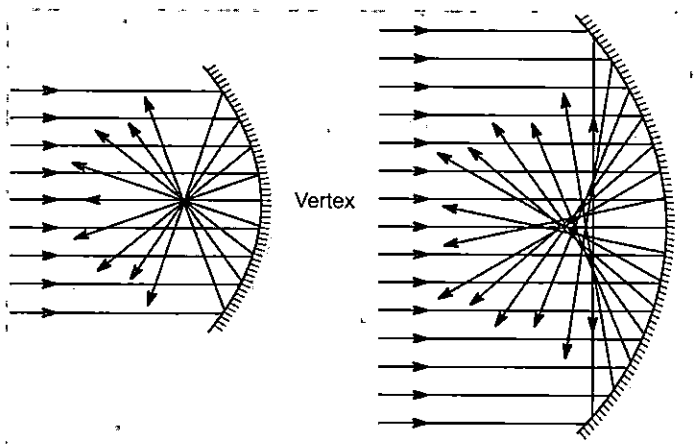


Fig. 1.60

The equation for the parabola is

$$y^2 = 4fx$$

where f is the distance from the vertex of the parabola to the focal point of the parabola. This distance is called **focal length.** The equation of the circle is

$$(x-R)^2 + y^2 = R^2$$

where R is the radius of the circle. On simplification we get

$$y^2 = 2Rx - x^2$$

For $x << R$ we can ignore x^2 compared to x^2. In this case the equation for the parabola and the equation for the circle are identical if we take $R = 2f$.

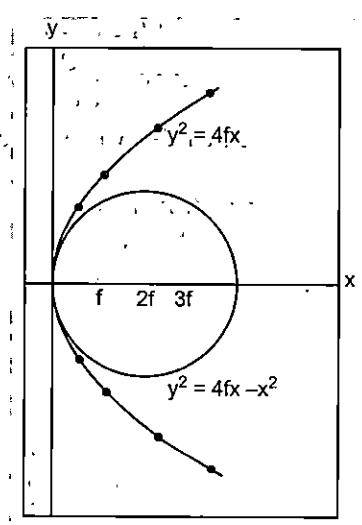


Fig. 1.61

The centre C of the spherical surface is the **optical centre** of the mirror. The point O at the middle of the mirror is called its **pole.** The normal to the mirror surface passing through the pole is called the **principal optical axis.** The normals drawn at other points of the mirror are called **auxiliary (or secondary) optical axes.** The rays passing in parallel to the principal optical axis near it are called **central rays.**

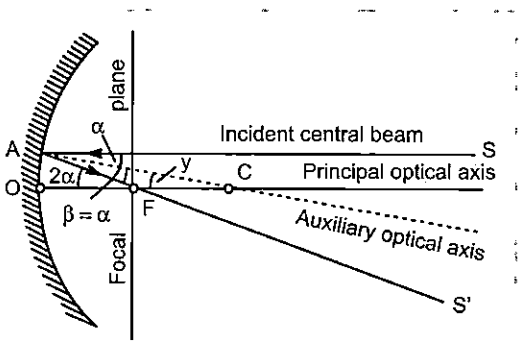


Fig. 1.62

The relation $F = R/2$ given above (where F is the focal length and R is the radius of a mirror) is not exact. It is the more exact, closer the incident central ray to the principal optical axis of the mirror. Indeed, for a concave spherical mirror in Fig. 1.62 in ΔAFC, the angles $\beta = \alpha$ and $\gamma = \alpha$, *i.e.*, $\beta = \gamma$. Consequently, ΔAFC is isosceles and $AF = FC$.

The focal length $OF \approx AF \cos\angle AFO = FC \cos 2\alpha$, *i.e.*, $F \approx (R - F)\cos 2\alpha$, whence

$$F = R\frac{\cos 2\alpha}{1+\cos 2\alpha} = R\frac{\cos^2\alpha - \sin^2\alpha}{2\cos^2\alpha},$$

or

$$F = R\frac{1-\tan^2\alpha}{2}$$

Thus, the spherical mirror does not exactly converge central rays at a single point (focal point). For a concave mirror, the further is a ray incident on a spherical mirror from the optical axis, the larger the displacement of the reflected ray from the source, for a convex mirror, the opposite is true.

The impossibility to converge the reflected rays at one point in this case is known as the **spherical aberration** of the mirror, which makes the image blurred. Spherical aberration can be reduced by using parabolic mirrors in which the radius of curvature increases for points of the surface at larger distances from the pole. But since the manufacture of parabolic mirrors involves considerable difficulties, spherical aberration is normally reduced by combining converging and diverging mirrors into a single optical system.

The point F at which ray SA parallel to the principal optical axis intersects it after having been reflected from the mirror is called the **focal point** (or **principal focus)** of the mirror. The distance OF from the focal point to the mirror pole is known as the **focal length** of the mirror and is denoted by F. The plane passing through the focal point normally to the principal optical axis is called the **focal plane.**

The focal point of a concave spherical mirror lies at the middle of the radius of the mirror, *i.e.*, its focal length is

$$F = R/2$$

The focal point of a convex mirror is virtual and lies on the principal optical axis behind the mirror at a distance $F = R/2$ from its pole.

RELATION BETWEEN FOCAL LENGTH AND RADIUS OF CURVATURE

Consider a ray of light parallel to the principal axis, incident on a spherical mirror at A. If the mirror is concave, it meets at F after reflection. If the mirror is convex it appears to come from F after reflection.

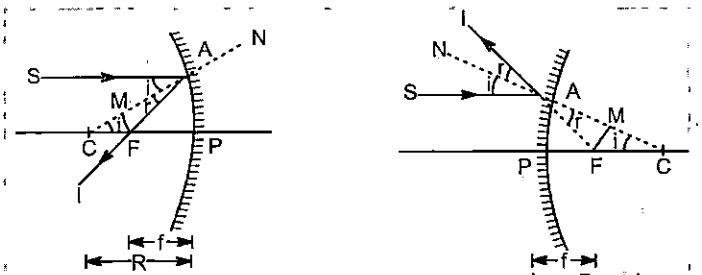


Fig. 1.63

From ΔFMC, $\dfrac{FC}{CM} = \sec i$

$$FC = \frac{R}{2}\sec i \qquad \left[CM = MA = \frac{R}{2}\right]$$

Now, $PF = PC - FC$

$$f = R - \frac{R}{2}\sec i \qquad ...(1)$$

For paraxial rays, $i \to 0$

$$f \approx \frac{R}{2} \qquad ...(2)$$

(i) The dependence of focal length on angle of incidence is a defect in spherical mirrors which is known as **spherical aberration**. This defect can't be completely eliminated from spherical mirror but it can be minimized by using mirror of small size or blocking marginal or paraxial rays using stops in case of large size mirrors.

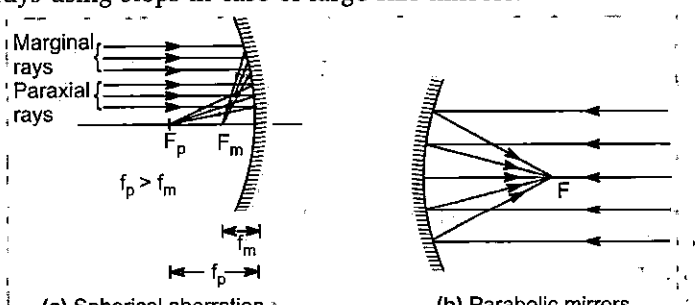


Fig. 1.64

(ii) Parabolic mirrors do not suffer from this defect because the normal at any point on a parabola bisects the angle between the line joining the point to the focus of the parabola and the line through the point parallel to its axis.

Paraxial Approximation:

Paraxial approximation assumes that all rays strike the mirror very close to the principal axis and have very small angle of incidence. Thus we can approximate

$$\tan\alpha \approx \alpha \qquad \tan\beta \approx \beta$$

We can further approximate.

$$QC \approx PC,\ QF \approx PF$$

As $\beta = 2\alpha$ or $\dfrac{AQ}{PC} = 2 \times \dfrac{AQ}{PF}$

or $$PF = \frac{1}{2}PR \quad i.e., \quad f = \frac{R}{2} \qquad ...(1)$$

where f is the focal length of the mirror

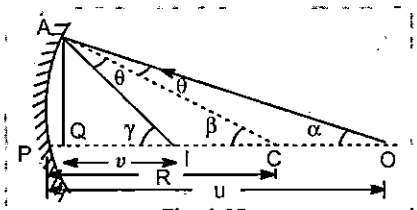


Fig. 1.65

The external angle equals the sum of the opposite two internal angles. Therefore.

$$\beta = \alpha + \theta \qquad ...(2)$$

and $$\gamma = \beta + \theta \qquad ...(3)$$

Eliminating the angle θ from equations (2) and (3), we get

$$2\beta = \alpha + \gamma \qquad ...(4)$$

Also $\tan\alpha = \dfrac{AQ}{QO'}$, $\tan\beta = \dfrac{AQ}{QC'}$, $\tan\gamma = \dfrac{AQ}{QI'}$

As stated earlier, we assume all angles $\alpha, \beta, \gamma, \theta$ are very small.

We can approximate

$$\tan\alpha \approx \alpha,\ \tan\beta \approx \beta \text{ and } \tan\gamma \approx \gamma$$

$$QO \approx PO, QC \approx PC \text{ and } QI \approx PI$$

Substituting these approximations in equation (1) we get,

$$\frac{2}{PC} = \frac{1}{PO} + \frac{1}{PI} \qquad ...(5)$$

or $$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

From equation (5), the position of the image can be calculated knowing the position of the object and the focal length of the mirror.

Mirror Formula

In terms of Cartesian sign convention mirror formula may be expressed as:

(i) $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

where u is the object distance from the pole
v is the image distance from the pole, and
f is the focal length of the mirror.

(ii) Lateral magnification ratio

$$m = \frac{h_i}{h_0} = \frac{-v}{u}$$

where h_i is the height of image, and
h_0 is the height of object.

The Cartesian Sign Convention

A sign convention facilitates the computations of the object and image distances, assesses the nature of image (real or virtual), its magnification and orientation.

The object is placed to the left of the optical surface (a mirror or refracting surface). The light is incident from left to right. The centre of the optical surface is called **vertex.** The vertex is taken as origin. The horizontal axis is called the **optic axis.**

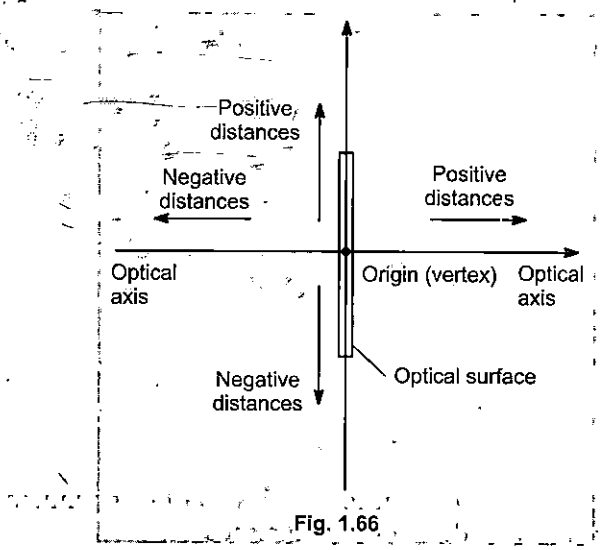


Fig. 1.66

(1) Distances measured to the right of the origin along optic axis are **positive** distances, since they are along the positive axis of the standard cartesian coordinate system.

(2) Distances measured to the left of the origin along the optic axis are **negative** distances, since they are along the negative axis of the standard cartesian coordinate system.

(3) **The sign convention for magnification:** Magnification is defined as the ratio of the size of image to the size of object.

$$|m| = \frac{\text{Image size}}{\text{Object size}}$$

This is referred to as **lateral magnification.** If the magnification m is positive the image of the object is **erect** (upright), meaning that the image has same orientation as the object. If the magnification is negative, the image is **inverted** (upside down).

Rules for Ray Diagrams

We can locate the image of any extended object graphically by drawing any two of the following four principle rays.

1. A ray, initially parallel to the principal axis is reflected through the focus of the mirror.

2. A ray, initially passing through the focus is reflected parallel to the principal axis.

3. A ray passing through the so line joining point object and its image cuts principal axis at centre of curvature.

4. A ray incident at the pole is reflected symmetrically. So line joining O and I' or O' and I will cut principal axis at pole.

(i) Spherical mirrors bring paraxial rays to an approximate focus at a point on the mirror axis.

(ii) The focal length of a spherical mirror is equal to half the radius of curvature of the mirror.

The mirror equation for spherical mirror is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$$

When the object point O is located infinitely far away from the mirror, then $u = -\infty$ and the position of the image is called the **focal length.** If we substitute $u = -\infty$ into the mirror equation,

$$\frac{1}{-\infty} + \frac{1}{v} = \frac{2}{R}$$

Since $v = f$, $\quad 0 + \frac{1}{f} = \frac{2}{f}, f = \frac{R}{2}$

Parallel incident rays intersect in case of concave mirror and appear to intersect in case of convex mirror.

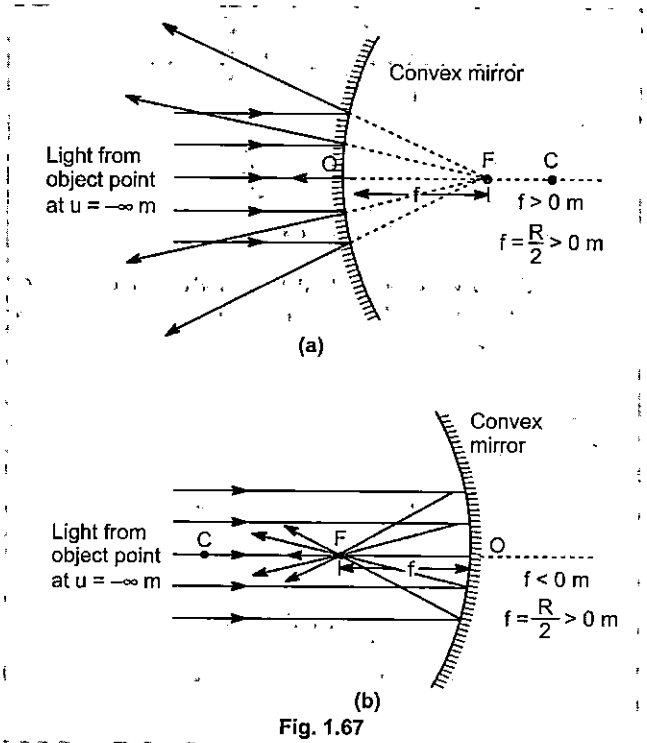


Fig. 1.67

The magnification is

$$m = \frac{\text{image size}}{\text{object size}} = -\frac{v}{u}$$

Note that the law of reflection is independent of the medium in which light is travelling. The mirror and magnification equation can be applied irrespective of the medium surrounding the mirror.

(1) A ray parallel to axis is reflected back through focal point as shown in Fig. 1.67 (a).

(2) A ray that passes through the focal point on the ray to the mirror is reflected back parallel to the mirror axis as shown in Fig. 1.67(b).

(3) A ray from the object is directed toward the centre of curvature of the mirror, after reflection the ray retraces its path because it strikes the mirror along the normal to the mirror.

(4) A ray that strikes the vertex of the mirror reflects at an equal angle on the other side of the mirror axis.

Any two of these rays are sufficient to locate the position of the image.

We can assess two things from ray diagrams :

(a) If the image is infront of the mirror, it is real; if the image is at the back of the mirror, the image is virtual.

(b) We are able to guess if the magnification is positive or negative and greater or less than unity.

RAY DIAGRAM FOR CONCAVE MIRROR

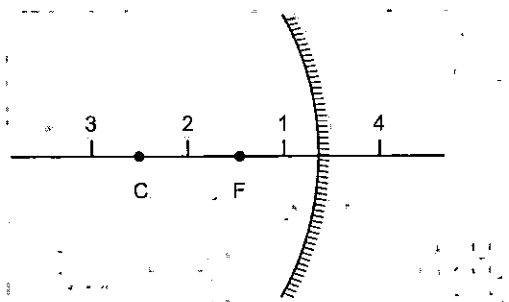


Fig.1.68

Fig. 1.68 shows a concave mirror with objects placed in form different regions. We now analyze the nature of image from ray diagram.

Region 1:

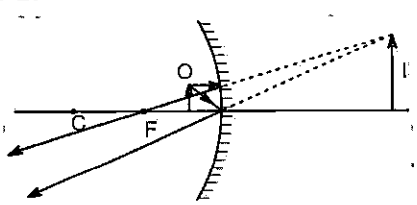


Fig.1.69

Object placed between pole and focus. Image is virtual, erect, magnified.

Note: If object is moved towards pole magnification goes on decreasing at pole magnification m tends to unity.

Region 2:

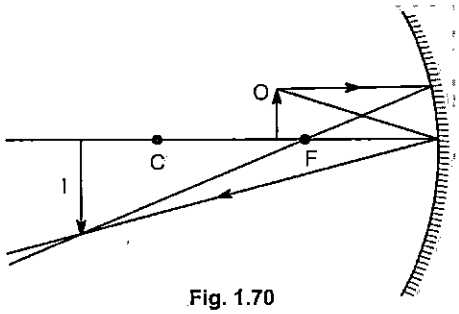


Fig. 1.70

Object placed between focus f and centre of curvature. Image is real, inverted, magnified.

Note: If object is moved from focus towards centre of curvature magnification goes on decreasing at centre of curvature it becomes unity.

Region 3:

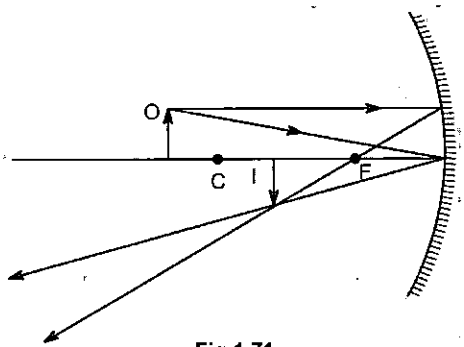


Fig.1.71

Object placed between centre of curvature and infinity. Image real, inverted, smaller (diminished)

Note: If object is moved away from centre of curvature towards infinity magnification goes on decreasing.

Region 4:

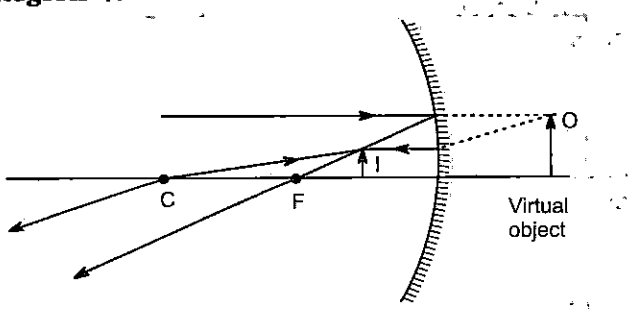


Fig.1.72

Object is virtual point object. Image real, erect, smaller (diminished)

Note: As object moves away from pole magnification goes on decreasing.

Concept: *A concave mirror always forms real image of a virtual object.*

Convex Mirror

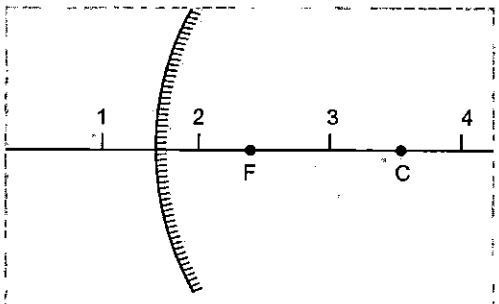


Fig.1.73

Figure shows convex mirror with four different object positions as shown in Fig. 1.73. Now we analyze the corresponding image formed.

Region 1:

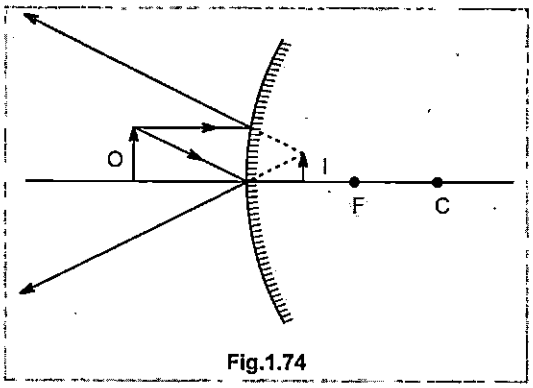


Fig.1.74

Object placed infront of mirror. For all the positions of object infront of mirror. Image is virtual, erect, smaller in size.

Note: As object moved towards pole magnification increases and tends to unity at pole.

Region 2:

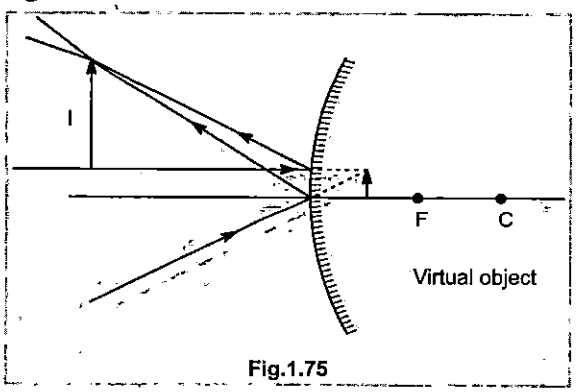


Fig.1.75

Virtual object placed between pole and focus. Image formed is real, erect, enlarged.

Note: As object is moved away from pole magnification increases.

Region 3:

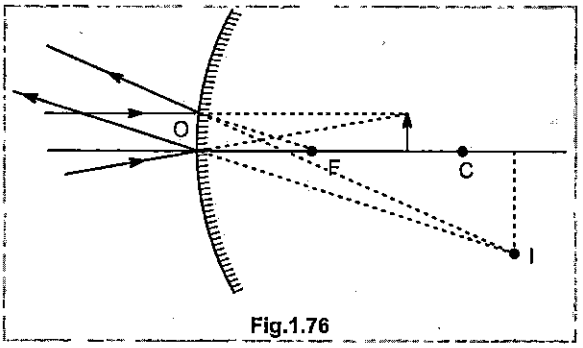


Fig.1.76

Virtual object placed between *F* and *C*. Image formed is virtual, inverted, enlarged.

Note: As object is moved towards *C* magnification decreases.

Region 4:

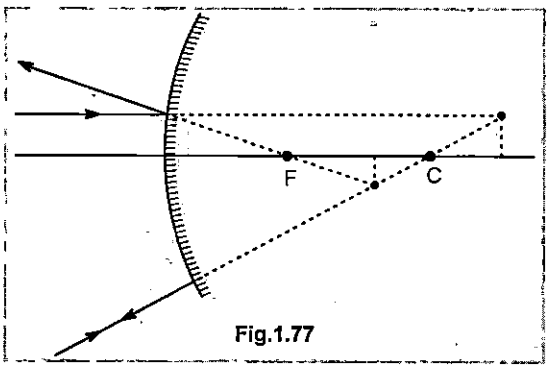


Fig.1.77

Virtual object is placed beyond *C*. Image is virtual, inverted, smaller.

Note: As object is moved away from *C* image size further decreases magnification decreases.

Concept: *A convex mirror can form virtual image of a virtual object. Also this virtual image is inverted.*

u vs v graph Concave Mirror

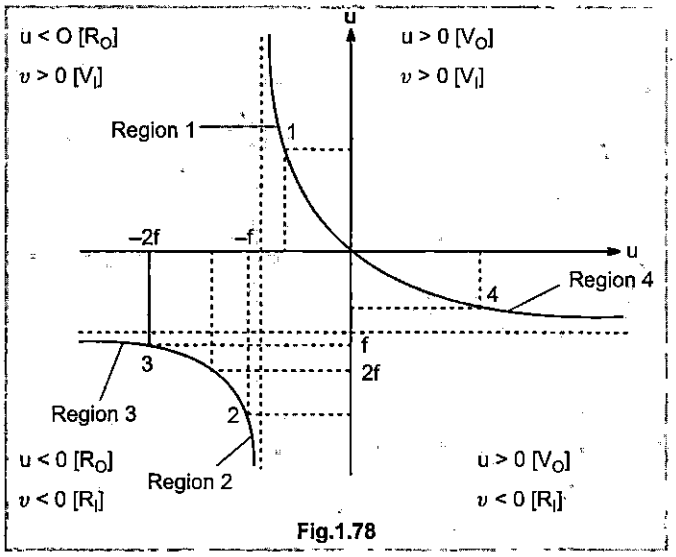


Fig.1.78

For interpreting graph we use sign convention.

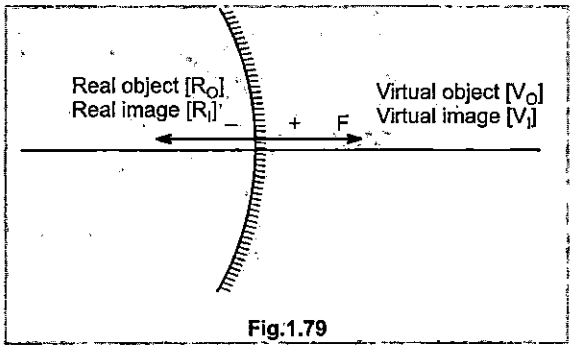


Fig.1.79

$m > 0$	means	erect image
$m < 0$	means	inverted image
$\|m\| > 1$	means	enlarged image
$\|m\| < 1$	means	smaller image

For point (1) in region (1)

$m > 0$ because u and v have opposite sign *i.e.*, image is inverted.

$|m| > 1$ because $|v| > |u|$ image is virtual because $v > 0$

For point (2) in region (2)

$m < 0$ because u and v have same sign, *i.e.*, image is inverted.

$|m| > 1$ because $|v| > |u|$ image is real because $v < 0$

For point (3) in region (3)

$m < 0$ because u and v have same sign *i.e.*, image is inverted $|m| < 1$ because $|v| < |u|$ image is real because $v < 0$.

For point (4) in region (4)

$m > 0$ because u and v have opposite sign *i.e.*, image is erect.

$|m| < 1$ because $|v| < |u|$. Image is real because $v < 0$.

Concept: *For all the points* $\frac{dv}{du}$ *represents longitudinal magnification for small linear object. Here* $\frac{dv}{du}$ *represents slope of curve at any point.*

v vs u graph Convex Mirror

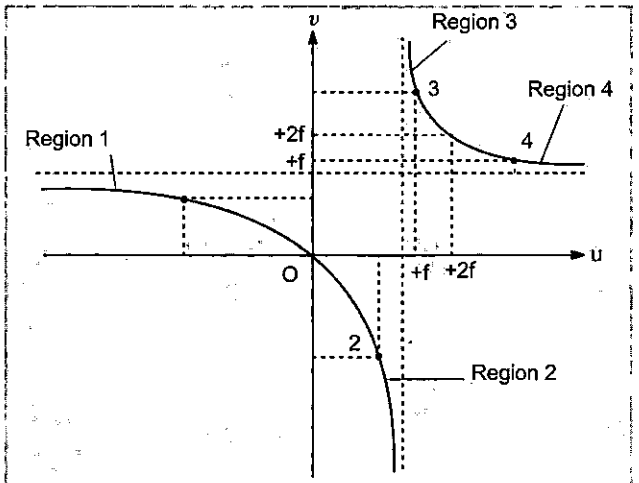


Fig.1.80

For interpreting graph we use sign convention.

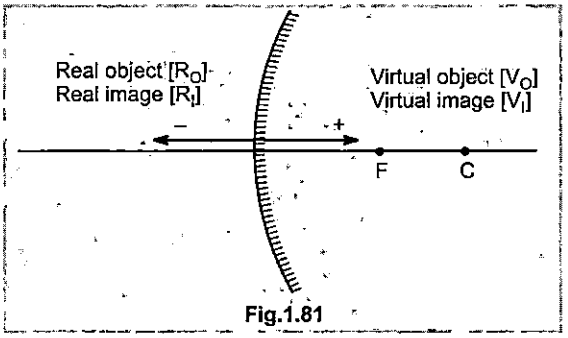


Fig.1.81

For point (1) in region (1)

$m > 0$ because v and u have opposite sign *i.e.*, image is erect.

$|m| < 1$ because $|v| < |u|$ image is virtual because $v > 0$

For point (2) in region (2)

$m > 0$ be cause u and v have opposite sign *i.e.*, image is erect $|m| > 1$ because $|v| > |u|$ image is real because $v < 0$.

For point (3) in region (3)

$m < 0$ because v and u have same sign *i.e.*, image is inverted $|m| > 1$ because $|v| > |u|$. Image is virtual because $v > 0$.

For point (4) in region (4)

$m < 0$ because v and u have same sign *i.e.*, image is inverted. $|m| < 1$ because $|v| < |u|$. Image is virtual because $v > 0$.

RAY DIAGRAMS FOR MIRRORS

Concave Mirror

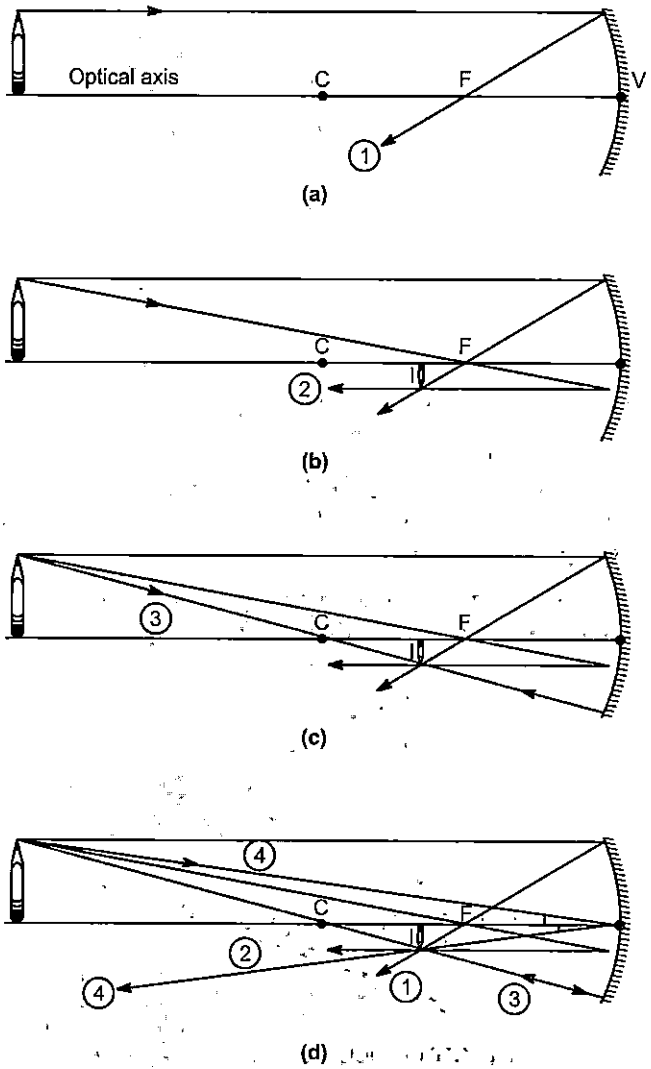


Fig. 1.82 (a)

Four Construction rays used to locate the image by concave mirror

Convex Mirror

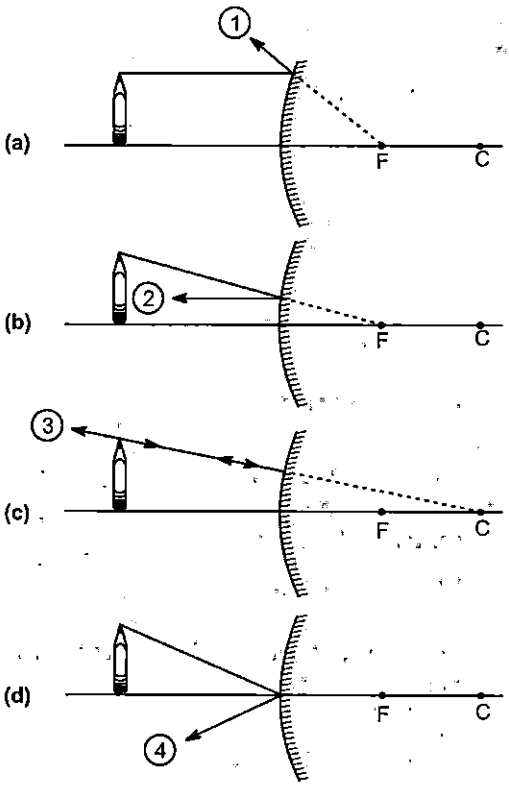


Fig. 1.82 (b)

Five construction rays used to locate the image by a convex mirror

CHARACTERISTICS OF IMAGE

Concave Mirror

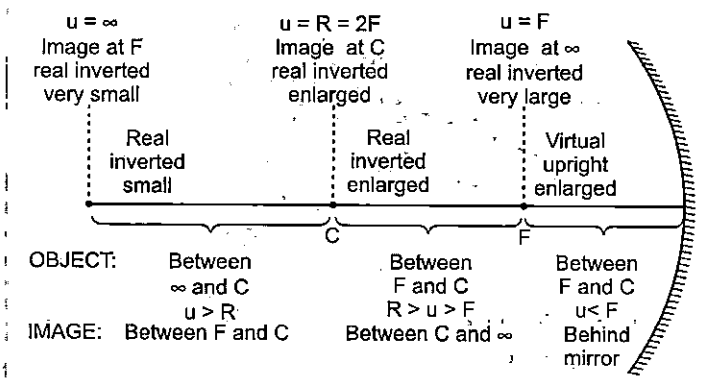


Fig. 1.82 (c)

Covex Mirror

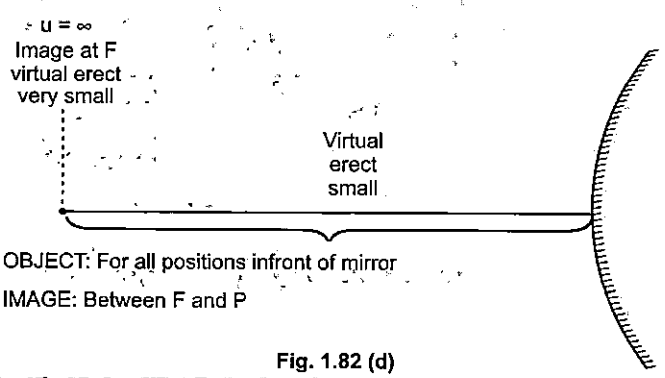


Fig. 1.82 (d)

Graphical method for determining the focal length of a concave mirror : It forms real and inverted image of an object placed beyond its focus. From mirror equation,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Using cartesian sign convention, we have

$$u=-x;\ v=-y \text{ and } f=-f$$

or $$\frac{1}{-y}+\frac{1}{-x}=\frac{1}{-f}$$

A graph between $\frac{1}{-v}$ and $\frac{1}{-u}$ is a **straight line,** as shown in Fig. 1.83 (a).

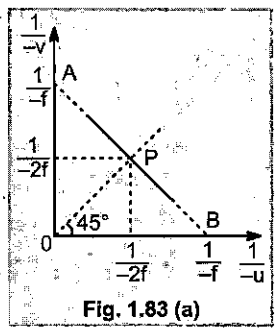


Fig. 1.83 (a)

Note that the slope of the straight line is –1 and the intercepts on the *horizontal* and *vertical* axes are equal. It is equal to $\frac{1}{-f}$.

A *straight line OP* at an angle 45° with the horizontal axis is drawn which intersects the line *AB* at *P*. The *coordinates* of the point *P* are $\left(\frac{1}{-2f},\frac{1}{-2f}\right)$

The *focal length* of the mirror can be calculated by measuring the coordinates of either of the points *A*, *B* or *P*.

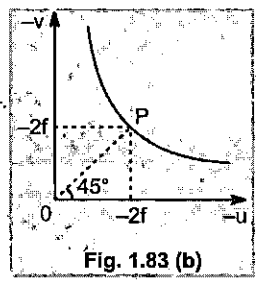


Fig. 1.83 (b)

Alternatively, a graph between $-v$ and $-u$ can also be plotted, which is a curve as shown in Fig. 1.83 (b). A line drawn at angle 45° from the origin intersects it at the point *P* whose coordinates are $(-2f,-2f)$. By measuring the coordinates of this point, the focal length of the mirror can also be measured.

Longitudinal Magnification for Very Short Object (m_l)

$$m_l=\frac{dv}{du}$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Differentiating both sides with respect to '*u*'.

$$-\frac{1}{v^2}\frac{dv}{du}-\frac{1}{u^2}=0$$

$$m_l=\frac{dv}{du}=\frac{-v^2}{u^2}$$ *(applicable only when object is very small.)*

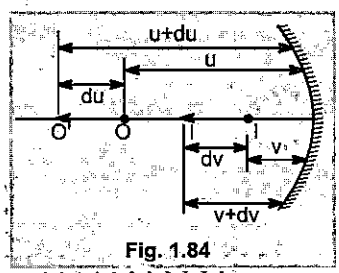


Fig. 1.84

Since m_l is always negative image is always longitudinally inverted,

$$m_l=-\left(\frac{-v}{u}\right)^2 \qquad \textit{(For small object)}$$

$$m_l=-(m_t)^2$$

For Large Object

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Put u_1, u_2 and solve for v_1, v_2,

$$m_l=\frac{||v_1|-|v_2||}{||u_2|-|u_1||}$$

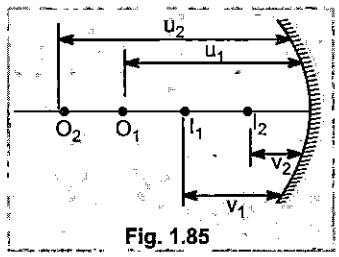


Fig. 1.85

Numerically $|v_1| \geq |v_2|$ or $|v_1| \leq |v_2|$ decides whether image is inverted or irrect.

Splitting of Mirror

A mirror is split into two parts that are displaced by small distance $2a$ as shown in figure.

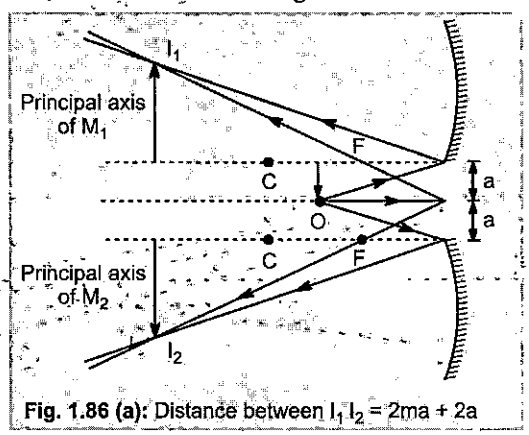


Fig. 1.86 (a): Distance between $I_1 I_2 = 2ma + 2a$

A point object is placed between *F* and *2F*. Each part forms image that is of size *ma* from its principal axis.

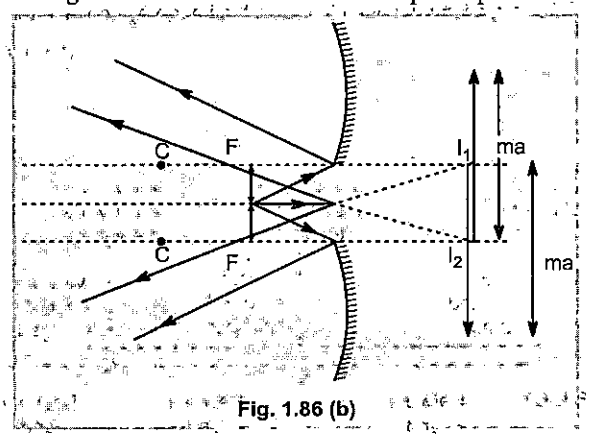


Fig. 1.86 (b)

Object is placed between focus and distance between $I_1 I_2 = 2ma - 2a$.

(i) Mirror is cut along its principal axis and two parts are separated perpendicular to the principal axis.

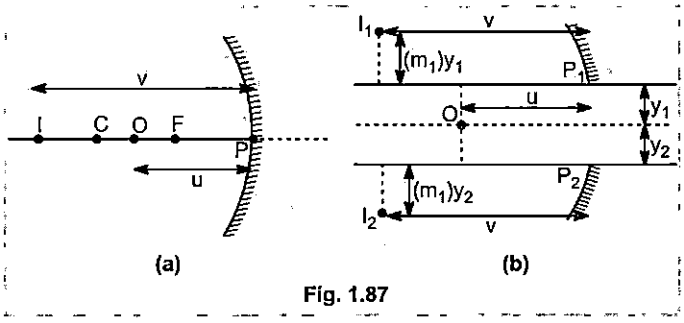


Fig. 1.87

$$m_t = -\frac{v}{u}$$

$$s = m_t y_1 + y_1 + m_t y_2 + y_2;\quad s = (m_t + 1)(y_1 + y_2)$$

Where 's' is the separation distance between two images I_1 and I_2 of the same object O formed due to two halves of the mirror.

(ii) Mirror is cut along its principal axis and two parts are separated parallel to the principal axis.

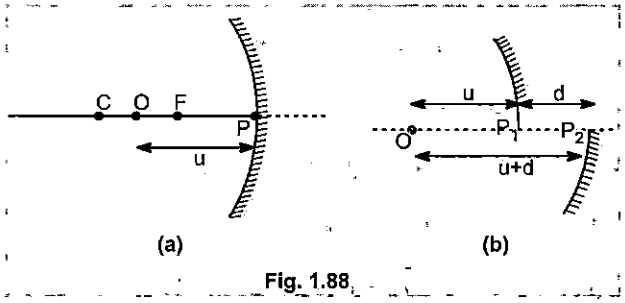


Fig. 1.88

Upper Half **Lower Half**

$$\frac{1}{v_1} - \frac{1}{u} = -\frac{1}{f} \qquad \frac{1}{v_2} - \frac{1}{(u+d)} = -\frac{1}{f}$$

$v_1 = ?$ $v_2 = ?$

Mirror is turned through small angle θ

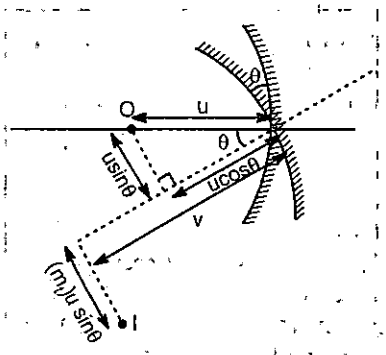


Fig. 1.89

$$\frac{1}{v} - \frac{1}{u\cos\theta} = -\frac{1}{f};\quad v = ?$$

$$m_t = -\frac{v}{u\cos\theta}$$

Newton's Formula

If the object and image distances are measured from the focus instead of the pole of the mirror. Then, the mirror formula reduces to a simple form called the ***Newton's formula.***

$$x_o x_i = f^2$$

where x_o is the object distance from the focus.

x_i is the image distance from the focus.

In case of spherical mirrors if object distance (x_1) and image distance (x_2) are measured from focus instead of pole, then $u = f + x$ and $v = f + x_2$ the mirror formula reduces to

$$\frac{1}{f + x_2} + \frac{1}{f + x_1}$$

or $$x_1 x_2 = f^2$$

which is known as Newton's formula.

While using mirror equation known quantities are to substituted with proper sign and quantities to be calculated (unknown quantities) are not be given any sign.

VELOCITY OF IMAGE IN SPHERICAL MIRROR

Consider pole of mirror to be origin of co-ordinate system and X-axis as the principal axis of mirror and Y-axis is perpendicular to principal axis. Incident rays point along $x > 0$.

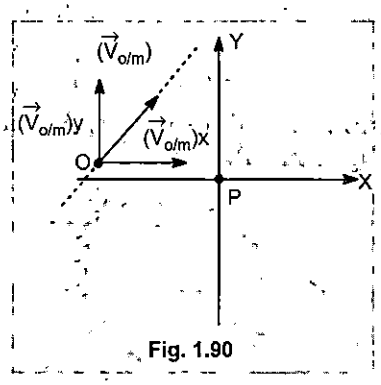


Fig. 1.90

From mirror equation, we have

$$\frac{1}{X_{I/m}} + \frac{1}{X_{O/m}} = \frac{1}{f}$$

Differentiating both side w.r.t., 't' we get

$$-\frac{1}{X_{I/m}^2}\frac{d}{dt}(X_{I/m}) - \frac{1}{X_{O/m}^2}\frac{d}{dt}(X_{O/m}) = 0$$

or $$\frac{d}{dt}(X_{I/m}) = -\left(\frac{X_{I/m}}{X_{O/m}}\right)^2 \frac{d}{dt}(X_{O/m})$$

$$(V_{I/m})_X = -m^2 (V_{O/m})_X$$

$$m = \frac{f}{f-u} = \frac{\text{Height of image}}{\text{Height of object}}$$

$$Y_{I/m} = \left(\frac{f}{f-u}\right)\cdot(Y_{O/m})$$

Differentiating w.r.t. 't' we get

$$\frac{d}{dt}(Y_{I/m}) = \frac{d}{dt}\left[\left(\frac{f}{f-u}\right)\cdot(Y_{O/m})\right]$$

$$(V_{I/m})_Y = \left(\frac{f}{f-u}\right)\cdot\frac{d}{dt}(Y_{O/m}) + (Y_{O/m})\cdot\frac{d}{dt}\left(\frac{f}{f-u}\right)$$

(Using product rule)

$$\text{or} \quad (V_{I/m}) = \left(\frac{f}{f-u}\right)(V_{O/m})_Y + (Y_{O/m})\frac{f}{(f-u)^2}\frac{du}{dt}$$

Case (I): If object is on principal axis, then $Y_{O/m} = 0$

$$(V_{I/m})_Y = \left(\frac{f}{f-u}\right)(V_{O/m})_Y$$

Case (II): If object is not on principal axis but moving parallel to principal axis then $(V_{O/m})_Y = 0$

$$V_{I/m} = (Y_{O/m})\frac{f}{(f-u^2)}\frac{du}{dt}$$

Note that $\frac{du}{dt}$ is negative if u is decreasing with time and it is taken positive if u is increasing with time.

Case (III): If object is on principal axis and moving along it then $Y_{O/m} = 0$ and $(V_{O/m})Y = 0\,(V_{I/m}) = 0$

Illustration 7: Find the velocity of image in situation as shown in Fig. 1.91 (a).

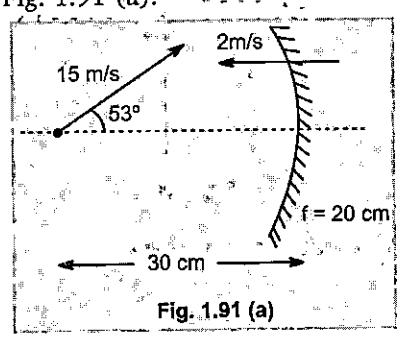


Fig. 1.91 (a)

Express velocity and mirror of object in unit vector notation $\vec{v}_o$ = Velocity of object = $(9\hat{i} + 2\hat{j})$ m/s

$\vec{v}_m$ = Velocity of mirror = $-2\hat{i}$ m/s

$$m = \frac{f}{f-u} = \frac{-20}{-20-(-30)} = -2$$

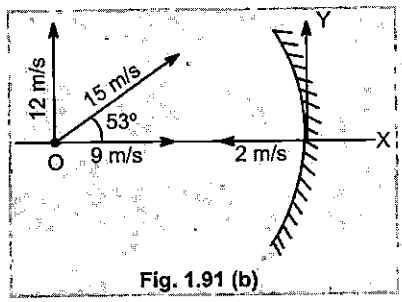


Fig. 1.91 (b)

As object lies on principal axis case (i) applies. For velocity component parallel to optical axis

$$(\vec{v}_{I/m})_{\parallel} = -m^2(\vec{v}_{O/m})_{\parallel}$$

$$(\vec{v}_{I/m})_{\parallel} = -(-2)^2 11\hat{i} = -44\hat{i}\ \text{m/s}$$

For velocity component perpendicular to optical axis

$$(\vec{v}_{I/m})_{\perp} = \left(\frac{f}{f-u}\right)(\vec{v}_{O/m})_g$$

$$= (-2)12\hat{j} = -24\hat{j}\ \text{m/s}$$

$\vec{v}_{I/m}$ = Velocity of image w.r.t. mirror

$$= (\vec{v}_{I/m})_{\parallel} + (\vec{v}_{I/m})_{\perp}$$

$$= (-44\hat{i} - 24\hat{j})\ \text{m/s}$$

Also, $\vec{v}_{I/m} = \vec{v}_I - \vec{v}_m$

or $\vec{v}_I = (-44\hat{i} - 24\hat{j}) - 2\hat{i}$

$$= (-46\hat{i} - 24\hat{j})\ \text{m/s}$$

Measurement of refractive index of a liquid by a concave mirror : A concave mirror of large radius or curvature is placed on a table with its principal axis *vertical*, as shown in Fig.1.92. A horizontal pin is placed with its tip on the principal axis of the mirror. The pin is moved till there is no parallax between the tip of the pin and its image, when the pin lies at the centre of curvature of the mirror.

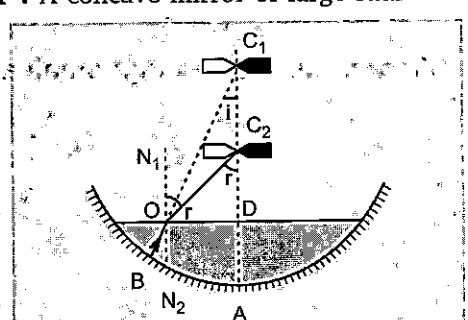


Fig. 1.92: Concave mirror with a drop of liquid

A small quantity of liquid whose refractive index is to be measured is poured into the mirror. The pin is moved down in order to remove the parallax between the tip of the pin and its image.

In Fig. 1.92, C_1 is the position of the pin when its image coincides with itself, without liquid and C_2 is the position of the pin when its image coincides with itself after pouring the liquid into the concave mirror.

The ray *BO* that is normal to the mirror passes through C_1 before pouring the liquid. It is refracted away from the normal when the liquid is poured, now it passes through C_2.

In Fig. 1.92 shown,

$$\sin i = \frac{OD}{OC_1}, \sin r = \frac{OD}{OC_2}$$

From Snell's law, $\mu = \frac{OD}{OC_2} \times \frac{OC_1}{OD} = \frac{OC_1}{OC_2}$

Taking paraxial ray assumption,

$$OC_1 \simeq DC_1;\ OC_2 \simeq DC_2$$

Thus $$\mu = \frac{DC_1}{DC_2}$$

If we neglect the depth of the liquid, we have

$$\mu = \frac{AC_1}{AC_2}$$

Example 30

A pencil is placed 20.0 cm infront of a concave spherical mirror of focal length $f = 15.0$ cm. Find the location of the image. State whether the image is real or virtual, erect or inverted and give its lateral magnification.

Solution: The ray diagram shows that the image is real and that, relative to the object, it is farther away from the mirror, inverted and larger.

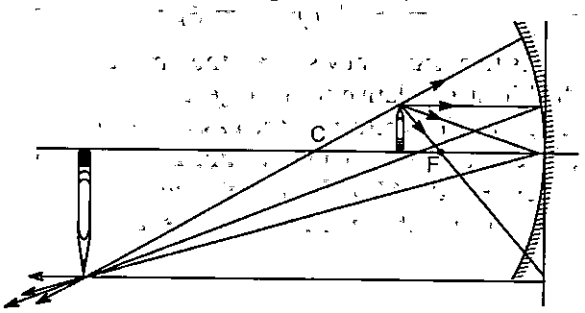


Fig. 1E.30

The given parameters are

$$f = -15\,\text{cm}$$

$$u = -20\,\text{cm}$$

From mirror equation,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \left(\frac{1}{-15}\right) - \frac{1}{(-20)} = -\frac{1}{60}$$

$$\therefore \quad v = -60\,\text{cm}$$

Because the image distance is negative, the image is real and is formed infront of the mirror, the reflected rays actually pass through the image.

The lateral magnification is

$$m = -\frac{v}{u} = -\frac{(-60)}{(-20)} = -3$$

The lateral magnification is negative indicating that the image is inverted with respect to the object.

Example 31

An object is located 6 cm infront of a mirror. The virtual image is located 4 cm away from the mirror and diminished. Find the focal length of the mirror.

Solution:

- The first question arises: is the mirror concave or convex or either is possible? Since both the mirrors form virtual images. The answer is, either type is possible. A concave mirror can form a virtual image if the object is within the focal point of the mirror. A convex mirror always forms a virtual image.
- The second question arises that the image is diminished in size as well as virtual, do these characteristics together indicate a concave or convex mirror? a concave mirror produces a diminished image only when the object is located beyond the centre of curvature of the mirror. However, the image in this case is real not virtual. *A convex mirror always produces an image that is virtual and smaller than the object.*
- The focal length of a convex mirror is positive. The given parameters are

$$u = -6\,\text{cm}$$

$$v = +4\,\text{cm}$$

From mirror equation,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{(+4)} + \frac{1}{(-6)} = \frac{1}{12}$$

$$f = +12\,\text{cm}$$

As expected, focal length is positive.

Example 32

When the position of an object reflected in a concave mirror of 0.25 m focal length is varied, the position of the image varies. Plot the image distance as a function of the object distance, taking the object distance from 0 to $+\infty$.

Solution: Fig. 1E.32 shows the required graph

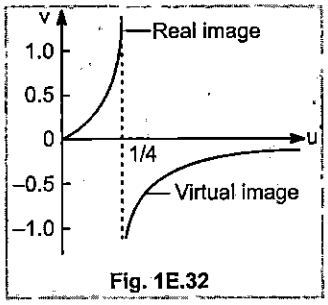


Fig. 1E.32

Example 33

If the pencil in Example 30 is at 10.0 cm infront of the concave mirror of focal length 15 cm, characterize the new image.

Solution: The ray diagram shows that the image is virtual, upright and larger relative to the object.

The given parameters are

$$u = -10\,\text{cm}$$
$$f = -15\,\text{cm}$$

From mirror equation,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{(-15)} - \frac{1}{(-10)} = \frac{1}{+30}$$
$$v = +30\,\text{cm}$$

The positive sign shows that image is behind the mirror, virtual; the rays appear to emanate from the image.

the lateral magnification is

$$m = -\frac{v}{u} = -\frac{(+30)}{(-10)} = +3$$

The magnification is positive indicating that the image is upright; $m > 1$ implies that the image is magnified.

Example 34

Because of its shape a convex mirror gives a wider field of view than do other types of mirrors. Such a mirror is mounted on an automobile. The radius of curvature of the mirror is 4.00 m. If a bus is located 6.00 m from the mirror, where is the image located ?

Solution: The focal length of the mirror is

$$f = R/2 = 2\,\text{m}$$

The given parameters are

$$f = +2.00\,\text{m}$$
$$u = -6.00\,\text{m}$$

From mirror equation, we have

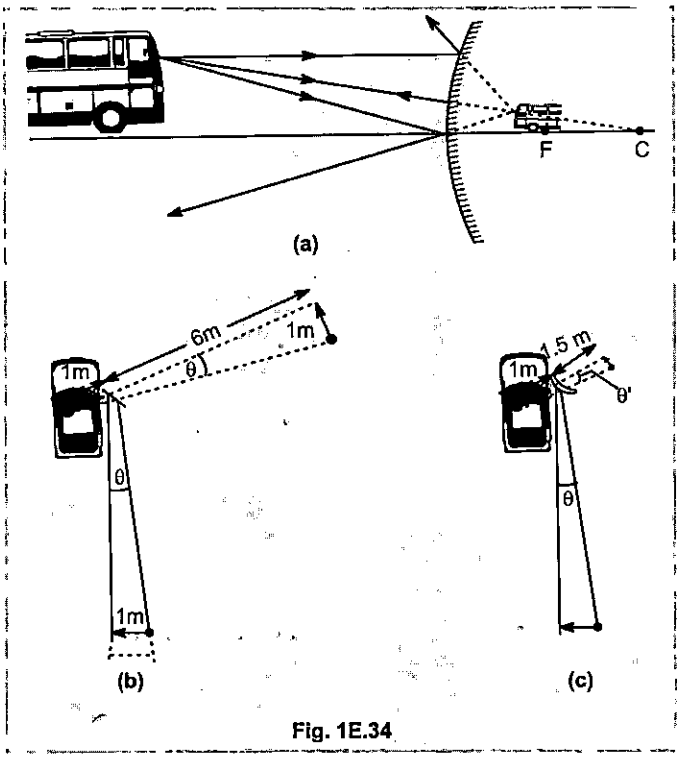


Fig. 1E.34

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{(+2)} - \frac{1}{(-6)} = \frac{2}{3}$$
$$v = +\frac{3}{2}\,\text{m}$$

The positive sign for v indicates that the image is behind the mirror and therefore is a virtual image.

The lateral magnification is

$$m = -\left(\frac{v}{u}\right) = -\frac{(+3/2)}{(-6)} = +0.25$$

The positive sign shows that the image is upright, diminished in size, relative to the object.

❒ The rear view mirror on the passenger side is often a convex mirror, having warning: "VEHICLES IN MIRROR ARE CLOSER THAN THEY APPEAR." The reason is that the virtual image is reduced in size and therefore looks smaller, just as the distant object would appear in a plane mirror.

We judge the apparent distance of objects by the angle they subtend at our eye. If the viewer's eye is at 1 m from a *plane side mirror* and looking at a 1 m wide portion of a bus behind him. Fig. 1E.34(b) shows that the image is 7 m from his eye and subtends an angle of $\theta = 1\text{m}/7\text{m} = 1/7$ rad. In the *convex mirror* the image is only 1.5 m behind the mirror, which is at 1 m from the viewer's eye. The size of the image is 0.25 m, the angle subtended is $0.25/2.5 = 1/10$ radian. The smaller angular size creates the illusion that the bus is farther behind than it actually is.

Example 35

A diamond ring is placed infront of a mirror of radius of curvature. The image is twice the size of the ring. Final the object distance of the ring.

Solution:

- The first question arises: is the mirror concave or convex or either is possible ? *The image formed by a convex mirror is always smaller than the object,* therefore the mirror must be concave.
- The next question arises : How many positions are there infront of a concave mirror, where the ring can be placed and produce an image that is twice the size of the object ? There are two places : (1) When the object is placed between centre of curvature and the focal point, the magnified image is real and inverted. (2) When the object is between the focal point and the mirror, the magnified image is virtual and upright.

In first case the image is inverted, so the magnification is $m = -2$. In the second case the image is upright, so the magnification is $m = +2$

From mirror equation and magnification equation,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \text{and} \quad m=-\frac{v}{u}$$

On solving magnification equation we have $v = -mu$; substituting this expression for v in mirror equation, we get

$$\frac{1}{u}+\frac{1}{(-mu)}=\frac{1}{f} \text{ or } u=\frac{f(m-1)}{-m}$$

Applying this result, we obtain

$$m=-2,\ u=\frac{f(m-1)}{m}=\frac{(-12)(-2-1)}{-2}=-18\,\text{cm}$$

$$m=+2,\ u=\frac{f(m-1)}{m}=\frac{(-12)(+2-1)}{+2}=-6\,\text{m}$$

Negative signs for object distances indicate that the object is real, lies infront of the mirror.

Example 36

A small plane mirror is placed 21 cm infront of a concave mirror of focal length 21 cm. An object is placed 42 cm infront of the concave mirror. If light from the concave mirror strikes the plane mirror, where is the final image ?

Solution: First we will obtain image position from concave mirror, by mirror equation.

The given parameters are

$$f=-21\,\text{cm}$$

$$u=-42\,\text{cm}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{(-21)}-\frac{1}{(-42)}=-\frac{1}{42}\,\text{cm}$$

$$v=-42\,\text{cm}$$

The image is 42 cm infront of the mirror. The image formed by the concave mirror is object for the plane mirror. Now we use the mirror equation for the plane mirror with $f=\infty, u=-[42-21]\,\text{cm}$.

$$\frac{1}{v}+\frac{1}{(-21)}=\frac{1}{\infty}$$

$$v=21\,\text{cm}$$

The position of the final image is 21 cm infront of the plane mirror.

Example 37

A concave mirror with aperture diameter 6 cm and radius of curvature 5 cm is covered with an opaque paper except a thin strip at the edge. At what distance from the mirror will a parallel beam of rays converge ?

Solution: The small strip at the edge of the concave mirror acts as a plane mirror; the law of reflection is obeyed.

The rays converge at point F

From figure, $O'C=\sqrt{5^2-3^2}=4\,\text{cm}$

$$OO'=OC-O'C=1\,\text{cm}$$

$$\tan i=\frac{O'A}{O'C}=\frac{3}{4}$$

Hence $i\simeq 37°$

$$\angle O'FA=74°$$

$$\tan 74°=\frac{O'A}{O'F}=\frac{3}{O'F}$$

$$O'F=\frac{3}{\tan 74°}=0.86\,\text{cm}$$

Hence $OF=OO'+O'F=1.86\,\text{cm}$

Example 38

The co-ordinates of the image of point object P formed by a concave mirror of radius of curvature 20cm (consider paraxial rays only) as shown in the figure is

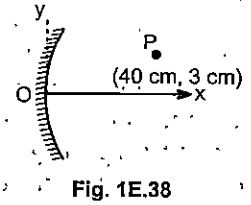


Fig. 1E.38

(a) 13.33 cm, –1 cm *(b) 13.33 cm, +1 cm*

(c) –13.33 cm, +1 cm *(d) –13.33 cm, –1 cm*

Solution: (a) $\frac{1}{v}-\frac{1}{40}=\frac{1}{-10}$

$$\frac{1}{v}=\frac{-1}{10}+\frac{1}{40}=\frac{-4+1}{40}$$

$$\Rightarrow \quad v=\frac{-40}{3}$$

$u=-40;\ f=-10$

$$v=\frac{uf}{u-f}=\frac{-40\times-10}{-40+10}=\frac{-40}{3}\text{ cm}$$

$$\frac{h_1}{3}=\frac{m=-\left(\frac{-40}{3}\right)}{-40}=\frac{-1}{3}$$

$$\Rightarrow \quad h_1=-1\text{ cm}$$

Example 39

A spherical mirror is polished on both sides. When the convex side is used as a mirror the image is erect with magnification 1/4. What is the magnification when the concave side is used as a mirror, the object remaining the same distance from the mirror ?

Solution:

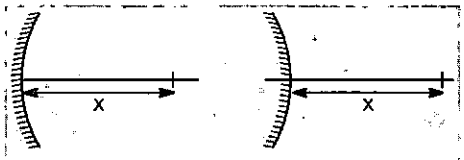


Fig. 1E.39

$$m=\frac{f}{f-u}=\frac{\frac{-R}{2}}{\frac{-R}{2}+\frac{3R}{2}}=-\frac{1}{2}$$

$$m=\frac{f}{f-u}=\frac{\frac{R}{2}}{\frac{R}{2}+x}=\frac{1}{4}$$

$$\Rightarrow \quad 4R=R+2x$$

$$\Rightarrow \quad x=\frac{3R}{2}$$

Example 40

A large convex spherical mirror in an amusement park is facing a plane mirror 10 m away. A boy of height 1m standing midway between the two sees himself twice as tall as in plane mirror as in spherical one. In other words, the angle subtended at the observer by the image in plane mirror is twice the angle subtended by the image in the spherical mirror. What is the focal length of the convex mirror ?

Solution: $\frac{1}{10}=\theta_1=0.1$ rad

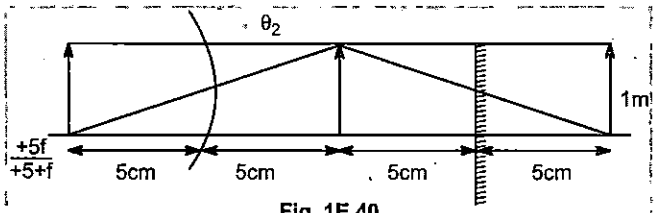


Fig. 1E.40

$$\frac{h_1}{1}=\frac{5f}{\frac{5+f}{5}}\quad\Rightarrow\quad h_1=\frac{f}{5+f}$$

$$\theta_2=\frac{\frac{f}{5+f}}{5+\frac{5f}{5+f}}=\frac{f}{25+10f}=\frac{0.1}{2}$$

$$2f=2.5+f$$

$$f=2.5\text{ m}$$

Example 41

A bright point S is on the principal optical axis of a concave mirror of radius R = 40 cm at d = 30 cm from its pole. At what distance (in cm) in front of the concave mirror should a plane mirror be placed so that after two reflections, the rays converge back at points S.

Solution: For mirror 1

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Big|\begin{matrix}y=-30\\f=-20\end{matrix}$$

$$\frac{1}{v}-\frac{1}{30}=-\frac{1}{20}$$

$$\frac{1}{v}=\frac{1}{30}-\frac{1}{20}$$

$v=-60$ cm

so $x=15$ cm

so distance between two mirrors = 45 cm.

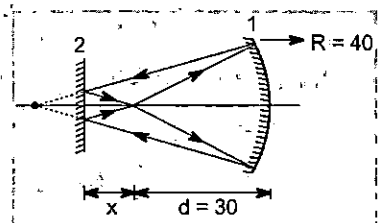
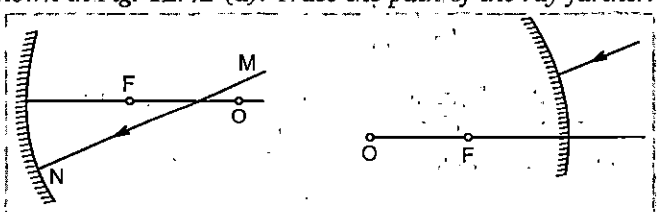


Fig. 1E.41

Example 42

(a) A ray of light falls on a concave spherical mirror, as shown in Fig. 1E.42 (a). Trace the path of the ray further.



(a) Fig. 1E.42 (b)

(b) A ray of light falls on a convex mirror, as shown in Fig. 1E.42 (b). Trace the path of the ray further.

Solution: (a) Consider a beam of rays parallel to the ray *MN*. The beam reflected from the mirror will converge at the secondary focus *F'*, which lies in the focal plane. Drawing the ray *DO||MN* parallel to the centre of the mirror, we find the secondary focus *F'*.

The ray *NK* is the one we are looking for [Fig.1E.42(c)]

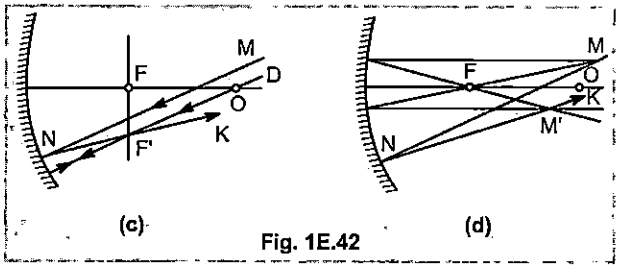


Fig. 1E.42

Method 2. Choose an arbitrary point *M* on the ray *MN* and with the aid of characteristic rays construct its image *M'*. The required ray *NK* passes through this point [see Fig. 1E.42 (d)]

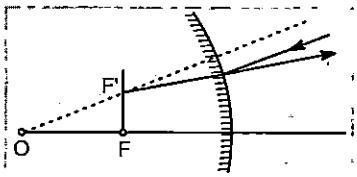


Fig. 1E.42 (e)

(b) The paths of the rays are shown in Fig. 1E.42 (e).

Example 43

Point S' is the image of a point source of light S in a spherical mirror whose optical axis is N_1N_2 [Fig. 1E.43(a)]. Find by construction the position of the centre of the mirror and its focus.

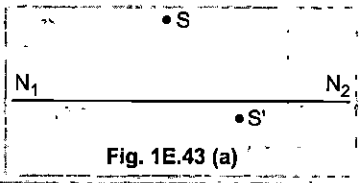


Fig. 1E.43 (a)

Solution: Since the ray incident on the mirror at its pole is reflected symmetrically w.r.t. the major optical axis, let us plot point S_1 symmetrical to *S'* and draw ray SS_1 until it intersects the axis at point *P* [Fig. 1E.43(b)]. This point will be the pole of the mirror.

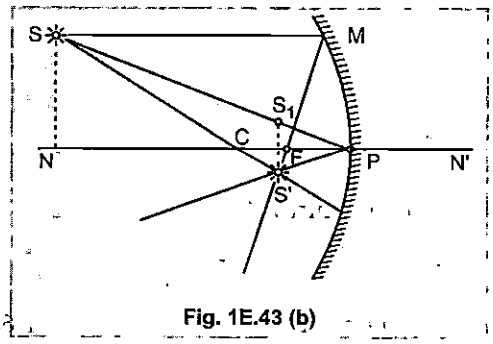


Fig. 1E.43 (b)

The optical centre *C* of the mirror can obviously be found as the point of intersection of ray *SS'* with axis *NN'*.

The focus can be found by the usual construction of ray *SM* parallel to the axis. The reflected ray must pass through focus *F* (lying on the optical axis of the mirror) and through *S'*.

Example 44

The positions of optical axis N_1N_2 of a spherical mirror, the source and the image are known [Fig. 1E.44(a)]. Find by construction the positions of the centre of the mirror, its focus and the pole for the cases : (a) A– source, B– image; (b) B– source, A – image.

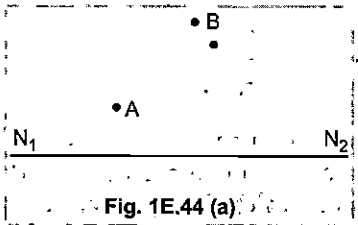


Fig. 1E.44 (a)

Solution: (a) Let us construct, as in the previous example, the ray *BAC* and find point *C* (optical centre of the mirror) [Fig. 1E.44 (b)]. Pole *P* can be found by constructing the path of the ray *APA'* reflected in the pole with the aid of symmetrical point *A'*. The position of the mirror focus *F* is determined by means of the usual construction of ray *AMF* parallel to the axis.

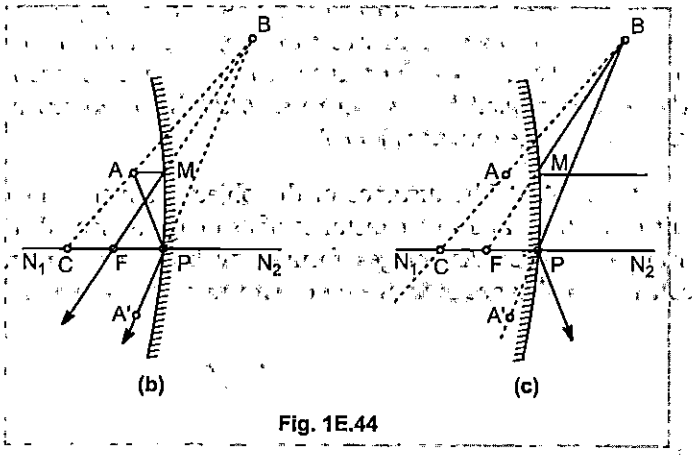


Fig. 1E.44

(b) This construction can also be used to find the centre *C* of the mirror and pole *P* [Fig. 1E.44 (c)]. The reflected ray *BM* will pass parallel to the optical axis of the mirror. For this reason, to find the focus, let us first determine point *M* at which straight line *AM*, parallel to the optical axis, intersects the mirror and then extend *BM* to the point of intersection with the axis at the focus *F*.

Example 45

A point source S is placed midway between two converging mirrors having equal focal length f as shown in figure. Find the values of d for which only one image is formed.

Solution:

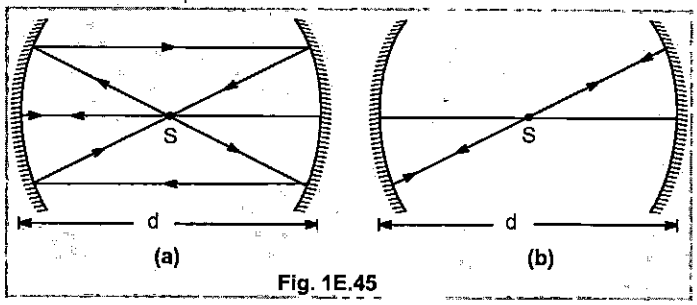


Fig. 1E.45

When S is placed at the common focus of mirrors, the rays after reflection from one mirror incident parallel on to the second mirror, which finally intersect at focus of the mirror.Thus there will be only one image. In this case the value of d will be $2f$. When S is placed at the centre of curvature, the image will form at the same point, so in this case the value of d will be $2f + 2f = 4f$ (see Fig. 1E.45)

Example 46

An object is placed in front of a convex mirror at a distance of 50cm. A plane mirror is introduced covering lower half of the mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no parallax between the images formed by two mirrors. What is the radius of curvature of the convex mirror ?

Solution: The distance of the object from the plane mirror is 30 cm and so the distance of its image is also 30 cm from the mirror. As images formed by both the mirrors concide, so distance of image for convex mirror is = 10 cm.

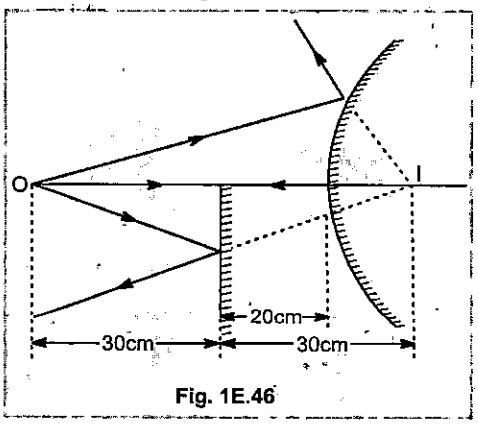


Fig. 1E.46

By mirror formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, we have

$$\frac{1}{10}+\frac{1}{-50}=\frac{1}{f}$$

which on solving gives $f = 12.5$ cm.

$\therefore$ Radius of curvature $R = 2f = 25$ cm.

Example 47

A converging mirror M_1, a point source S and a diverging mirror M_2 are arranged as shown in Fig.1E.47. The source is placed at a distance of 30 cm from M_1. The focal length of each of the mirrors is 20 cm. Consider only the images formed by a maximum of two reflections. It is found that one image is formed on the source itself.

(a) Find the distance between the mirrors.

(b) Find the location of the image formed by the single reflection from M_2.

Solution: For mirror M_1:

$$u = -30\,\text{cm},$$

$$f = -20\,\text{cm}$$

By mirror formula, $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, we have

$$\frac{1}{-30}+\frac{1}{v}=\frac{1}{-30}$$

which on solving gives, $v = -60$ cm.

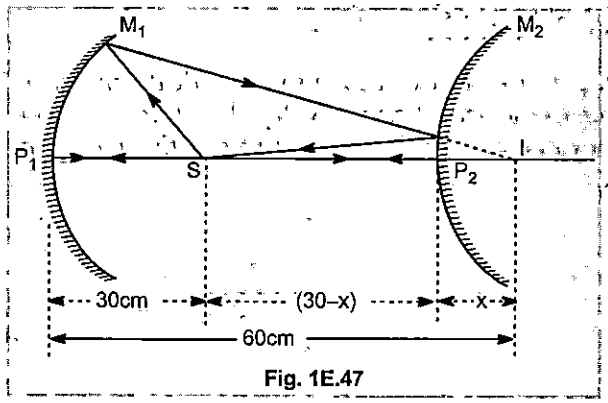


Fig. 1E.47

For mirror M_2: The image formed by mirror M_1 is a virtual object for mirror M_2. Let it is a distance x from the pole of mirror M_2. Thus

$$u_2 = +x$$

$$v_2 = -(30 - x)$$

Again by mirror formula, we have

$$\frac{1}{x}+\frac{1}{-(30-x)}=\frac{1}{20}$$

which on solving gives $x = 10$ cm or 60 cm, $x = 60$ cm is not possible, thus $x = 10$ cm.

(a) Thus the separation between the mirrors = 60 – 10 = 50 cm.

(b) The image formed by mirror M_2 is at a distance 10cm.

Example 48

A gun of mass M fires a bullet of mass m with a horizontal speed v. The gun is fitted with a concave mirror of focal length f facing toward the receding bullet. Find the speed of separation of the bullet and image just after the gun was fired.

Solution: If v' is the recoil velocity of the gun, then by conservation of linear momentum,

$$O = mv' + Mv'$$

$$\therefore \quad v' = \frac{mv}{M}$$

The velocity of the bullet with respect to the mirror

$$\left[\vec{\mathbf{v}}_{\text{bullet}}\right]_{\text{mirror}} = \left[\vec{\mathbf{v}}_{\text{bullet}}\right]_g - \left[\vec{\mathbf{v}}_{\text{mirror}}\right]_g$$

$$= v - v'$$

$$= v - \left(-\frac{mv}{M}\right)$$

$$= \left(1 + \frac{m}{M}\right)v$$

or $$\frac{du}{dt} = \left(1 + \frac{m}{M}\right)v$$

By the defination, the image velocity,

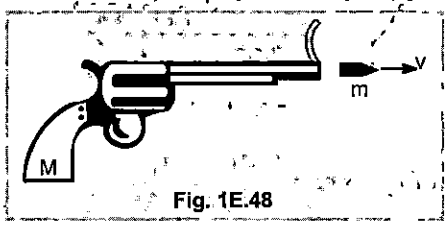


Fig. 1E.48

$$u_{I/M} = -m^2(v_{O/M})$$

At the instant of firing, bullet and its image are at the pole of the mirror, so $m = 1$.

The velocity of image w.r.t. mirror = velocity of bullet w.r.t. mirror

The speed of separation between bullet and its image

$$= 2\left(\frac{du}{dt}\right)$$

$$= 2\left(1 + \frac{m}{M}\right)v.$$

Example 49

Two concave mirrors of equal radii of curvature R are fixed on a stand facing opposite directions. The whole system has a mass m and is kept on a frictionless horizontal table [Fig. 1E.49 (a)]

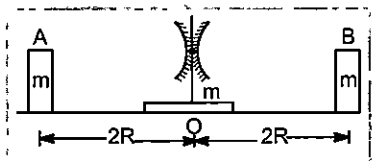


Fig. 1E.49 (a)

Two block A and B, each of mass m, are placed on the two sides of the stand. At t = 0, the separation between A and the mirror is 2R and also the separation between B and the mirror is 2R. The block B moves towards the mirror at a speed v. All collisions which take place are elastic. Taking the original position of the mirrors standard system to be x = 0 and x-axis along AB, find the position of the images of A and B at;

(a) $t = \frac{R}{v}$, *(b)* $t = \frac{3R}{v}$, *(c)* $t = \frac{5R}{v}$.

Solution: (a) At $t = \frac{R}{v}$.

For block A, $u = -2R$

$$\therefore \quad \frac{1}{v} + \frac{1}{-2R} = \frac{2}{-R}$$

or $$v = \frac{-2R}{3}$$

For block B: The distance travels by block B in time $\frac{R}{v}$ is R

Thus $$u = -R$$

$$\frac{1}{v} + \frac{1}{-R} = \frac{2}{-R}$$

or $$v = -R$$

The x-coordinate of the image of the block with respect to the mirror will be +R.

(b) At $$t = \frac{3R}{v}.$$

The block B will collide with the stand after time $\frac{2R}{v}$.

After collision block B becomes at rest and mirror starts moving with the same velocity v. In the remaining time R/v, the distance moved by the mirror is R.

The position of blocks and mirror are shown in Fig. 1E.49 (b)

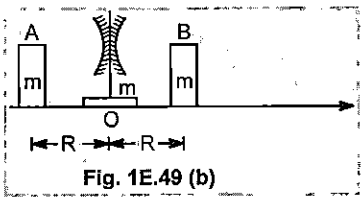


Fig. 1E.49 (b)

At this time the blocks lie at the centre of curvature of the respective mirrors. Their images will form at the centres of curvature. So their co-ordinates are:

For block A, $x = -R$

For block B, $x = +R$

(c) At $t = \dfrac{5R}{v}$.

The block B will collide to the mirror after a time $\dfrac{2R}{v}$. Thereafter mirror starts moving towards block A with velocity v. At $t = \dfrac{4R}{v}$, the mirror will collide with block A and stops after collision. The positions of blocks and mirror are shown in Fig. 1E.49 (c).

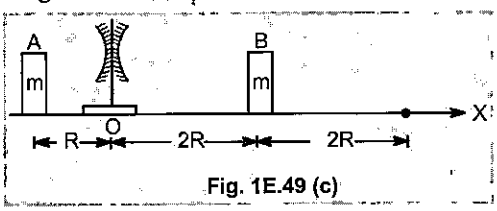


Fig. 1E.49 (c)

For block A; Its image will form on the same place. Therefore the positions of the blocks are

$$x_A = -3R$$

For block B; $u = -2R$

$$\frac{1}{v} + \frac{1}{-2R} = \frac{2}{-R}$$

$$v = -\frac{2R}{3}$$

The co-ordinates of B are $-\left(2R - \dfrac{2R}{3}\right)$

$$= \frac{-4R}{3}$$

Example 50

Two concave mirrors each of radius of curvature 40 cm are placed such that their principal axes are parallel to each other and at a distance of 1 cm to each other. Both the mirrors are at a distance of 100 cm to each other. Consider first reflection at M_1 and then at M_2, find the coordinates of the image thus formed. Take location of object as the origin.

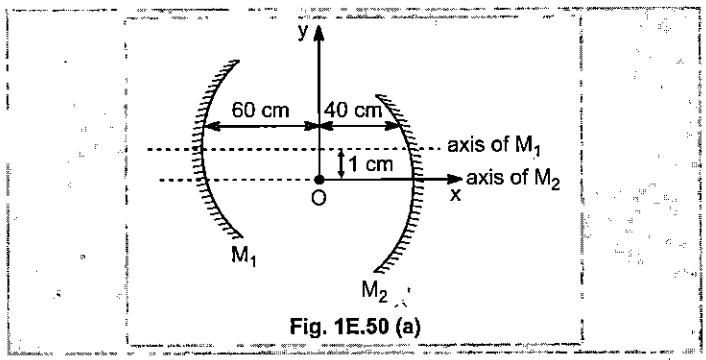


Fig. 1E.50 (a)

Solution: Using mirror formula for first reflection:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-20} = \frac{1}{v} + \frac{1}{-70}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{60} - \frac{1}{20} \Rightarrow v = -30\,\text{cm}$$

Using mirror formula for second reflection

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-20} = \frac{1}{v} + \frac{1}{-70}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{70} - \frac{1}{20} = \frac{2-7}{140}$$

$$\Rightarrow \quad v = -\frac{140}{5} = -28\,\text{cm}$$

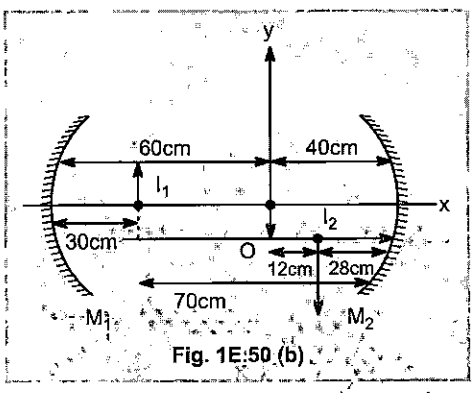


Fig. 1E.50 (b)

Height of $I_2 \Rightarrow m = \left(\dfrac{-30}{-60}\right) = \dfrac{I_1}{-1}$

$$\Rightarrow \quad I_1 = \frac{1}{2}\,\text{cm}$$

Height of first image from x-axes $= 1 + \dfrac{1}{2} = \dfrac{3}{2}\,\text{cm}$

Height of $I_2 \Rightarrow m = \left(\dfrac{-28}{-70}\right) = \dfrac{2I_2}{3}$

$$\Rightarrow \quad I_2 = \frac{3 \times 28}{2 \times 70}, \quad I_2 = -0.6\,\text{cm}$$

Co-ordinate of $I_2 = (12 - 0.6)$

Example 51

A point object is placed at the centre of curvature of a concave mirror (taken as origin). A plane mirror is also placed at a distance of 10 cm from the object as shown. Consider two reflection first at plane mirror and then at concave mirror. Find the coordinate of the image thus formed.

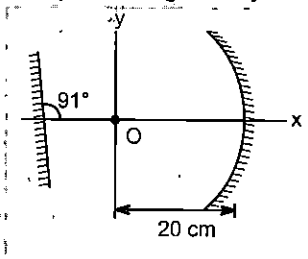


Fig. 1E.51-(a)

Solution: Distance of I_1 from mirror = 40 cm

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{-10}=\frac{1}{v}+\frac{-1}{-140}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{40}-\frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1-4}{40}$$

$$\Rightarrow \quad v=-40/3\,\text{cm}$$

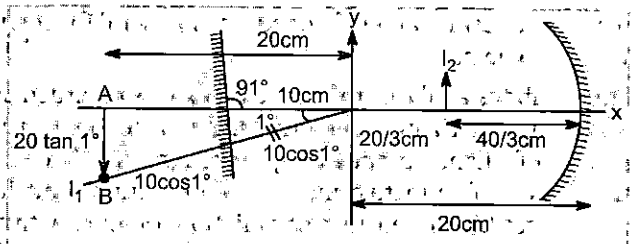


Fig. 1E.51 (b)

Using magnification formula

$$m=\frac{I}{0}=-\frac{v}{u}$$

$$\Rightarrow \quad \frac{I}{-20\tan 1^\circ}=-\frac{-40/30}{-40}$$

$$\Rightarrow \quad I=\frac{20\tan 1^\circ}{3}=\frac{20}{3}\tan\frac{\pi}{180}$$

$$\simeq\frac{20}{3}\times\frac{\pi}{180}=\frac{\pi}{27}$$

Co-ordinate of image $=\left(\frac{20}{3},\frac{\pi}{27}\right)$

Example 52

A convex mirror of focal length $f = 10$ cm is placed infront of a glass slab of thickness $t = 5$ cm at a distance (nearer surface) $x = 5$ cm. An object lies on the common axis of the system at a distance $d = 15$ cm from the mirror. Find the position of image. Draw the ray diagram showing the formation of the image. μ of slab = 1.5.

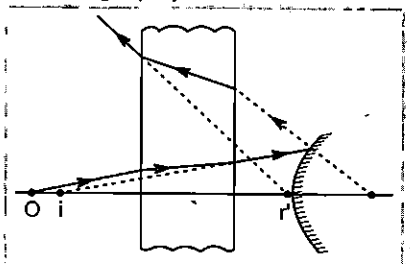


Fig. 1E.52

Solution: Since the rays are diverging, the function of the slab is to bring the object closer by $t-\frac{t}{\mu}=5-\frac{5}{1.5}=\frac{5}{3}$.

So we may consider the object is at a distance $15-\frac{5}{3}=\frac{40}{3}$ cm.

Considering reflection at the mirror,

$$\frac{1}{v}+\frac{1}{-40/3}=\frac{1}{+10}\Rightarrow v=+\frac{40}{7}\,\text{cm}$$

Since the rays are again diverging, the slab will bring the image closer by 40/3 cm.

$\therefore$ Final image is at a distance $\frac{40}{7}-\frac{40}{3}=-40\times\frac{4}{21}$

$=-7.6$ cm

The negative sign shows that it will be infront of the mirror.

REFRACTION

When a light ray obliquely encounters an abrupt change in medium, the direction of light ray changes permanently at the interface; this bending is called **refraction**. At the boundary the media are assumed to be homogeneous isotropic and transparent. A **homogeneous medium** means uniform composition and structure throughout the material. An **isotropic medium** means identical or invariant in all directions; otherwise it is **anisotropic.**

Refractive Index

Refractive index depends on the optical properties of the medium into which light penetrates. The refractive index obtained when light gets from vacuum into a medium is known as the *absolute refractive index for a given medium.*

Let the absolute refractive index of the first medium be n_1 and of the second medium be n_2. Considering the refraction at the interface between the first and second media, we can make sure that the refractive index n for the transition from the first to the second medium (*relative refractive index*) is equal to the ratio of the absolute refractive indices of the second and first media:

$$n = \frac{n_2}{n_1} \qquad ...(1)$$

(Fig. 1.93). On the contrary, when the light propagates from the second to the first medium, the relative refractive index is

$$n' = \frac{1}{n} = \frac{n_1}{n_2} \qquad ...(2)$$

These relations between the relative refractive index for two media and their absolute refractive indices could be derived theoretically without new experiments in the same way as it can be done for the reversibility principle (See reversibility of light rays).

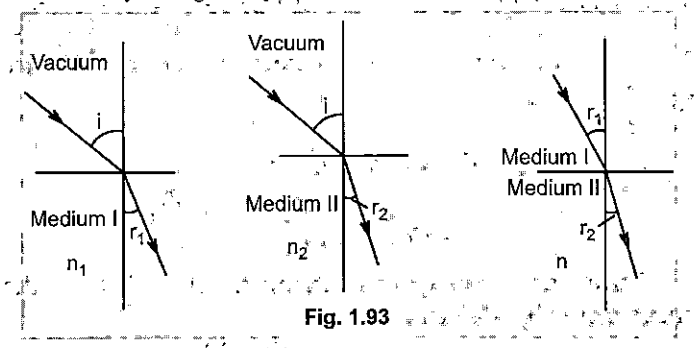


Fig. 1.93

A medium having a larger refractive index is referred to as an *optically denser medium*. The refractive index of various media is normally determined relative to air. The absolute refractive index of air n_{air} is 1.003. Thus, the absolute refractive index n_{abs} of a medium is connected with its refractive index relative to air (n_{rel}) through the formula

$$n_{abs} = n_{air}\, n_{rel} = 1.003\, n_{rel} \qquad ...(3)$$

Table 1.1 contains relative refractive indices determined for some cases of refraction of light at the interface between air and a corresponding medium.

Refractive index depends on the wavelength of light, *i.e.*, on its *colour*. Different refractive indices correspond to different colours. This phenomenon, which is known as *dispersion*, plays an important role in optics.

Relative refractive index for two media:

$$n_1 = \frac{\sin i'}{\sin r_1}, n_2 = \frac{\sin i}{\sin r_2}, n = \frac{\sin r_1}{\sin r_2} = \frac{n_2}{n_1}.$$

Table 1.1: Refractive Indices of Some Substances Relative to Air

Liquid		Solid	
Substance	***n***	**Substance**	***n***
Carbon disulphide	1.632	Diamond	2.417
Ethyl alcohol	1.362	Glass (heavy flint)*	1.80
Glycerine	1.47	Glass (light crown)*	1.57
Liquid helium	1.028	Ice	1.31
Liquid hydrogen	1.12	Ruby	1.76
Water	1.333	Sugar	1.56

We shall often deal with this phenomenon in the following chapters. The data contained in table 1.1 refer to yellow light.

It is interesting to note that the law of reflection can formally be written in the same form as the law of refraction. It should be recalled that we agreed to measure angles from the normal to the corresponding ray. Consequently, the angle of incidence i and the angle of reflection i' should be taken with opposite signs. Then the law of reflection should be written in the form

$$i' = -i$$

or

$$\frac{\sin i}{\sin i'} = -1 \qquad ...(4)$$

Comparing this formula with the law of refraction, we see that the law of reflection can be treated as a special case of the law of refraction for $n = -1$. This formal similarity of the laws of reflection and refraction is very useful for solving practical problems.

In the above analysis, refractive index was regarded as a constant characterizing a medium and independent of the intensity of light propagating through it. Such an interpretation of refractive index is quite natural, but it is unjustified for high intensities of radiation attained with the help of modern lasers. In this case, the properties of a medium through which a high-power optical radiation propagates depend on the radiation intensity. The medium is said to become nonlinear. The nonlinearity of the medium is manifested, in particular, in the fact that a high-intensity light wave changes the refractive index in it. The dependence of the refractive index on the radiation intensity J has the form

$$n = n_0 + aJ.$$

Here n_0 is an ordinary refractive index and aJ is the nonlinear refractive index, where a is the proportionality factor. The additional term in the formula can be either positive or negative.

* Crown and flint are the grades of optical glass

(a) A ray entering a material of larger index of refraction bends toward the normal.

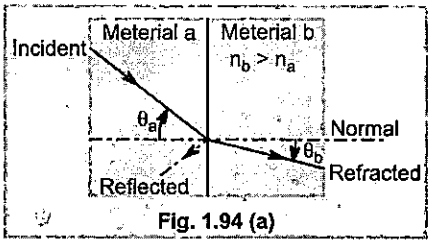


Fig. 1.94 (a)

(b) A ray entering a material of smaller index of refraction bends away from the normal.

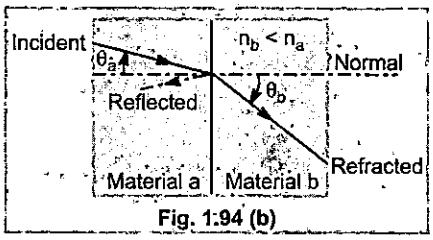


Fig. 1.94 (b)

(c) A ray oriented along the normal does not bend, regardless of the materials.

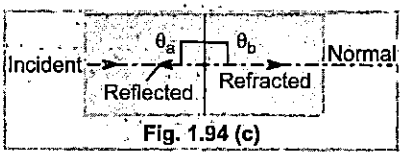


Fig. 1.94 (c)

Isotropic optical medium: Refractive index of medium in all directions is same (not directional property).

Anisotropic medium: Refractive index of same medium in different directions is different.

Optical density is according to velocity of propagation of wave in the medium. Velocity of propagation decreases with increase in optical density of medium and *vice-versa*.

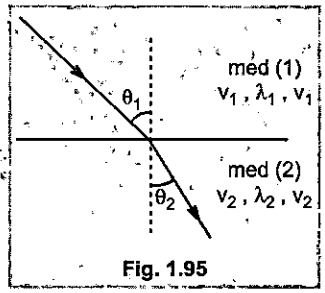


Fig. 1.95

Frequency of incident and refracted ray is same.

$$v_1 = v_2$$

By convention ${}_1\mu_2 = \mu_{12} = \frac{\mu_2}{\mu_1}$

R.I. of medium (2) w.r.t. medium (1) is given by

$${}_1\mu_2 = \mu_{12} = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2}$$

$$= \frac{\lambda_1}{\lambda_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

$${}_2\mu_1 = \mu_{21} = \frac{1}{\mu_{12}}$$

$$\mu_{21} \times \mu_{12} = 1$$

General form of Snell's Law

The Snell's law describes the relation between angle of incidence θ_1 and angle of refraction θ_2.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = \text{constant} \quad \ldots. (1)$$

$$\mu_{12} = \frac{\mu_2}{\mu_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

$$\mu_1 v_1 = \mu_2 v_2 = \mu_3 v_3 = \ldots\ldots = \text{const.} \quad \ldots(2)$$

$$\Rightarrow \quad v \propto \frac{1}{\mu}$$

$$\mu_1 \lambda_1 = \mu_2 \lambda_2 = \mu_3 \lambda_3 = \ldots\ldots = \text{const.} \quad \ldots(3)$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\mu}$$

Eqns. (1) and (2) are always applicable,

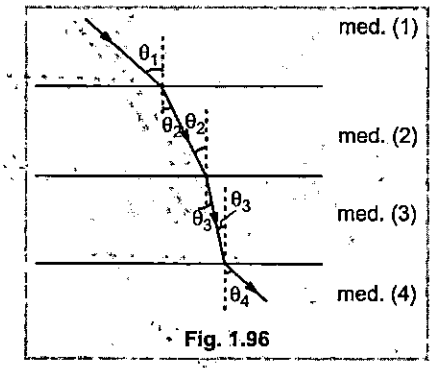


Fig. 1.96

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = \mu_4 \sin\theta_4 = \ldots = \text{const.}$

Applicable only when all refractory surfaces are simple plane and parallel to each other.

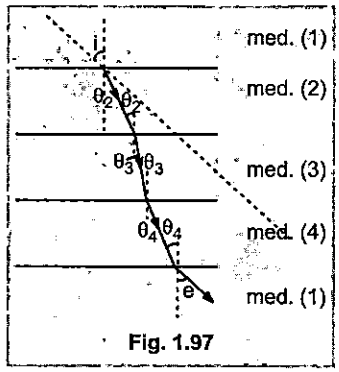


Fig. 1.97

$\mu_1 \sin i = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = \mu_4 \sin\theta_4 = \mu_1 \sin e$

$\mu_1 \sin i = \mu_1 \sin e$

$i = e$

If all refractory surfaces are simple plane and parallel and nature of initial and final medium are same, then initial incident and final emergent rays are parallel to each other irrespective of nature and number of medium present

between them. Final emergent ray may have some lateral shift.

where n_1 and n_2 are refractive indices of the two media.

The refractive index is never less than 1 because $v < c$; n varies with wavelength of light, except in vacuum. So a particular wavelength is specified, for a given refractive index.

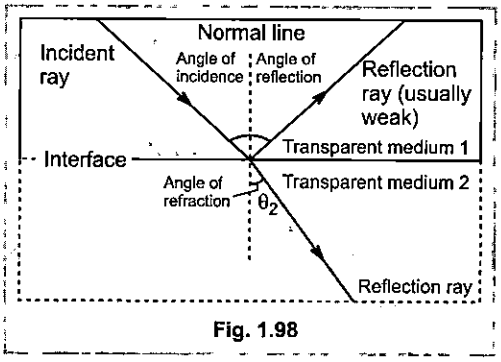


Fig. 1.98

❐ When light travels from medium 1 to 2 its refractive index is written as

$$= \frac{n_2}{n_1} \quad \text{.... (4)}$$

$_1n_2$

Medium of incident ray → Medium of refracted ray

$_1n_2$ is refractive index of medium 2 w.r.t. medium 1 referred to as **relative refractive index**. If the medium 1 is air or vacuum the refractive index relative to it is referred to as **absolute refractive index**. It is customary to drop the first subscript in case of absolute refractive index. Thus n_{glass}, stands for absolute refractive index of glass, with incident ray in air or vacuum and refracted ray in glass.

When light travels from one medium to the other, its frequency does not change but its wavelength and velocity does. Consider observers at points A and B as shown in Fig. 1.99. Wavefronts pass the observer at point A in medium 1 with a certain frequency. The Frequency with which the wavefronts pass the observer at B in medium 2 must be equal to frequency at which they pass point A. If this were not so, then the wavefronts would accumulate at the boundary or they would be destroyed or created at the boundary; hence the frequency must be constant.

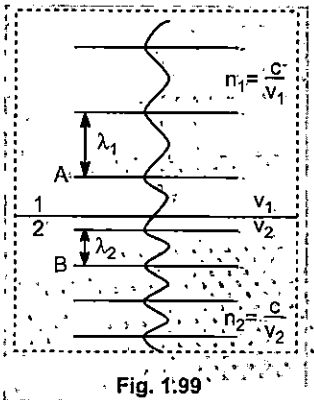


Fig. 1.99

As $v_1 = \nu\lambda_1$ and $v_2 = \nu\lambda_2$

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

Thus we have

$$n_1\lambda_1 = n_2\lambda_2 \text{ and } n_1v_1 = n_2v_2 \quad \text{.... (5)}$$

From Snell's law,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \quad \text{.... (6)}$$

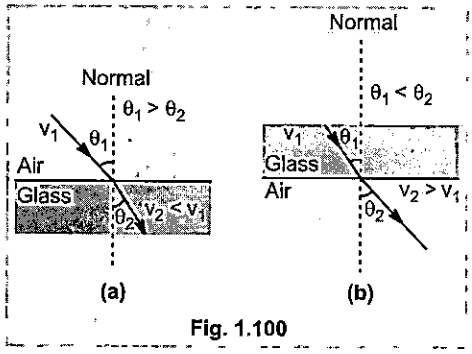


Fig. 1.100

❐ The path of light ray through a refracting surface is reversible, *e.g.*, in the figure the incident ray travels from point A to point B. If the ray originated at B, it would travel along line BA to reach A.

$$_1n_2 = \frac{1}{_2n_1}$$

From eqn. (5) we have

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

When light moves from a medium in which its speed is high **(rarer medium)** to a medium in which its speed is lower **(denser medium)**, as shown in Fig. 1.100, the angle of refraction θ_2 is less than the angle of incidence θ_1 and the beam is bent toward the normal. If the ray moves from a material in which it moves slowly into another medium in which it moves fast as shown in Fig. 1.100 (b), θ_2 is greater than θ_1.

❐ When light travels in air its speed is 3×10^8 m/s, it is reduced to 2×10^8 m/s in glass with refractive index 3/2. When the light emerges from glass its velocity is restored. Inside the glass light encounters electrons bound to an atom; this causes the electron to oscillate. An oscillating charge is a source of further electromagnetic radiations; the neighbouring atoms absorb this electromagnetic radiations; The finer details of these absorptions and radiations are the subject matter of quantum mechanics. Overall, the process consists of transmission of light from one atom to the other. Although light travels from one glass atom to other at 3×10^8 m/s, due to absorption and radiation the average speed through the material is slowed to 2×10^8 m/s

HUYGEN'S PRINCIPLE

When a wave is incident on a boundary between media, some of its energy is reflected and some is transmitted. For example, when light travelling in air is incident on a transparent media such as glass, it is partially reflected and partially transmitted. We can analyze this phenomenon on the basis of a geometrical method developed by Huygens. **Huygen's principle** states that : *"Each point on an advancing wavefront can be considered to be a source of secondary waves or wavelets and the line or surface tangent to all these wavelets defines a new position of the wavefront.*

Huygen's principle is applied to incident and transmitted wavefronts at a media boundary as shown in Fig. 1.101 (a). The wave speeds are different in the two media. In this case, $v_1 > v_2$. (The speed of light varies in different media and, in general is less in denser media.

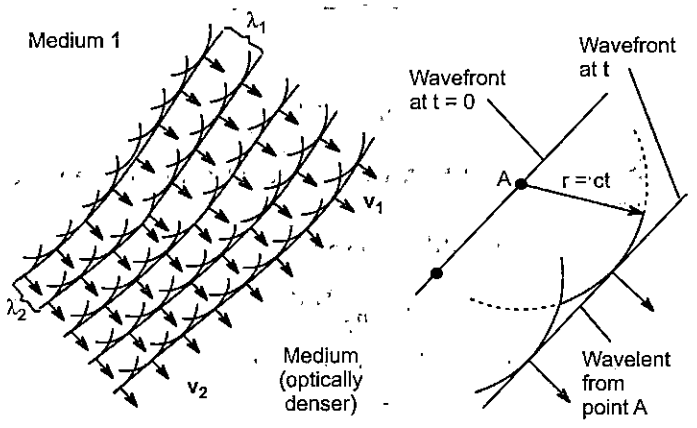


Fig. 1.101

The distances the wavefronts travel in a time t are v_1t in medium 1 and v_2t in medium 2 [Fig. 1.101 (b)]. As a result of the smaller wave speed in the second medium, the direction of the transmitted wavefront is different from that of the incident wavefront. The particles of the second medium are set in motion by the incident wave disturbance, thus the frequency of wave is the same in both media. However, the wavelengths are different because of the different wave speeds ($v = \lambda f$)

The change in the direction of wave propagation is described by the **angle of refraction.** In Fig. 1.101 (b), the angle of incidence is θ_1 and the angle of refraction is θ_2. From the geometry of two parallel rays, where d is the distance between the normals to the boundary at the points where the rays are incident,

$$\sin\theta_1 = \frac{v_1 t}{d} \text{ and } \sin\theta_2 = \frac{v_2 t}{d}$$

Combining these two equations in ratio form gives

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \text{ (Snell's law)}$$

This expression is known as **Snell's law.**

REVERSIBILITY OF LIGHT RAYS

While analyzing refraction observed when light is incident on the interface between two media, we assumed that light propagates in a certain direction indicated by arrows. Let us now see what happens when light propagates in the reverse direction. For the case of reflection, this means that the incident ray is directed not from the left and downwards as in Fig. 1.102 (a) but from the right and downwards as in Fig. 1.102 (b).

For the case of refraction, we shall consider the propagation of light not from the first to the second medium as in Fig. 1.102 (c), but from the second to the first medium as in Fig. 1.102 (d).

Accurate measurements show that both for reflection and refraction, the angles between the rays and the normal to the interface remain unchanged, and only the direction of arrows is reversed.

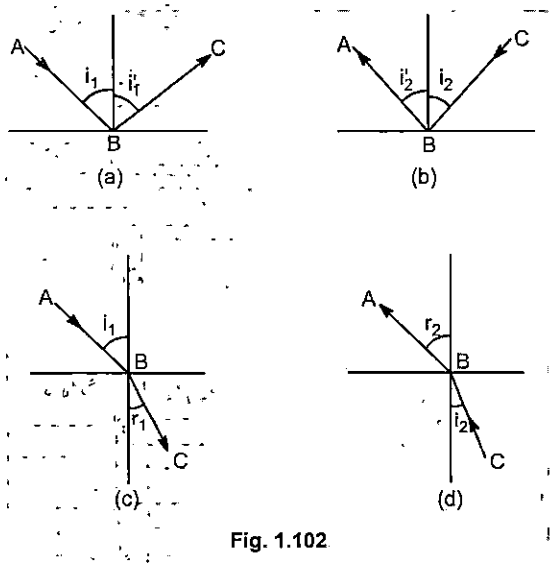


Fig. 1.102

Therefore, if a light ray is incident along *CB* [see Fig. 1.102 (b)], the reflected ray will propagate along *BA*, *i.e.*, the *incident and reflected rays have changed places* in comparison with the former case. The same is observed for

refraction of a light ray. Let *AB* be an incident ray and *BC* a refracted ray [see Fig. 1.102 (c)]. If the light is incident along *CB* [see Fig. 1.102 (d)], the refracted ray will propagate along *BA*, *i. e.*, the *incident and refracted rays have changed places.*

Thus, both in reflection and refraction, light can follow the same path in the opposite directions (Fig. 1.102). This property of light is known as the *reversibility of light rays.*

The reversibility of light rays indicates that if the refractive index is n for the transition from the first to the second medium for the reverse transition it will be $1/n$. Indeed, let the light be incident at angle i and refracted at angle r so that $n = \sin i/\sin r$. If for the reverse direction of the rays, the light is incident at angle r, it must be refracted at angle i (reversibility). In such a case, the refractive index $n' = \sin r/\sin i$, and hence $n' = 1/n$. For example, when light passes from air to glass, $n = 1.50$, while for the reverse transition from glass to air, $n' = 0.67 = 1/1.50$.

The reversibility of light rays is preserved in multiple reflections and refractions which may take place in any order. This follows from the fact that in each reflection or refraction, the direction of a light ray can be reversed.

Thus, *if a light ray emerging from any system of reflecting and refracting media is reflected at the last stage exactly in the backward direction, it will pass through the entire system in the reverse direction and return to its source.*

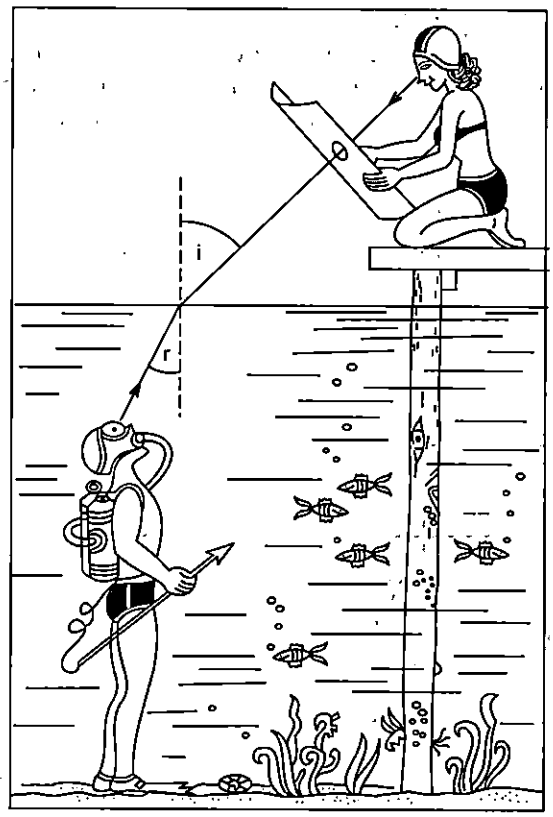


Fig. 1.103: To the reversibility of light rays in refraction.

Example 53

A parallel plate of thickness d and index of refraction n_2 is surrounded by a medium with index of refraction n_1. A light beam passes from medium 1 to medium 2, with an angle of incidence θ_1 as shown in Fig. 1E.53 (a) Show that the emerging ray is parallel to the incident ray. (b) What is the lateral shift of the ray as shown in the figure ?

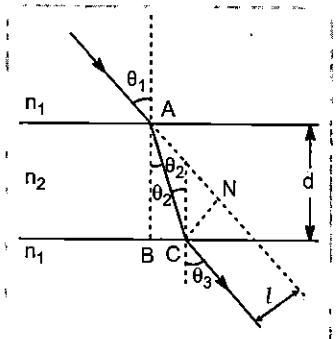


Fig. 1E.53 (a)

Solution: (a) First we apply Snell's law at the upper surface:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 \quad \ldots(1)$$

Now we apply Snell's law at the lower surface.

$$n_2 \sin\theta_2 = n_1 \sin\theta_3$$

$$\sin\theta_3 = \frac{n_2}{n_1}\sin\theta_2 \quad \ldots(2)$$

Now substituting eqn. (1) in eqn. (2), we have

$$\sin\theta_3 = \left(\frac{n_2}{n_1}\right)\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin\theta_1$$

Therefore $\theta_3 = \theta_1$; the ray emerges parallel to initial ray however it is displaced.

In ΔACN, $\quad l = AC\sin(\theta_1 - \theta_2) \quad \ldots(3)$

In ΔABC, $\quad AC = \dfrac{d}{\cos\theta_2} \quad \ldots(4)$

From eqns. (3) and (4),

$$l = \frac{d}{\cos\theta_2}\sin(\theta_1 - \theta_2)$$

$$= d\sin\theta_1\left(1 - \frac{\sin\theta_2\cos\theta_1}{\sin\theta_1\cos\theta_2}\right) \quad \ldots(5)$$

$$= d\sin\theta_1\left[1 - \frac{(n_1/n_2\sin\theta_1)\cos\theta_1}{\sin\theta_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2}}\right]$$

$$= d\sin\theta_1\left[1-\frac{n_1\cos\theta_1}{(n_2^2-n_1^2\sin^2\theta_1)^{1/2}}\right] \quad \ldots (6)$$

If θ_1 and θ_2 are small then $\sin\theta_1 \simeq \theta_1$ and $\cos\theta_1 \simeq \cos\theta_2 \simeq 1$; thus we get

$$l = t\theta_1\left(1-\frac{1}{n}\right)$$

$$= t\theta_1\left(\frac{n-1}{n}\right) \quad \ldots (7)$$

(b) Solution of part (a) is true for any number of parallel media.

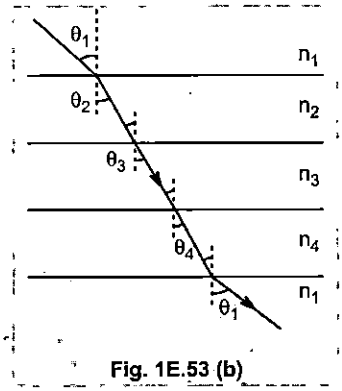


Fig. 1E.53 (b)

$$n_1\sin\theta_1 = n_2\sin\theta_2$$
$$n_2\sin\theta_2 = n_3\sin\theta_3$$
$$n_3\sin\theta_3 = n_4\sin\theta_4$$
$$n_4\sin\theta_4 = n_1\sin\theta'$$

Combining these, we get

$$\theta_1 = \theta'$$

❒ If $n_1 = 1$ and $n_2 = n$, the eqn. (6) reduces to

$$l = \frac{t}{n}\left(1-\frac{1}{n^2+1}\right)$$

❒ Eqn. (7) is valid for rays close to normal line.

Example 54

A swimmer has dropped a coin in a pool 2.0 m deep. The coin does not look that deep. Why ? How deep does the coin appear to be when he looks straight down into the water ? Take $n_w = 4/3$.

Solution: The ray diagram shows that the rays travelling upward from the coin on the bottom are refracted away from the normal as they exit the water. The rays appear to be diverging from a point higher in the water.

From Snell's law,

$$\sin\theta_1 = n_2\sin\theta_2$$

For small angles, $\sin\theta \simeq \tan\theta \simeq \theta_1$. With θ in radians, we have

$$\theta_1 \simeq \tan\theta_1 = \frac{x}{d'} \text{ and } \theta_2 \simeq \tan\theta_2 = \frac{x}{d}$$

Substituting these in Snell's law,

$$\theta = n_2\theta_2$$
$$\frac{n}{d'} \simeq n_2\frac{x}{d}$$
$$d' \simeq \frac{d}{n_2} = \frac{2}{(4/3)} = 1.50\,\text{m}$$

Note that the above derivation assumes that rays are very close to the normal line.

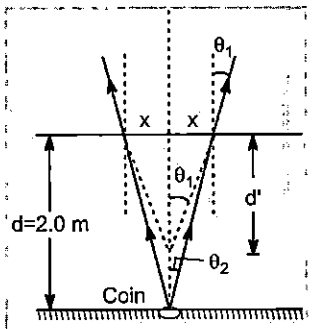


Fig. 1E.54

Example 55

Fig.1E.55 shows two perpendicular mirrors that form two sides of a vessel filled with water. (a) A light ray is incident normal to the water surface. Show that the emerging ray is parallel to the incident ray. Assume that there are two reflections at the mirror surfaces. (b) Show that the result is true for oblique rays.

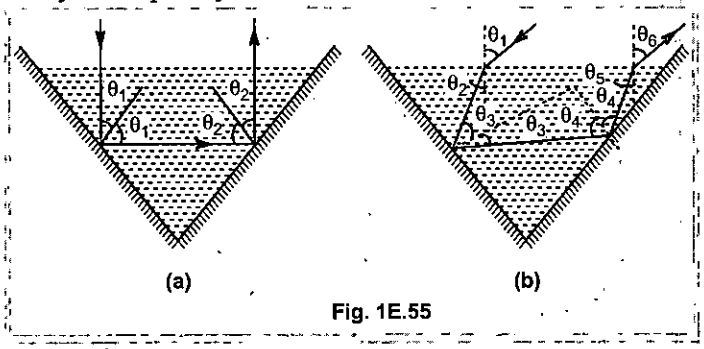


Fig. 1E.55

Solution: (a) The normal rays are not deviated. The angle of incidence at first mirror is $\theta_1 = 45°$; it will be reflected at 45°. This means that the ray is travelling horizontally after reflection. The angle of incidence at the second mirror is $\theta_2 = 45°$; the angle of reflection will also be 45°. Thus the ray leaves the mirror vertically and it will come out without deviation.

(b) From Snell's law at air-water surface, $\sin\theta_1 = n\sin\theta_2$. The normal to the water surface and normal to first mirror make an angle 45°. If normals n_1 and n_2 are produced, the triangle formed will have interior angle 135° [see Fig. 1E.55(c)].

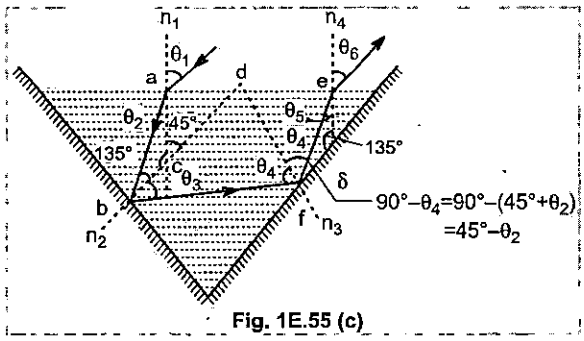


Fig. 1E.55 (c)

From figure, $\theta_3 + \theta_2 + 135° = 180°$

$$\theta_3 = 45° - \theta_2$$

The angle of reflection is also $45° - \theta_2$

From the triangle formed by n_2 and n_3,

$$\theta_3 + \theta_4 + 90° = 180°$$

$$\theta_4 = 90° - \theta_3 = 90° - 45° + \theta_2 = 45° + \theta_2$$

The angle of reflection at the second mirror is also $45° + \theta_2$. The angle made by n_4 with the inclined surface is 135°. From Δefg,

$$90° - \theta_4 + 135° + \theta_5 = 180°$$

$$(45° - \theta_2) + 135° + \theta_5 = 180°$$

$$\theta_5 = \theta_2$$

Now we apply Snell's law

$$\sin\theta_6 = n \sin\theta_5$$

As $\theta_5 = \theta_2$,

This means $\theta_6 = \theta_1$

The exiting ray is parallel to the incident ray.

Apparent Depth

A point object O is placed in a medium of refractive index μ_1. Another medium of refractive index μ_2 has its boundary at PA.

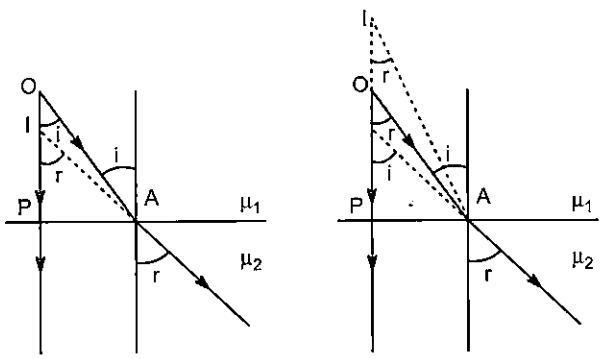


Fig. 1.104

If i and r are small,

$$\sin i \approx \tan i = \frac{PA}{PO}$$

and $$\sin r \approx \tan r = \frac{PA}{PI}$$

Thus, $$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

$$= \left(\frac{PA}{PO}\right)\cdot\left(\frac{PI}{PA}\right) = \frac{PI}{PO}.$$

Suppose medium 2 is air and an observer looks at the image from this medium [Fig. 1.104 (a)]. The real depth of the object inside medium 1 is PO whereas the depth as it appears to the observer is PI referred as apparent depth. Writing $\mu_2 = 1$ and $\mu_1 = \mu$.

$$\frac{1}{\mu} = \frac{\text{apparent depth}}{\text{real depth}}$$

or, $$\mu = \frac{\text{real depth.}}{\text{apparent depth}}$$

or apparent depth $= t/\mu$

The image shifts closer to eye by an amount $OI = PO - PI$

$$= \left(\frac{PO - PI}{PO}\right)PO = \left(1 - \frac{PI}{PO}\right)PO$$

or, $$\Delta t = \left(1 - \frac{1}{\mu}\right)t,$$

Similarly if observer in $\mu_2 = \mu$

and object in medium $\mu_1 = 1$

$$\frac{\mu}{1} = \frac{\text{apparent depth}}{\text{real depth}}$$

or apparent depth $= \mu$(real depth)

$$= \mu t$$

Case (i) Let object distance is u and image distance v. Light travels from n_1 to n_2. All the distance are measured from surface. Distance measured in direction of light is positive and apposite to it negative.

We can write above result as

$$\frac{n_2}{v} - \frac{n_1}{u} = 0$$

Let a coin lie at the bottom of a pool, at a depth t.

$$u = -t, \qquad n_2 = n_{air} \simeq n$$

$$n_1 = n_{water} = n$$

$$\frac{1}{v} - \frac{n}{(-t)} = 0$$

$$v = -\frac{t}{n}$$

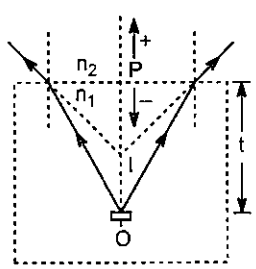


Fig. 1.105

The negative sign shows that image is formed at same side as the object. The swimming pools appear more shallow than they actually are because when the light rays begin from the bottom of the pool ($n_1 = 1.33$) and enter air ($n_2 = 1$), they are bent away from the normal.

Case (ii) Similarly if we have seen an object in air from water, it looks farther away, because when rays from an object change the medium they bend toward normal.

$$\frac{n}{v} - \frac{1}{(-t)} = 0$$

$$v = -nt$$

The negative sign shows that image lies on the side of the object.

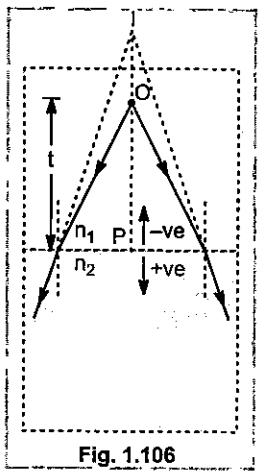


Fig. 1.106

❒ In general,

$$\frac{n_2}{v} - \frac{n_1}{(-u)} = 0 \text{ [sign substituted]}$$

$$v = -\frac{n_2}{n_1}u$$

When $n_2 > n_1$, object is in rarer medium, viewer in denser medium, $v > u$.

When $n_2 < n_1$, object is in denser medium, viewer in rarer medium, $v < u$

❒ For an object in water, viewer in air, the shift in the position of object,

Shift = real depth – apparent depth $= t - t/n$

Similarly in case (ii), Shift $= (nt - t)$

❒ When an object is placed near a glass slab and viewed from the other side of the slab, the object seems to be shifted by $(t - t/n)$ in the direction of light.

❒ When a glass slab comes in the way of converging rays, the point of convergence is shifted by $(t - t/n)$ in the direction of light, *i.e.*, away from the glass slab.

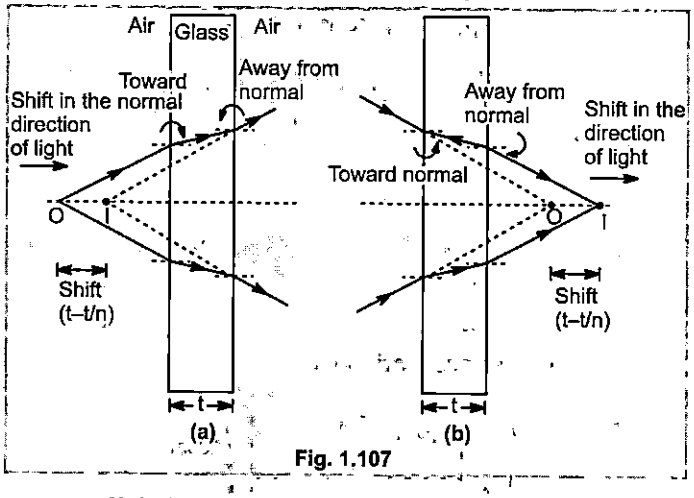


Fig. 1.107

Condition (1)

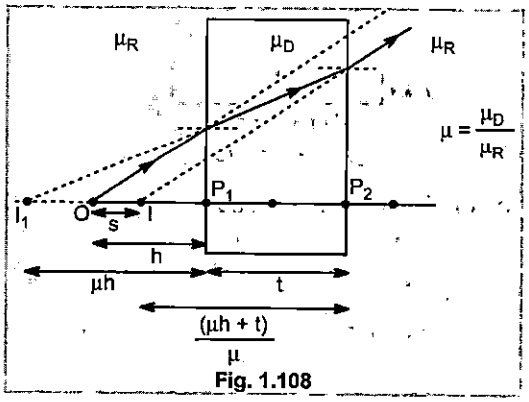


Fig. 1.108

Shift, $h + t - \dfrac{(\mu h + t)}{\mu}$

$$s = h + t - h - \frac{t}{\mu};$$

$$s = t\left(1 - \frac{1}{\mu}\right)$$

Condition (2)

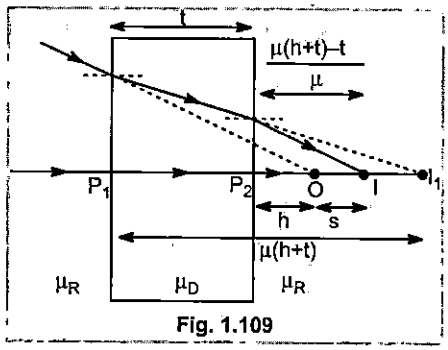


Fig. 1.109

Shift, $s = \dfrac{\mu(h+t) - t}{\mu} - h$

$$s = h + t - \frac{t}{\mu} - h$$

$$s = t\left(1 - \frac{1}{\mu}\right)$$

In both the conditions (1) and (2) shift is same and in the direction of incident ray. Shift does not depend on 'h'. It only depends on 't' and μ.

Condition (3)

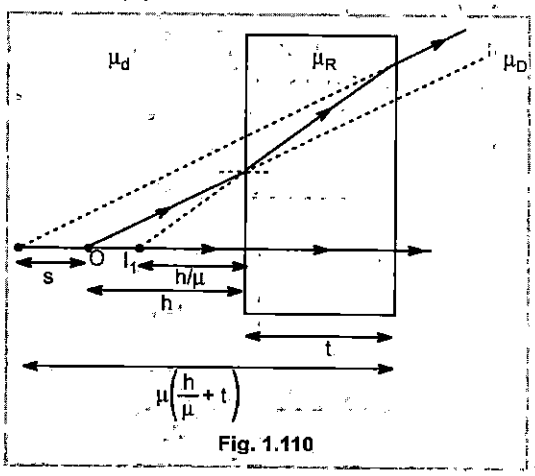


Fig. 1.110

Shift, $s = \mu\left(\dfrac{h}{\mu} + t\right) - t - h$

$$s = h + \mu t - t - h;$$

$$s = (\mu - 1)t$$

Condition (4)

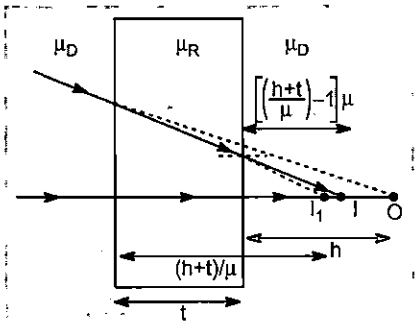


Fig. 1.111

Shift, $s = h - \left\{\frac{h+t}{\mu} - t\right\}\mu$

$$s = h - h - t + \mu t; s = (\mu - 1)t$$

In both conditions (3) and (4), shift is same and in the opposite direction of incident ray. Again shift does not depend on 'h'. It only depends on 't' and μ.

If there are number of media of different depths, one over other and their boundaries is/are simple plane and parallel to each other;

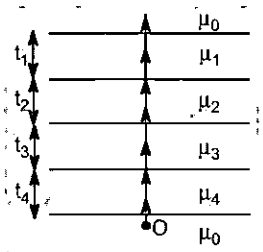


Fig. 1.112

Total Shift

$$s = s_1 + s_2 + s_3 + s_4 + \ldots$$

$$s = t_1\left(1 - \frac{\mu_0}{\mu_1}\right) + t_2\left(1 - \frac{\mu_0}{\mu_2}\right) + t_3\left(1 - \frac{\mu_0}{\mu_3}\right) + t_4\left(1 - \frac{\mu_0}{\mu_4}\right) + \ldots$$

If combination of media are replaced by a single medium of same outer physical size and shift produced is same as above. Then R.I. of that medium is equivalent refractive index of combination.

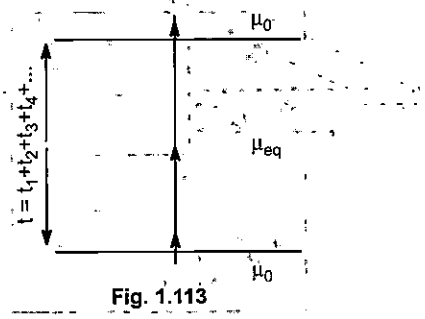


Fig. 1.113

$$s = (t_1 + t_2 + t_3 + t_4 + \ldots)\left(1 - \frac{\mu_0}{\mu_{eq}}\right)$$

$$(t_1 + t_2 + t_3 + t_4 + \ldots)\left(1 - \frac{\mu_0}{\mu_{eq}}\right)$$

$$= t_1\left(1 - \frac{\mu_0}{\mu_1}\right) + t_2\left(1 - \frac{\mu_0}{\mu_2}\right) + t_3\left(1 - \frac{\mu_0}{\mu_3}\right) + \ldots$$

$$\mu_{eq} = \frac{t_1 + t_2 + t_3 + \ldots}{\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \frac{t_3}{\mu_3} + \ldots} = \frac{\Sigma(t)}{\Sigma(t/\mu)}$$

If $t_1 = t_2 = t_3 = \ldots$

$$\frac{1}{\mu_{eq}} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3} + \ldots$$

PARTIAL REFLECTION AND PARTIAL REFRACTION AT THE BOUNDARY OF TWO MEDIA

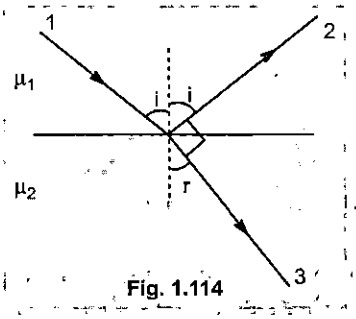


Fig. 1.114

If partially reflected ray (1) and partially refracted ray (2) are mutually perpendicular (this is possible only for unique value of angle of incidence 'i')

$$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90° - i)} = \frac{\sin i}{\cos i}$$

$$\tan i = \frac{\mu_2}{\mu_1}; i = \tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

$$i = \tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

This angle 'i' is called "Brewster's angle" which will be further discussed in polarization.

Illustration 8:

Case (i) Interface stationary

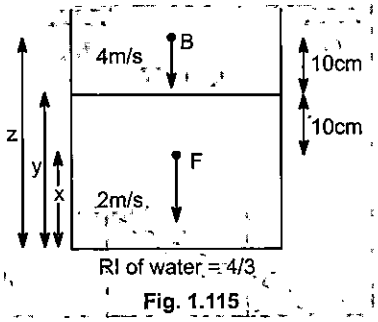


Fig. 1.115

Position of bird as seen by fish $X_{B/F} = \mu(y - z) + (y - x)$

$$\frac{d}{dt}(X_{B/F}) = \dot{X}_{B/F}$$

$\dot{X}_{B/F}$ represents derivative of $X_{B/F}$ wrt time.

$$\dot{X}_{B/F} = \mu[\dot{z} - \dot{y}] + [\dot{y} - \dot{x}]$$

$$= 4/3[-4] + (-C - 2)$$
$$= \frac{-10}{3}\text{ m/s}$$

Note that $\dot{z} = -4\text{m/s}$ as z is decreasing

$\dot{y} = 0$ as y is constant

$\dot{x} = -z$ m/s as x is decreasing.

Method 2: Set reference point on the surface

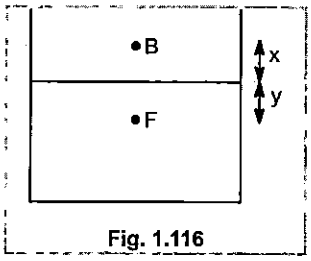


Fig. 1.116

$$X_{B/F} = \mu x + y$$
$$\dot{X}_{B/F} = \mu \dot{x} + \dot{y}$$
$$= \frac{4}{3}[-4] + \dot{z}$$
$$= \frac{-10}{3}\text{m/s}$$

Note that $\dot{x}$ have refers to speed of bird relative to surface.

$\dot{x} < 0$ if x decreases

$\dot{y} > 0$ is y increases

Case (ii) Suppose interface is also moving find speed of bird as seen by fish.

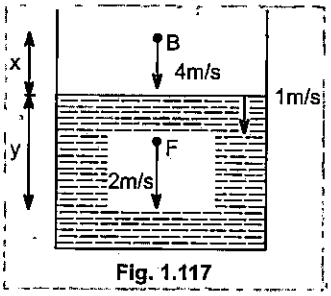


Fig. 1.117

$$\dot{X}_{B/F} = \mu x + y$$
$$\dot{X}_{B/F} = \mu \dot{x} + \dot{y}$$
$$= \frac{4}{3}[-3] + [+1]$$
$$= -3\text{ m/s}$$

Here $\dot{x} = -3$ m/s as relative to surface bird is coming near to surface.

Similarly $\dot{X}_{B/F} = \left(\frac{y}{\mu}\right) + x$

$$\dot{X}_{F/B} = \frac{\dot{y}}{\mu} + \dot{x}$$
$$= \frac{1}{4/3} + (-3)$$
$$= -9/4\text{m/s}$$

Case (iii) Suppose base of container is silvered. Find speed of bird as seen by fish in mirror and speed of fish as seen by bird in mirror.

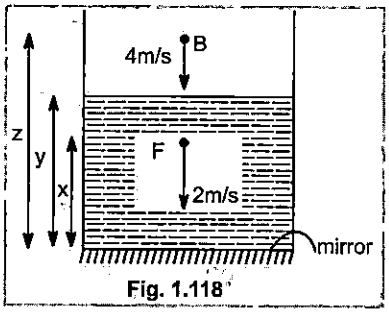


Fig. 1.118

$$X_{B/F} = \underbrace{\mu(z-y)}_{\text{from mirror}} + y + x$$

$$\dot{X}_{B/F} = \mu(\dot{z} - \dot{y}) + \dot{y} + \dot{x}$$
$$= \frac{4}{3}[(-4) - 0] + 0 + (-2)$$
$$= \frac{-16}{3} - 2$$
$$= \frac{-22}{3}\text{ m/s}$$

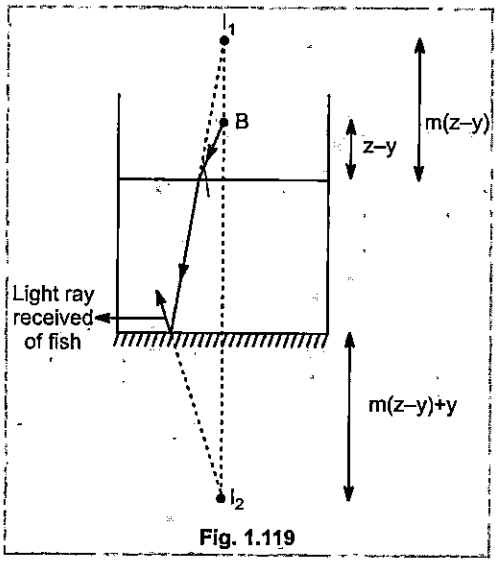


Fig. 1.119

Example 56

See the figure

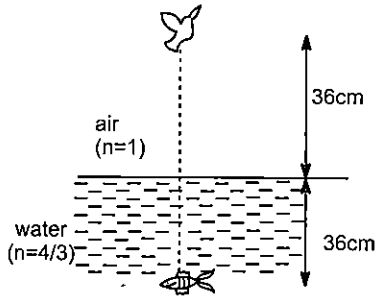


Fig. 1E.56

(i) At what distance will the bird appear to the fish.

(ii) At what distance will the fish appear to the bird.

Solution: (i) For fish: $d_B = 36 + 48 = 84$ cm

$d_B = 36 + 48 = 84$cm

(ii) For bird: $d_F = 27 + 36 = 63$ cm

Example 57

A concave mirror is placed inside water with its shining surface upwards and principal axis of concave mirror. Find the position of final image.

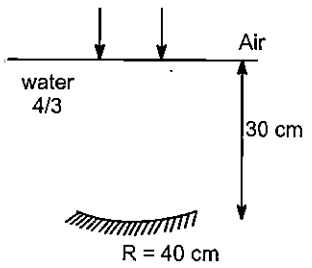


Fig. 1E.57 (a)

Solution: The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞. Its image A [in Fig 1E.57 (b)] will be formed at focus which is 20 cm from the mirror. Now for the interface between water and air, $d = 10$ cm.

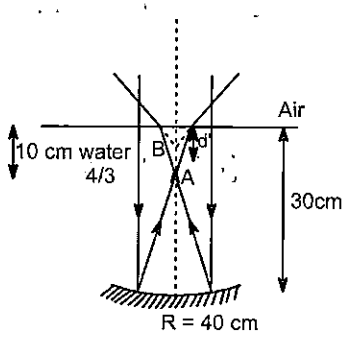
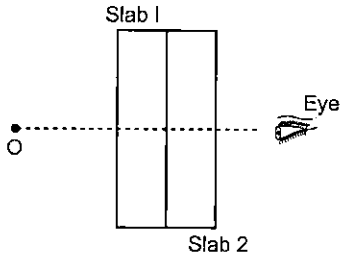


Fig. 1E.57 (b)

$$\therefore \quad d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5\,\text{cm}.$$

Refraction Across Multiple Slabs:

In Fig.1.123 an object is placed in front of two slabs in contact. The thickness and refractive indices of the slabs are t_1, μ_a and t_2, μ_b respectively. Where will the final image of the object appear to be ?



A object placed in front of two glass slabs in contact

Fig. 1.123

A light ray emerging from O now refracts at three surfaces. The first is between air and μ_a, the second between μ_a and μ_b while the third is between μ_b and air. Let us solve the problem taking one step at a time.

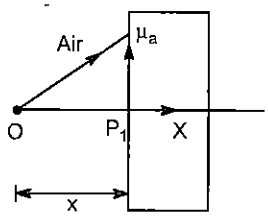


Fig. 1.124

Ist interface

Here, $\mu_1 = 1; \quad \mu_2 = \mu_a$

$d_1 = -x, \quad d_2 = ?$

but $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$

Therefore, the image distance $d_2 = -\mu_a x$ ...(1)

2nd interface

Here,

$\mu_1 = \mu_a,$

$\mu_2 = \mu_b$

$d_1 = -(\mu_a x + t_1);$

$d_2 = ?$

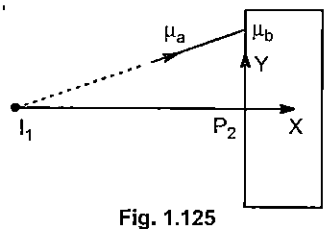


Fig. 1.125

Since $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$

the image distance $d_2 = -\mu_b\left(x + \dfrac{t_1}{\mu_a}\right)$ ...(2)

3rd Interface

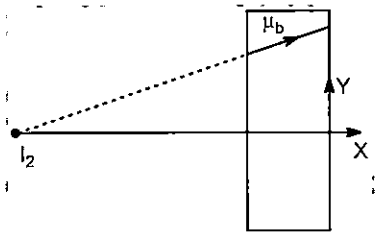


Fig. 1.126

Here, $\mu_1 = \mu_b, \quad \mu_2 = 1$

$$d_1 = -\mu_b\left(x + \frac{t_1}{\mu_a}\right) + t_2, d_2 = ?$$

and the final image distance from the 3rd interface is

$$d_2 = -\left[x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_a}\right]$$

Therefore, the net shift in the position of the image is

$$s = -\left(x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b}\right) - [-(x + t_1 + t_2)]$$

or $$s = t_1\left(1 - \frac{1}{\mu_a}\right) + t_2\left(1 - \frac{1}{\mu_b}\right)$$

Looking at the above result we realize that the net shift in the position of the image is simply the sum of the individual shifts at each of the slabs if they were independently placed in air.

Example 58

A tank contains three layers of immiscible liquids [Fig. 1E.58 (a)]. The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is an oil with refractive index 3/2 and thickness 9 cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.

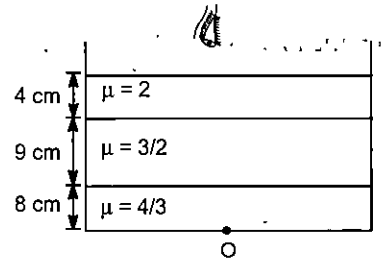


Fig. 1E.58 (a)

Solution: Case-1: Method of Interfaces:

A ray of light from the object undergoes refraction at three interfaces. (1) Water-oil, (2) Oil-glycerine (3) Glycerine-air. The co-ordinate system for each of the interfaces is shown in Fig. 1E.58 (b).

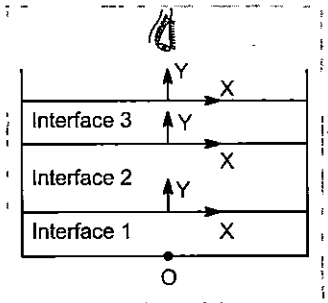


Fig. 1E.58 (b)

Water-Oil Interface:

$$d_1 = -8\text{cm}, \quad \mu_1 = 4/3, \quad \mu_2 = 1.5$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$$

or $$d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get } d_2 = -9\text{cm}$$

Oil-Glycerine Interface:

$$d_1 = -(9+9) = -18\text{cm}, \mu_1 = 1\cdot 5, \mu_2 = 2$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1 \text{ we get } d_2 = -24\text{cm}$$

Glycerine-Air interface:

$$d_1 = -(4+24) = -28\text{cm}, \mu_1 = 2, \mu_2 = 1$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \quad \text{or} \quad d_2 = \frac{\mu_2}{\mu_1} d_1$$

we get $d_2 = -14$ cm

Thus the final image is 14 cm below the clycerine-air interface.

GLASS SLAB

Single Glass Slab in Air

(i) When a glass slab of thickness and refractive index μ is placed in the path of a convergent beam as shown in the Fig. 1.127 then the point of convergence is shifted by

$$s = t\left(1 - \frac{1}{\mu}\right)$$

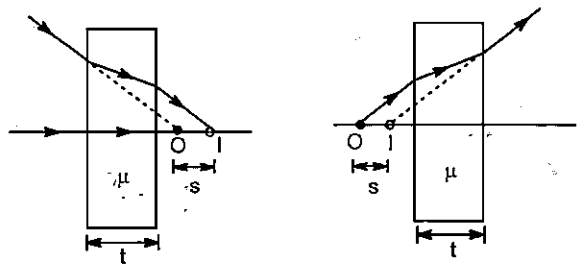


Fig. 1.127

(ii) When the same glass slab is placed in the path of a diverging beam, the point of divergence is shifted by

$$s = t\left(1 + \frac{1}{\mu}\right)$$

Important *It is important to note that the shift (s) is always on the direction of light.*

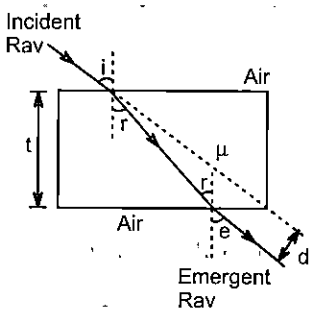
If the slab is made of air and the surrounding medium is of refractive index μ, then the apparent shift would be

$$s = t(\mu - 1)$$

Concept: *The refracting surfaces of a glass slab are parallel to each other. When a light ray travels through a glass slab, it is refracted twice at the two parallel forces and finally emerges out parallel to its incident direction. The light ray undergoes zero deviation, δ = 0.*

Angle of emergence = Angle of incidence

$$e = i$$



A glass slab produces lateral displacement

Fig. 1.128

The lateral displacement of the ray is the perpendicular distance between the incident and the emergent ray and is given by $d = \left(\frac{t}{\cos r}\right)\sin(i - r)$

Example 59

A convergent beam is incident on two slabs placed in contact as shown in Fig. 1E.59 (a). Where will the rays finally converge?

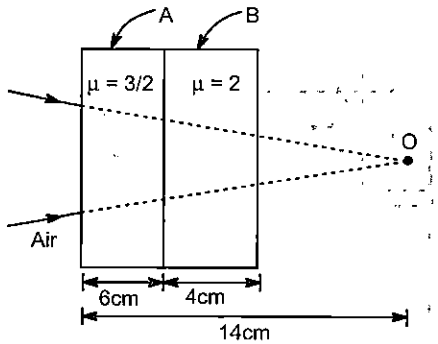


Fig. 1E.59 (a)

Solution: A ray of light from the object undergoes refraction at three interfaces. (1) Air-Medium *A*, (2) Medium *A*-Medium *B*, (3) Medium *B*-Air. The co-ordinate system for each of the interfaces is shown in Fig. 1E.59 (b).

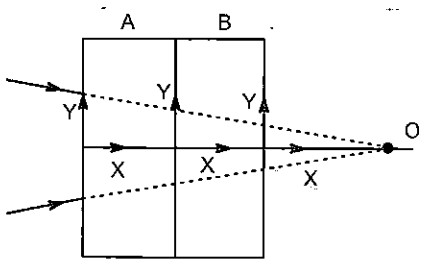


Fig. 1E.59 (b)

Air-Medium A Interface:

$$d_1 = +14\text{cm}, \quad \mu_1 = 1, \mu_2 = 1.5$$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$

or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get $d_2 = +21$ cm

Medium A-Medium B Interface:

$$d_1 = (21 - 6) = 15\text{cm}, \quad \mu_1 = 1.5, \mu_2 = 2$$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$

or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get $d_2 = +20\,\text{cm}$

Medium B- Air Interface:

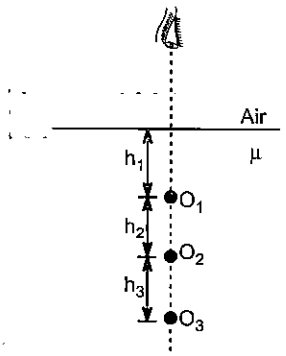
$$d_1 = (20 - 4) = +16\text{cm}, \quad \mu_1 = 2, \mu_2 = 1$$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$

or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get $d_2 = +8\,\text{cm}$

Thus the final image is 8cm in front of the medium *B*-air interface.

Concept: *The Fig. 1.129 shows three objects O_1, O_2 and O_3 located on a vertical line in a liquid of refractive index μ. These objects are observed from air a long the vertical line. The distance between the images of O_1 and O_2 is $\frac{h_2}{\mu}$ and the distance between the images of O_1 and O_3 is $\frac{h_2 + h_3}{\mu}$.*



Three points in a denser medium are observed from a rarer medium.

Fig. 1.129

Note: That the distance between any two points in a denser medium is reduced by a factor μ.

Example 60

The image of an object kept at a distance 30 cm in front of a concave mirror is found to coincide with itself. If a glass slab (μ = 1.5) of thickness 3 cm is introduced between the mirror and the object, then

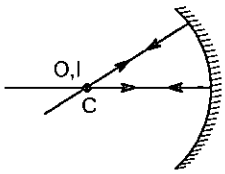


Fig. 1E.60 (a)

(i) Identify, in which direction the mirror should be displaced so that the final image may again coincide with the object itself.

(ii) Find the magnitude of displacement.

Solution: (i) Since the apparent shift occurs in the direction of incident light, therefore, the mirror should be displaced away from the objects.

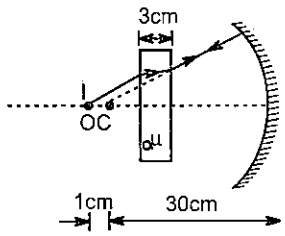
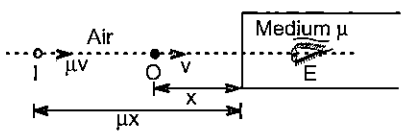


Fig. 1E.60 (b)

(ii) The magnitude of displacement is equal to the apparent shift, *i.e.*,

$$s = t\left(1-\frac{1}{\mu}\right) = 3\left(1-\frac{1}{3/2}\right) = 1 \text{ cm}$$

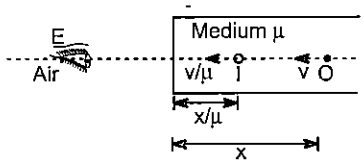
Concept: *The Fig. 1.130 (a) shows an object O moving toward the plane boundary of a denser medium. To an observer in the denser medium the object appears to be more distant but moving faster. If the speed of the object is v, then the speed of the image will be μv.*



The image of an object in the rarer medium appears to move faster

Fig. 1.130 (a)

The Fig. 1.130 (b) shows an object O moving toward the plane boundary of a rarer medium. To an observer in the rarer medium the object appears to be closer but moving slowly. If the speed of the object is v, then the speed of the image will be $\frac{v}{\mu}$.



The image of an object in the denser medium appears to move slower.

Fig. 1.130 (b)

Example 61

A bird in air is diving vertically over a tank with speed 5 cm/s base of tank is silvered. A fish in the tank is rising upward along the same line with speed 2 cm/s. Water level is falling at rate of 2 cm/s. [Take μ water = 4/3]

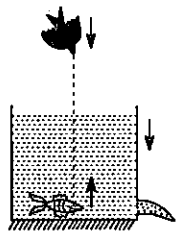


Fig. 1E.61 (a)

	Column-I		Column-II (in cm/s)
(a)	Speed of the image of fish as seen by the bird directly	(q)	8
(b)	Speed of the image of fish formed after reflection in the mirror as seen by the bird	(q)	6
(c)	Speed of image of bird relative to the fish looking upwards	(r)	3
(d)	Speed of image of bird relative to the fish looking downwards in the mirror.	(s)	4

Solution: $\frac{dy_3}{dt} = -5$

$\frac{dy_1}{dt} = 2$

$\frac{dy_2}{dt} = -2$

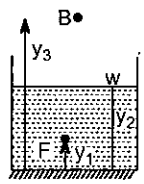


Fig. 1E.61 (b)

(a) Distance of image of fish as seen by bird directly

$$x_1 = (y_3 - y_2) + \left(\frac{y_2 - y_1}{\mu}\right)$$

Speed $$\frac{dx_1}{dt} = \frac{dy_3}{dt} + \frac{dy_2}{dt}\left(\frac{1}{\mu} - 1\right) - \frac{dy_1}{dt}\left(\frac{1}{\mu}\right)$$

$$= -5 - 2\left(\frac{3}{4} - 1\right) - (2)\frac{3}{4} = -6\,\text{cm/s}$$

(b) Distance of image of fish as seen by bird after reflection

$$x_2(y_3 - y_2) + \left(\frac{y_2 + y_1}{\mu}\right)$$

Speed $$\frac{dx_2}{dt} = \frac{dy_3}{dt} + \left(\frac{dy_2}{dt}\right)\left(\frac{1}{\mu} - 1\right) + \frac{dy_1}{dt}\left(\frac{1}{\mu}\right)$$

$$= -5 + (-2)\left(\frac{3}{4} - 1\right) + 2\left(\frac{3}{4}\right)$$

$$= -5 - \frac{3}{2} + 2 + \frac{3}{2} = -3\,\text{cm/s}$$

(c) Distance of bird as seen by fish directly

$$x_3 = (y_2 - y_1) + \mu(y_3 - y_2)$$

Speed $$\left(\frac{dx_3}{dt}\right) = \mu\frac{dy_3}{dt} + \frac{dy_2}{dt}(1 - \mu) - \frac{dy_1}{dt}$$

$$= \frac{4}{3}(-5) + (-2)\left(-\frac{1}{3}\right) - 2 = -8\,\text{cm/s}$$

(d) Distance of bird as seen by fish after reflection

$$x_4 = (y_3 - y_2)\mu + y_2 + y_1$$

Speed $$\frac{dx_4}{dt} = \mu\frac{dy_3}{dt} + \frac{dy_2}{dt}(1 - \mu) + \frac{dy_1}{dt}$$

$$= -\frac{20}{3} + (-2)\left(-\frac{1}{3}\right) + 2 = -4\,\text{cm/s}$$

Measurement of Refractive Index of a Liquid by a Travelling Microscope

Using the fact that $\mu = \dfrac{\text{real depth}}{\text{apparent depth}}$

We may determine the refractive index of a glass slab.

The travelling microscope is first focused on a fine mark on a piece of paper and the reading on the scale is noted as Y_1. Then, the glass slab is placed on the paper.

The microscope is now shifted up and is focused again on the mark so that it is distinctly visible. The reading of the microscope is noted as Y_2.

Now, a little lycopodium powder is scattered on the upper surface of the glass block. The microscope is focused on the powder and third the reading of the scale is noted as Y_3.

The real thickness of the glass slab is $Y_3 - Y_1$ and the *apparent thickness* of the glass slab is $Y_3 - Y_2$. Thus,

$$m = \frac{Y_3 - Y_1}{Y_3 - Y_2}$$

The same method can also be used to find the refractive index of a liquid. Any mark on the bottom of a glass vessel is first focused and then liquid is poured into it and the same mark is again focused. *Finally,* a little lycopodium powder is sprinkled on the surface of the liquid and is focused as before.

TOTAL INTERNAL REFLECTION AND CRITICAL ANGLE

A material with a higher index of refraction than the other is known as **optically denser,** the medium with lower index of refraction is **optically rarer,** quite independent of the real relative density of the materials in kg/m^3.

When light passes from an optically denser to rarer medium, say from water to air, an interesting effect occurs. Let the denser medium have refractive index n_d (here water) and the rarer medium, $n_r; n_d > n_r$

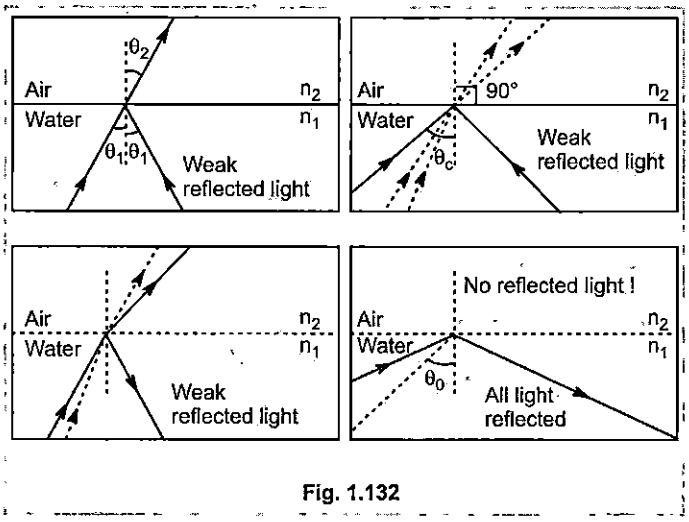


Fig. 1.132

From Snell's law, $n_d \sin\theta_1 = n_r \sin\theta_2$

Since the light is travelling from denser medium to rarer medium, the refracted ray is bent away from the normal line, that is, $n_d > n_r$; we must have $\theta_2 > \theta_1$.

When the angle of incidence θ_1 is increased, the angle of refraction θ_2 also increases. If we keep increasing θ_1, eventually the angle of refraction θ_2 becomes 90°. The angle of incidence for which the angle of refraction is 90° is called the **critical angle** θ_c, the refracted ray grazes the surface.

When the angle of incidence is equal to the critical angle, Snell's law yields

$$n_d \sin C = n_r \sin 90°$$

$$\sin C = \frac{n_r}{n_d}$$

If, *e.g.*, $n_d = 2$ while $n_1 = 1$, from Snell's law $2\sin\theta_i = \sin\theta_r$ and provided $\sin\theta_1 \le 1/2$, the equation can be satisfied. But for incident angles greater than this critical angle $C = 30°$, the $\sin\theta_r$ would have to be greater than 1, and that cannot be; hence there is no transmitted beam.

When the incidence angle is greater than the critical angle, 100% of the energy in the incident beam is reflected back into the incident medium and so the effect is called **total internal reflection.**

At the critical angle, the refracted light propagates along the interface, the wave is limited to the boundary, its energy flows back and forth across the interface with no average transmission into the second medium.

When light is incident on the interface between two media, the luminous energy splits into two parts: one part is reflected, while the other part penetrates through the interface into the second medium. Considering the example of light propagation from air to glass, *i.e.*, towards the optically denser medium, we learned that the fraction of reflected energy depends on the angle of incidence. In this case, the fraction of reflected energy sharply increases with the angle of incidence. However, even at very large angles of incidence close to 90°, when the light ray almost slips along the interface, a part of the luminous energy still goes to the second medium.

If light propagating in a medium is incident on the interface between this medium and an optically less dense medium, *i.e.*, one having a smaller absolute refractive index. In this case, the fraction of the reflected energy increases with the angle of incidence, but this increase obeys another law: *starting from a certain value of the angle of incidence, the entire luminous energy is reflected from the interface.* This phenomenon is known as the *total internal reflection.*

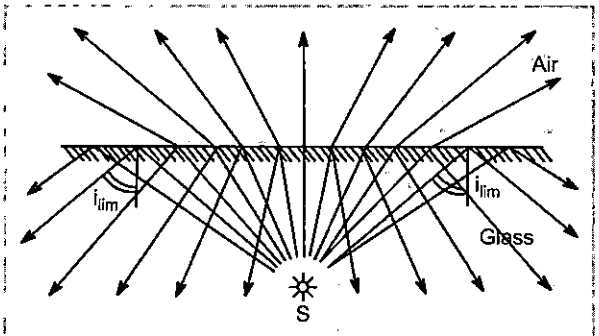


Fig. 1.133: Total internal reflection : the thickness of rays corresponds to the fraction of the reflected luminous energy or that transmitted through the interface

Let us consider again the incidence of light on the glass-air interface. Let a light ray be incident from glass on the interface at different angles (Fig. 1.133). If we measure the fraction of the reflected luminous energy and the fraction of the luminous energy transmitted through the interface, we obtain the values compiled in table 1.2. (the refractive index of glass is $n = 1.555$). The value i_{cr} of the angle of incidence starting from which the entire luminous energy is reflected from the interface is called the *critical angle of total internal reflection.* For the grade of glass for which table 1.2 is complied ($n = 1.555$), the critical angle is approximately equal to 40°.

Table 1.2: Fraction of Reflected Energy for Various Angles of Incidence of Light Propagating from Glass to Air.

Angle of incidence *i*	0°	10°	20°	30°	35°	38°	39°	39°30'	40°	50°	60°	70°	80°
Angle of refraction *r*	0°	15°40'	32°	51°	63°	73°20'	79°	82°	90°	–	–	–	–
Fraction of reflected Energy (in %)	4.7	4.7	5.0	6.8	12	23	36	47	100	100	100	100	100

Total internal reflection can be observed at the boundaries of air bubbles in water. They shine since the solar rays incident on them are completely reflected, without penetrating the bubbles. This is especially noticeable for the air bubbles which are always present on pedicles and leaves of seaweed and which, when observed in solar rays, appear as if made of silver, *i.e.*, are a good reflector of light.

Total internal reflection is used in the construction of glass *reflecting and erecting prisms.* The operation of these prisms is explained by Fig. 1.134. The critical angle for a prism is 35-40° depending on the refractive index of a given grade of glass. Therefore, the angles of incidence and emergence of light rays can easily be selected for such prisms. Reflecting prisms successfully play the role of mirrors and are preferable since their reflecting properties remain unchanged, while metal mirrors grow dull with time because of the oxidation of metals. It should be noted that an erecting prism is simpler in construction than any erecting mirror system. Erecting prisms are used, in particular, in periscopes.

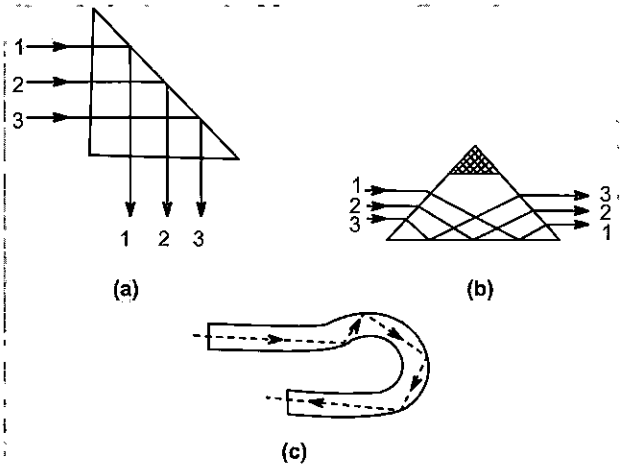


Fig. 1.134

Critical Angle and Colours

When the light is just emerging into the air from glass, or water near the critical position, it is observed to be coloured. This is because the critical angle values are different for the different colours. It was shown that ${}_a\mu_m = \frac{1}{\sin c}$, where c is the critical angle from the medium into the air.

But $\mu_{\text{Blue light}} > \mu_{\text{Red light}}$, and so sin c, and hence c, is smaller for blue light than for red light. Thus when the light, in passing out, is near the critical position, the blue portion reaches it first and passes along the surface whilst the red portion is being refracted into the air (see Fig. 1.135).

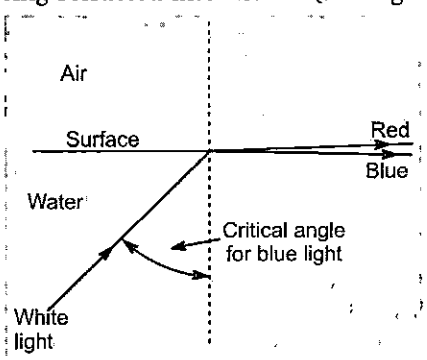


Fig. 1.135

This effect is observable for all the colours in the usual order violet to red.

SOME EXPLANATION OF SOME REFRACTION PHENOMENON

1. The altitude of a star or planet appears to be greater than it really is, for light from either is refracted in its path to the earth, owing to a variation in optical density of air layers. As the physical density of the air increases, or decreases, so its optical density increases, or decreases. The denser air being usually nearer the surface of the earth, the path of light when received there is more oblique than a direct line from the heavenly body to the earth (as shown in Fig. 1.136).

2. The apparent flattening of the sun and moon when near the horizon is due to refraction. Rays from the lower part of the sun or moon travel at greater length through the earth's atmosphere than rays from the higher part, and so are more refracted. Thus the lifting-up effect is greater for the lower part of the sun or moon than for the top. The extremities of the sides are equally affected, and so the moon appears elliptical instead of circular.

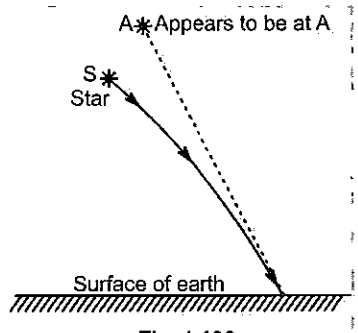


Fig. 1.136

3. Stars are a great distance away from the earth, and so a very small proportion of the light radiated by them reaches us. Light received from a star travels along paths which are very variable, with a consequent variation in time taken. Thus the light received from a star by an observer on the earth is not uniform in intensity; at one instant little light is received, at another, light is received by more than one path, and so we get a twinkling effect. Planets are of greater angular size than stars, *i. e.*, they subtend at the eye a much larger angle than do stars, thus more light is received at any instant from a planet by an observer on the earth. Refraction effects are therefore inappreciable and so planets do not twinkle. This is borne out by the fact that stars, as seen in large telescopes, do not twinkle, for the amount of light collected by the telescope and then viewed is approximately constant.

4. An observer under water sees (a) objects outside as if they were in a cone of semi-vertical angle $48\frac{1}{2}°$, and lifted-up, and (*b*) objects in the water away from him by total internal reflection (see Fig. 1.137). If a glass of water

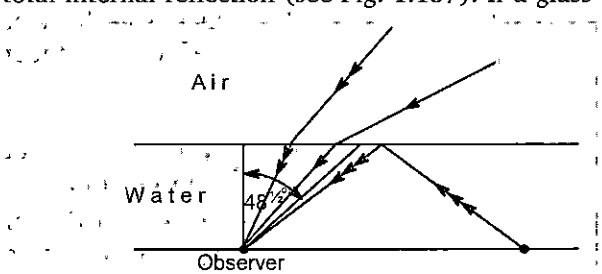


Fig. 1.137: Viewing objects from under water

be held in the air above the level of a bright light, the water surface, viewed from below, appears like one of mercury of silver, and a coin or spoon in the water can be seen in this surface by total internal reflection.

5. Hold an empty test-tube slant-wise in water and look at it from above (Fig. 1.138). Light travelling in water towards the test-tube at an angle of incidence greater than the critical angle cannot pass into the air in the test-tube and so is reflected upwards. The total reflection causes the test-tube to appear as if silver-coated. Pour water into the test-tube; the effect disappears. The bright metallic appearance of cracks in plate-glass windows when looked at obliquely is due to such total internal reflection in the glass at the cracks.

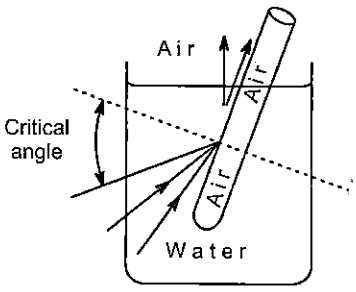


Fig. 1.138

6. Mirages: Pools of water appear to be on the surface of a tarred road or a stone promenade on a hot afternoon. This occurs when the sun is shining along the direction of either of them and is at a certain height. Rays of light, travelling through air layers of varying optical density, gradually change in direction. As a result they strike some layers at angles greater than the critical angle.

In hot countries the resultant effect is more common. The layers of air near the earth become very hot and rare, and, at the same time, less optically dense than the air above. Thus some rays of light from a distant object travel as shown in Fig. 1.139, giving an inverted image.

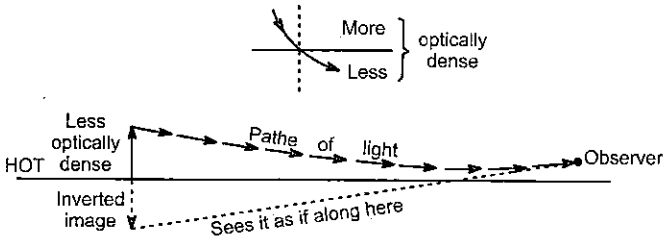


Fig. 1.139: Mirage produced in Hot Regions.

Some light, when at the critical angle position, travels along parallel to the ground and gives the appearance of bright pools of water. This optical illusion is called mirage.

7. In arctic regions, and over the sea in hot weather, mirages are seen in the sky. The air near the ground is cooler and more optically dense than the air above and the effect is as shown in Fig. 1.140.

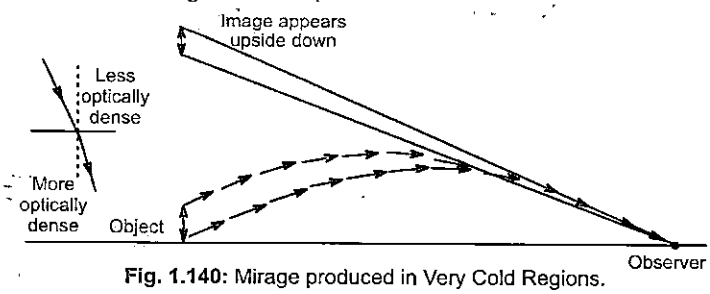


Fig. 1.140: Mirage produced in Very Cold Regions.

8. After passing through a plane-parallel (parallel-sided) plate (Fig. 1.141), the rays leave it at the same angle i at which they enter it. The plate displaces a ray of light parallel to itself over the distance

$$\Delta = d\sin i\left(1-\sqrt{\frac{1-\sin^2 i}{n^2-\sin i}}\right)$$

where d = thickness of the plate

i = angle of incidence of the rays

n = refractive index of the plate material relative to the surrounding medium.

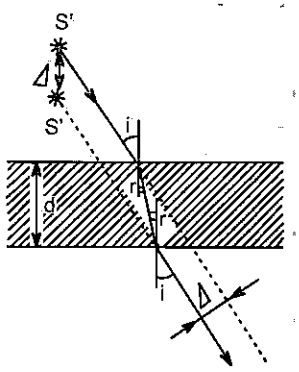


Fig. 1.141

The luminous point S of a source or an illuminated object seems to be brought closer to the surface of the plate by the distance

$$\Delta' = d\left(1-\sqrt{\frac{1-\sin^2 i}{n^2-\sin^2 i}}\right)$$

With normal incidence of the rays ($i = 0$), we have $\Delta = 0$, and $\Delta' = d(n-1)/n$.

Example 62

Investigate how a thin film of water on a glass surface affects the critical angle for total reflection. Take $n = 1.5$ for glass and $n = 1.33$ for water. (a) What is the critical angle for total internal reflection at the glass-water interface ? (b) Is there any range of incident angles that are greater than θ_C for glass to air refraction and for which light rays will leave the glass and the water and pass into the air ?

Solution: (a) Critical angle for glass-water interface,

$$\theta_C^{g,w} = \sin^{-1}\left(\frac{n_w}{n_g}\right) = \sin^{-1}\left(\frac{1.33}{1.5}\right) = 62.5°$$

(b) Critical angle for glass-air interface,

$$\theta_C^{g,a} = \sin^{-1}\left(\frac{n_a}{n_g}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$$

and for water-air interface,

$$\theta_C^{w,a} = \sin^{-1}\left(\frac{n_a}{n_w}\right) = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.8°$$

Suppose a ray in the glass strikes the glass-water surface with an angle of incidence $\theta^g < \theta_C^{g,w}$, *i.e.*, angle of incidence is less than critical angle for this interface. Light refracts into water with an angle of refraction θ^w such that, from Snell's law,

$$\sin\theta^w = \frac{1.5}{1.33}\sin\theta^g$$

This ray will strike the water-air surface with the angle of incidence θ^w. If $\theta^w > \theta_C^{w,a}$ the ray will reflect at water-air interface. If $\theta_C^{g,w} > \theta^g > \theta_C^{g,a}$ we might think that light will refract into air.

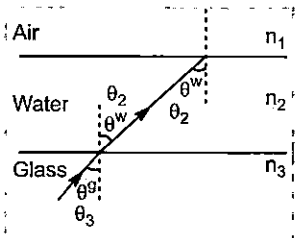


Fig. 1E.62

For $\theta^g > \theta_C^{g,a}$, we have

$$\sin\theta^w = \frac{1.5}{1.33}\sin\theta^g > \frac{1.5}{1.33}\sin\theta_C^{g,a}$$

$$= \frac{1.5}{1.33}\times\frac{1}{1.5} = \frac{1}{1.33}$$

$$\theta^w > \sin^{-1}\left(\frac{1}{1.33}\right) = 48.8° = \theta_C^{w,a}$$

Thus the light is totally reflected at the water-air surface.

We can generalise this result. We consider three media such that

$$n_3 > n_2 > n_1$$

$$\sin\theta_C^{3,2} = \frac{n_2}{n_3},\ \sin\theta_C^{3,1} = \frac{n_1}{n_3}$$

Let $\theta_C^{3,1} < \theta_3 < \theta_C^{3,2}$

The ray from medium 3 penetrates into medium 2 and strikes the interface of medium 2 and 1 with the angle of incidence θ_2. Now

$$\sin\theta_2 = \frac{n_3}{n_2}\sin\theta_3 > \frac{n_3}{n_2}\sin\theta_C^{3,1}$$

$$= \frac{n_3 n_1}{n_2 n_3} = \frac{n_1}{n_2} \text{ as } \sin\theta_C^{2,1} = \frac{n_1}{n_2}$$

Hence $\theta_2 > \theta_C^{2,1}$ and so the ray is totally reflected at the interface 2-1. No light will penetrate into 1.

Example 63

A light ray enters a glass slab at an angle of incidence θ_1. What can be the minimum value for the index of refraction of the glass if the ray does not emerge at face BC, irrespective of angle θ_1 ?

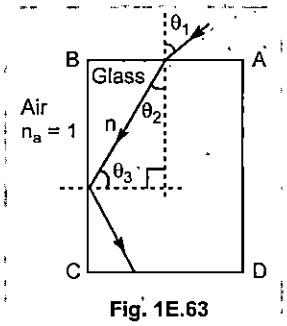


Fig. 1E.63

Solution: Let the refractive index of glass be n.

From Snell's law at face AB,

$$\sin\theta_1 = n\sin\theta_2$$

$$\sin\theta_2 = \frac{\sin\theta_1}{n} \quad \ldots (1)$$

The normals at faces AB and BC intersect at right angles; thus

$$\theta_2 + \theta_3 = 90° \quad \text{or} \quad \theta_3 = 90°-\theta_2 \quad \ldots (2)$$

If the ray has to suffer total internal reflection at face BC, it must be incident at least at critical angle $\theta_3 = C$, where

$$\sin\theta_3 = \frac{1}{n} \quad \ldots (3)$$

From eqns. (2) and (3)

$$\sin(90°-\theta_2) = \frac{1}{n} \text{ or } \cos\theta_2 = \frac{1}{n} \quad \ldots (4)$$

On squaring and adding eqns. (1) and (4), we get

$$\frac{\sin^2\theta_1}{n^2} + \frac{1}{n^2} = 1$$

or $$n = \sqrt{\sin^2\theta_1 + 1}$$

This is the required n for a given angle of incidence. But the problem states that irrespective of θ_1 the rays should reflect at face BC. In such cases we must work out what is the least angle of incidence at BC. This angle is taken to be the critical angle.

From eqn. (1), when θ_1, is maximum, θ_2 is also maximum.

From eqn. (2), θ_3 is minimum when θ_2 is maximum. Thus least angle θ_3 is formed at face BC when the ray is incident at maximum angle at face AB. The maximum angle at face AB is 90°.

Thus, $$n = \sqrt{(\sin 90°)^2 + 1} = \sqrt{2}$$

As $$\sin\theta_3 = \frac{1}{n}$$

if θ_3 is maximum, n must be least.

Any ray that is incident at face AB at an angle less than 90° will make an angle greater than θ_3 at face BC; hence it will reflect at face BC. If $n > \sqrt{2}$, as $\sin\theta_3 = \frac{1}{n}$, the angle θ_3 will decrease, *i.e.*, the required critical angle is decreased.

Method 2: The condition for total internal reflection at the second surface is

$$n\sin\theta_3 \geq 1$$

As $\theta_2 + \theta_3 = 90°$, so $n\cos\theta_2 \geq 1$

$$n^2(1-\sin^2\theta_2) \geq 1$$

From Snell's law, $$\sin\theta_2 = \frac{\sin\theta_1}{n}$$

$$n^2\left(1-\frac{\sin^2\theta_1}{n^2}\right)\geq 1$$

$$n^2-\sin^2\theta_1\geq 1$$

$$n^2\geq 1+\sin^2\theta_1$$

The largest value of n corresponds to $\theta_1 = 90°$.

Hence $n \geq \sqrt{2}$

Concept: *Deviation* (δ) - *The Fig. 1.83 shows a light ray travelling from a denser to rarer medium at an angle θ_1 smaller than the critical angle θ_c. The deviation δ of the light ray is given by $\delta = \theta_2 - \theta_1$. Since $\mu_D \sin\theta_1 = \mu_R \sin\theta_2$, therefore,*

$$\sin\theta_2 = \frac{\mu_D}{\mu_R}\sin\theta_1 = \mu\sin\theta_1;$$

$$\theta_2 = \sin^{-1}(\mu\sin\theta_1)$$

$$\delta = \sin^{-1}(\mu\sin\theta_1) - \theta_1$$

This is a non-linear increasing equation. The maximum value of δ occurs when $\theta_1 = \theta_c$, and is equal to,

$$\delta_{max} = \frac{\pi}{2} - \theta_c$$

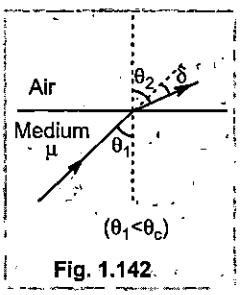


Fig. 1.142

If the light is incident at an angle $\theta_1 > \theta_c$, then the angle of deviation is given by,

$$\delta = \pi - 2\theta_1$$

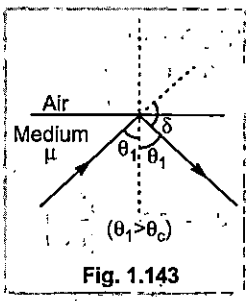


Fig. 1.143

This is a linearly decreasing function. The maximum value of δ occurs when $\theta_1 = \theta_c$ and is equal to,

$$\delta_{max} = \pi - 2\theta_c$$

The variation of δ with the angle of incidence θ_1 is plotted in Fig. 1.144.

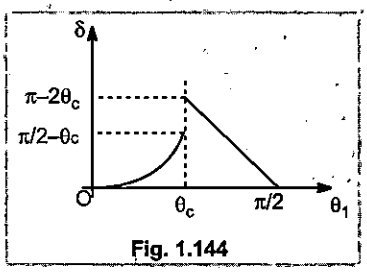


Fig. 1.144

Area of Spherical Cap

Area (A) of the spherical surface within a cone whose vertex is at the centre of sphere and semi-vertex angle θ is given by,

$$A = \int dA = 2\pi R^2\int_0^{\theta}\sin\theta d\theta$$

$$A = 2\pi R^2(1-\cos\theta)$$

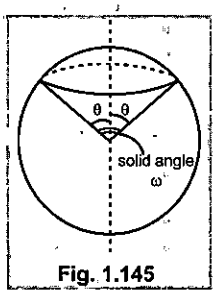


Fig. 1.145

Solid angle (ω) substended by this cap is given by,

$$\omega = \frac{A}{R^2} = 2\pi(1-\cos\theta)$$

Circle of Illumination

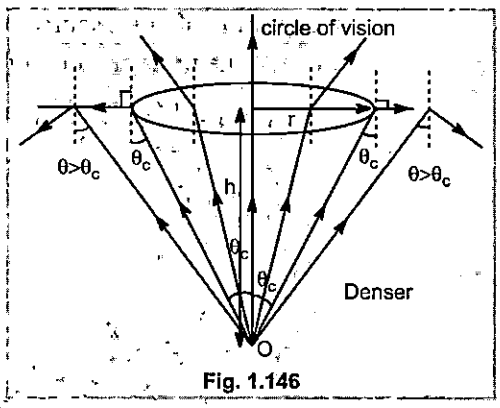


Fig. 1.146

$$\sin\theta_c = \frac{1}{\mu};\quad \tan\theta_c = \frac{1}{\sqrt{\mu^2-1}}$$

$$\mu = \frac{\mu_D}{\mu_R}$$

$$\tan\theta_c = \frac{r}{h}$$

$$r = h\tan\theta_c$$

$$r = \frac{h}{\sqrt{\mu^2-1}}$$

Observed at position 'O' in denser medium can see whole of world in rarer medium through this circle of illumination only. According to reversibility of path of light observer in rarer medium can see the object 'O' in denser medium only through rays coming out from this circle of illumination. If this circle of vision is covered with opaque

disc, then above mentioned observation will not be possible. Radius of circle of vision of red colour is maximum and that of voilet colour is minimum this is because μ depends on λ.

Only those light rays emerge from denser to rarer which are confined in the conc whose vertex is at 'O' and forms vertex angle $2\theta_c$ are transmitted.

If a point source of light is placed at position 'O' in denser medium then fraction of light energy produced by source emerges from boundary and it is independent of depth 'h'.

Because of total internal reflection, ray will be reflected back into denser medium if $\theta > \theta_c$, only that portion of incident light rays will emerge through boundary which are confined to cone of vertex angle $2\theta_c$ at the position of point source.

Fraction of light emerging from boundary

$$= \frac{\text{Area of spherical cap}}{\text{Area of whole of the sphere}}$$

$$= \frac{2\pi R^2(1-\cos\theta_c)}{4\pi R^2}$$

$$= \frac{2\pi(1-\cos\theta_c)}{4\pi}$$

$$= \frac{\text{solid angle at the position of point source due to cone}}{\text{total solid angle at the position point of source}}$$

$$= \frac{1}{2}(1-\cos\theta_c) = \frac{1}{2}\left(1-\sqrt{1-\sin^2\theta_c}\right)$$

$$= \frac{1}{2}\left[1-\sqrt{1-\left(\frac{1}{\mu}\right)^2}\right]$$

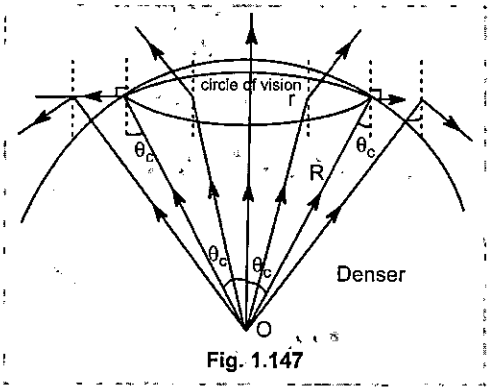


Fig. 1.147

% of light energy emerging from boundary,

$$\frac{1}{2}\left[1-\sqrt{1-\left(\frac{1}{\mu}\right)^2}\right]\times 100$$

Illustration 9: If monochromatic light is incident on a plane interface to two media (I) and (II) of refractive index μ_1 and μ_2 ($\mu_2 > \mu_1$) at an angle θ as shown in Fig. 1.148 (a) and θ is infinitely greater than the critical angle for this interface, then total internal reflection takes place. Now if transparent medium (III) of refractive index μ_3 is placed on the interface as shown in Fig. 1.148 (b), then for any value of μ_3 all light will ultimately be reflected back again into medium (II).

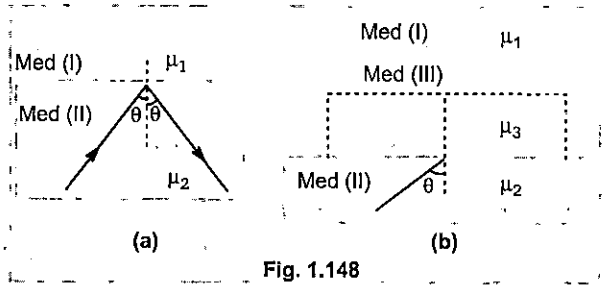


Fig. 1.148

Condition-I

$$(\mu_3 < \mu_1)$$

Initially $\theta > \theta_c$

$$\theta > \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$$

Now in the presence of III medium,

$$\theta_c' = \sin^{-1}\left(\frac{\mu_3}{\mu_2}\right)$$

Since $\mu_3 < \mu_1$,

So $\frac{\mu_3}{\mu_2} < \frac{\mu_1}{\mu_2}$

$$\sin^{-1}\left(\frac{\mu_3}{\mu_2}\right) < \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$$

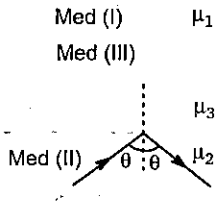


Fig. 1.149

Angle of incidence θ is greater than θ_c and θ_c is greater than θ_c', so θ is greater than θ_c'. Total internal reflection takes place at the interface of medium II and III and light ray will be reflected back to medium II.

Condition-II

$$(\mu_3 > \mu_1)$$

Initially $\theta > \theta_c$

$$\theta > \sin^{-1}\left[\frac{\mu_1}{\mu_2}\right]$$

$$\theta_c' = \sin^{-1}\left(\frac{\mu_3}{\mu_2}\right)$$

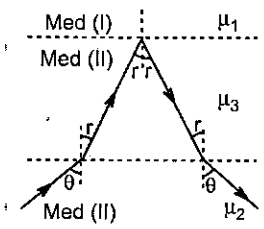


Fig. 1.150

Since $\mu_3 > \mu_1$,

So $\frac{\mu_3}{\mu_2} > \frac{\mu_1}{\mu_2}$; $\sin^{-1}\left(\frac{\mu_3}{\mu_2}\right) > \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$

$$\theta_c' > \theta_c$$

Since θ is infinitesimally greater than θ_c, so θ is less than θ_c',

Ray refracts to medium II as shown in Fig. 1.150. at an angle 'r',

$$\mu_2 \sin\theta = \mu_3 \sin r$$

$$\sin r = \frac{\mu_2}{\mu_3}\sin\theta \qquad \because \theta_c < \theta$$

$$\sin r > \frac{\mu_2}{\mu_3}\sin\theta_c; \quad \sin r > \frac{\mu_2}{\mu_3}\frac{\mu_1}{\mu_2}$$

$$\sin r > \frac{\mu_1}{\mu_3}$$

$$\sin r > \sin\theta_c''; \quad r > \theta_c''$$

θ_c'' critical angle for the interface of medium II and medium I. Total internal reflection takes place at the upper interface and finally ray will be transmitted back to medium II.

OPTICAL FIBRE / OPTICAL PIPE

The optical fibres functions on the principle of total internal reflection. It consists of very long fine quality glass or quartz fibre known of the thickness or the diameter of the strand is approx. 10^{-4} cm. and the refractive index of the main fibre is approximately 1.7. The fibres are coated with a layer of material of lower refractive index (1.5).

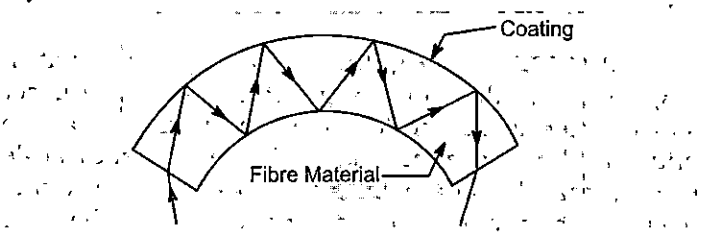


Fig. 1.151

When light is incident on one end of the fibre at small angle, it propagates down the fibre without emerging through its curved surface of lateral surface.

The angle of incidence at the boundary of fibre and its coating is larger than the critical angle of the fibre material with respect to its coating. The light undergoes repeated total internal reflections along the fibre and finally emerges out at the other end, even if the fibre is bent.

Minimum Refractive Index Required

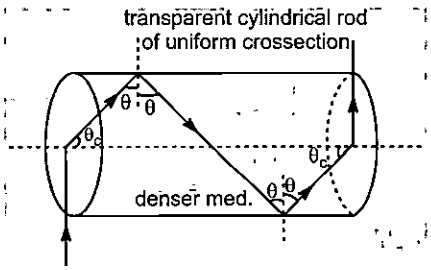


Fig. 1.152

$$\theta + \theta_c = 90° \qquad \theta = 90° - \theta_c$$

If ray does not emerge from cylindrical curved surface, then total internal reflection must takes place.

$$\theta > \theta_c$$

$$90° - \theta_c > \theta_c$$

$$90° > 2\theta_c$$

$$\theta_c < 45°$$

$$\sin\theta_c < \sin 45°;$$

$$\frac{1}{\mu} < \frac{1}{\sqrt{2}}$$

$$\mu > \sqrt{2}$$

Successive total internal reflections take place at curved surface and finally ray emerges from other end of cylinder through flat surface.

Here $\mu_{(min)} = \sqrt{2}$

Minimum Angle of Incidence Required

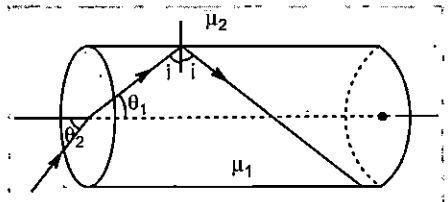


Fig. 1.153

$$\mu_1 > \mu_2$$

$$i > \theta_c \qquad \sin i > \sin\theta_c$$

$$\sin i > \frac{\mu_2}{\mu_1} \qquad ...(1)$$

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \qquad (\theta_1 + i = 90°)$$

$$\mu_1 \sin(90° - i) = \mu_2 \sin\theta_2$$

$$\mu_1 \cos i = \mu_2 \sin\theta_2$$

$$\cos i = \frac{\mu_2 \sin\theta_2}{\mu_1}$$

$$\sin i = \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}\sin^2\theta_2} \qquad ...(2)$$

From eqns. (1) and (2) we get,

$$\sqrt{1 - \frac{\mu_2^2}{\mu_1^2}\sin^2\theta_2} > \frac{\mu_2}{\mu_1} \Rightarrow 1 - \frac{\mu_2^2}{\mu_1^2}\sin^2\theta_2 > \frac{\mu_2^2}{\mu_1^2}$$

$$\frac{\mu_2^2}{\mu_1^2}\sin^2\theta_2 < 1 - \frac{\mu_2^2}{\mu_1^2} \Rightarrow \sin^2\theta_2 < \frac{\mu_1^2 - \mu_2^2}{\mu_2^2}$$

$$\sin^2\theta_2 < \frac{\mu_1^2 - \mu_2^2}{\mu_2^2} \Rightarrow \theta_{2(max)} = \sin^{-1}\left[\left(\frac{\mu_1}{\mu_2}\right)^2 - 1\right]^{1/2}$$

This is the maximum angle of incidence for which a ray can be propagated down the fibre. $\theta_{2(max)}$ is called the angle of acceptance.

Illustration 10: Solid sphere is made of transparent material and its refractive index w.r.t. surrounding is

$\mu(\mu > 1)$. Maximum surface area of sphere visible from point A on the surface of sphere is given by,

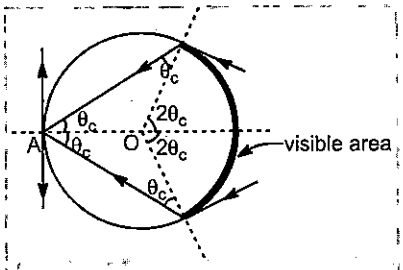


Fig. 1.154

$$\text{Visible area} = 2\pi r^2(1-\cos 2\theta_c)$$

$$= 2\pi r^2 2\sin^2\theta_c = \frac{4\pi r^2}{\mu^2}$$

Illustration 11: Optical rod of uniform cross-section bent in the U-shape. Rays incident normally at face A will eventually come out of face B provided,

$$\left(\frac{R}{r} > \frac{2}{(\mu-1)}\right)$$

Working $\theta > \theta_c$; $\sin\theta > \sin\theta_c$

$$\sin\theta > \frac{1}{\mu_D/\mu_R}; \quad \sin\theta > \frac{1}{\mu}$$

$$\frac{R}{R+2r} > \frac{1}{\mu}$$

$$\mu R > R + 2r \qquad (\mu-1)R > 2r$$

$$\frac{R}{r} > \frac{2}{\mu-1}$$

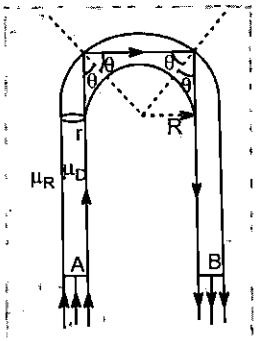


Fig. 1.155

Example 64

A ray of light is incident normally at face AB of a glass prism $n = 3/2$. Find the largest value for the angle ϕ so that the ray is totally reflected at face AC if the prism is immersed (a) in air and (b) in water.

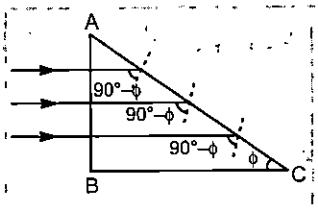


Fig. 1E.64

Solution: (a) No deviation occurs at face AC; hence the angle of incidence at surface AC is $90°-\phi$. For total internal reflection at the second surface,

$$n_g \sin(90°-\phi) \geq n_a$$

$$n_g \cos\phi \geq n_a$$

As ϕ increases from zero, $\cos\phi$ decreases. Thus ϕ has the largest value when

$$n_g \cos\phi = n_a$$

or $$\cos\phi = \left(\frac{n_a}{n_g}\right)$$

or $$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{2}{3}\right) \simeq 49°$$

(b) If the prism is immersed in water,

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{8}{9}\right) = 29°$$

Example 65

Consider a glass slab of refractive index n covered by a thin sheet of wet paper. When a beam of light strikes the paper at normal incidence, a distinctive circular pattern is formed. This illuminated circular pattern is surrounded by a dark ring and again a circular bright ring. Find the radius of the bright ring.

Solution: In the Fig. 1E.65 shown rays incident at an angle less than the critical angle pass through the upper surface whereas that incident at an angle greater than the critical angle are reflected. The rays incident at critical angle $\theta_2 = C$, graze the surface. Imagine a cone with semivertical angle θ_2, the rays enclosed in it come out the others are reflected back to the lower surface where they escape into the wet paper and are scattered, forming the brighter surrounding area.

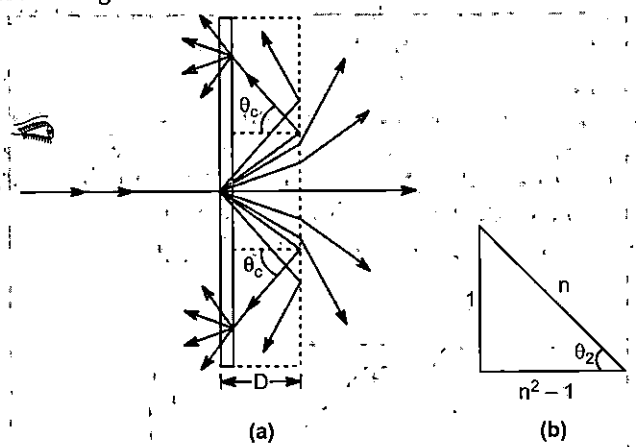


Fig. 1E.65

From Snell's law,

$$n\sin\theta_2 = 1\sin 90°$$

$$\sin\theta_2 = \frac{1}{n} \qquad \ldots (1)$$

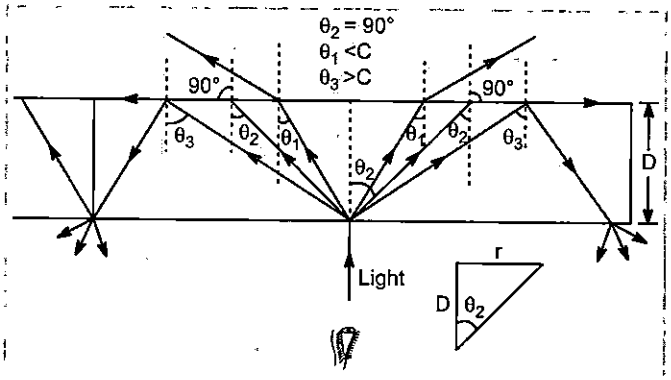


Fig. 1E.65 (c)

From the figure, radius of illuminated circle,

$$r = D\tan\theta_2$$

$$r = \frac{D}{\sqrt{n^2-1}}$$

Similarly if an illuminated point object is kept at a depth h in water, the light rays that are confined in a cone of angle of vertex $\theta = 2C$ will emerge and the circle of illumination will have radius

$$r = \frac{h}{\sqrt{h^2-1}}$$

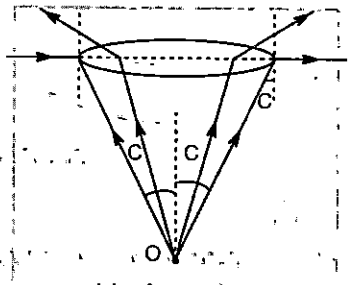


Fig. 1E.65 (d)

Example 66

Consider a solid glass cube of edge length 10 mm and refractive index n = 1.5. A black spot lies at the geometrical centre of the cube. What fraction of the cube surface must be covered so as to prevent the spot from being seen ?

Solution: The light that passes through each face forms a circle of radius r.

Applying the result of previous problem,

$$r = \frac{L}{2\sqrt{n^2-1}}$$

$$= \frac{10}{2\sqrt{(1.5)^2-1}}$$

$$= 4.47\text{mm}$$

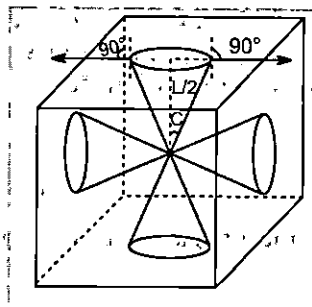


Fig. 1E.66

Area through which light can escape = πr^2.

Six circles of illumination are possible; thus the fraction of the surface are through which light can escape,

$$f = \frac{6\pi r^2}{6L^2} = \frac{\pi r^2}{L^2} = \frac{\pi(4.47)^2}{(10)^2} = 0.63$$

Example 67

A point source of light is placed a distance h below the surface of water. What is the fraction of light energy that escapes directly from the water surface ? Ignore any absorption or scattering of light energy.

Solution: The light that escapes is confined within a cone of apex angle $2C$, where C is critical angle for total internal reflection. Imagine a sphere with source of light as its centre and suppose that the surface area A of the sphere is inside the cone. At each point of the sphere the intensity is same, therefore the fraction of total energies passing out is

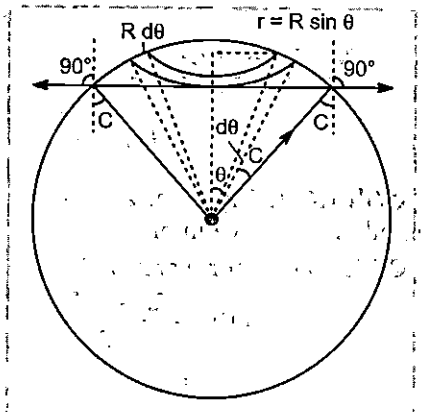


Fig. 1E.67

$$f = \frac{A}{4\pi R^2}$$

where R is the radius of the sphere.

Now we will derive A in terms of critical angle C. Consider a differential ring of angular thickness $d\theta$ on the spherical surface. From the figure, the radius of this ring is $r = R\sin\theta$. Its thickness is $R\,d\theta$, and circumference $2\pi R\sin\theta$. Hence its area,

$$dA = 2\pi R^2\sin\theta\,d\theta$$

Hence

$$A = 2\pi R^2\int_0^C \sin\theta\,d\theta$$

$$= 2\pi R^2(1-\cos C)$$

The critical angle is given by

$$\sin C = \frac{1}{n}$$

$$\cos C = \sqrt{1-\sin^2 C} = \sqrt{1-\left(\frac{1}{n^2}\right)}$$

Thus

$$f = \frac{2\pi R^2(1-\cos C)}{4\pi R^2} = \frac{1}{2}\left[1-\sqrt{1-\frac{1}{n^2}}\right]$$

Example 68

An isotropic source of light is deep inside a lake. The surface of the lake is covered by a layer of oil of refractive index 1.2. Find the percentage of light (a) escaping from the lake surface and (b) totally internally reflected in the oil layer. Assume that both oil and water are 100% transparent, refractive index of water is 1.33.

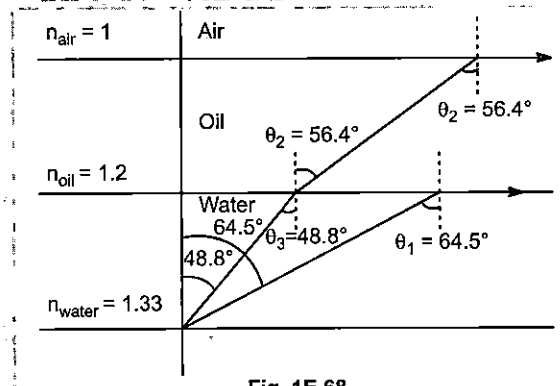


Fig. 1E.68

Solution: Light refracts at water-oil interface and then oil-air interface in order to come out.

Critical angle for water-oil boundary is given by

$$\sin\theta_1 = \frac{1.2\sin 90°}{1.33} = \frac{1.2}{1.33}$$

$$\theta_1 = 64.5°$$

Critical angle for oil-air boundary is given by

$$\sin\theta_2 = \frac{1}{1.2}$$

$$\theta_2 = 56.4°$$

Let a ray incident at angle θ_2 at oil-air interface make an angle θ_3 at water-oil interface.

From Snell's law,

$$n_{water}\sin\theta_3 = n_{oil}\sin\theta_2$$

$$\sin\theta_3 = \frac{1.2\sin 56.4°}{1.33} = \frac{1}{1.33}$$

$$\theta_3 = 48.8°$$

Proceeding similar to previous example, we can find the solid angle subtended by the vertex of the cone of angle α as

$$\int_0^{\alpha} \frac{2\pi\sin\theta r d\theta}{r^2} = 2\pi(1-\cos\alpha)$$

Percentage of light escaping from the lake surface

$$= \frac{2\pi}{4\pi}(1-\cos 48.8°)\times 100$$

$$= 17\%$$

Percentage of light totally internally reflected at water oil surface

$$= \frac{1}{2}(1-\cos 64.5°)\times 100$$

$$= 28.5\%$$

Therefore percentage of light totally reflected at oil-air surface

$$= (28.5-17)\% = 11.5\%$$

Example 69

Glass mugs are often made with thick sides, so that they appear to hold more than they really do. When filled with say apple juice it appears to extend all the way to the outer surface of the mug. Fig. 1E.69 shows a view from above of the path of a ray from a point P in the juice at the inner surface of the mug. Take refractive index of glass and juice to be 3/2 and 4/3 respectively.

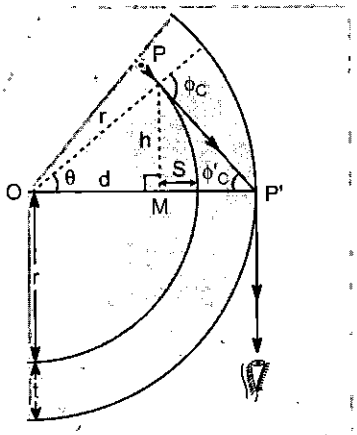


Fig. 1E.69

(a) Find t/r

(b) Find the ratio of the apparent volume of juice to the actual volume of juice in the mug.

Solution: (a) The figure is drawn so that the ratio of the wall thickness t to the inner radius r is as large as possible. In this case the light from point P appears to be coming from P' at the edge. For thicker walls light from point P would appear to be coming from inside the edge. For thinner walls, light from point P could not be seen from this direction, and light apparently coming from P' would be originating at another point in the juice.

At this point the ray is tangential and will refract at critical angle ϕ_C in the glass; similarly at P' once again critical angle ϕ_C is formed.

Critical angle at juice-glass interface,

$$\sin\phi_C = \frac{8}{9},\ \phi_C = 62.73°$$

Critical angle at glass-air interface,

$$\sin\phi'_C = \frac{2}{3},\ \phi'_C = 41.81°$$

From ΔOPM, $\sin\theta = \frac{h}{r}$ ... (1)

From $\Delta PMP'$, $\tan\phi'_C = \frac{h}{S+t}$ ... (2)

where $S = r-(d+t) = r-(r\cos\theta+t)$

Angle ϕ_C is exterior angle of triangle OPP'.

Hence $\theta+\phi'_C = \phi_C$

$$\theta = \phi_C - \phi'_C = 20.91°$$

Dividing eqn. (1) by (2), we get

$$\frac{\sin\theta}{\tan\phi'_C} = \frac{r-(r\cos\theta+t)}{r} = 1-\cos\theta+\frac{t}{r}$$

$$\frac{t}{r} = \frac{\sin\theta}{\tan\phi'_C}+\cos\theta-1$$

On substituting numerical values, we get

$$\frac{t}{r} = 0.33$$

(b) Actual volume $= \pi r^2 l$

Apparent volume $= \pi(r+t)^2 l$

Hence $\dfrac{\text{Apparent volume}}{\text{Actual volume}} = \dfrac{\pi(r+t)^2 l}{\pi r^2 l}$

$$= \left(1+\frac{t}{r}\right)^2 = 1.77$$

Example 70

A rod made of glass, refractive index 1.5 and of square cross-section, is bent into the shape shown in Fig. 1E.70(a). A parallel beam of light falls normally on the plane flat surface A. Referring to the diagram d is the width of a side and R is the radius of inner semicircle. Find the maximum value of ratio d/R so that all light entering the glass through surface A emerges from the glass through surface B.

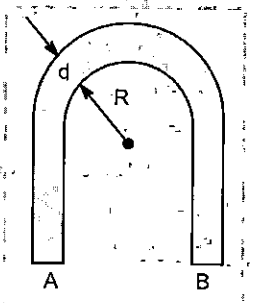


Fig. 1E.70 (a)

Solution: Fig. 1E.70 (b) shows three rays 1, 2, 3 incident on plane face A. We can see that angle of incidence at curved surface is least for ray 3. If ray 3 reflects at the curved surface, then all the rays will reflect as their angle of incidence is greater than angle θ_3.

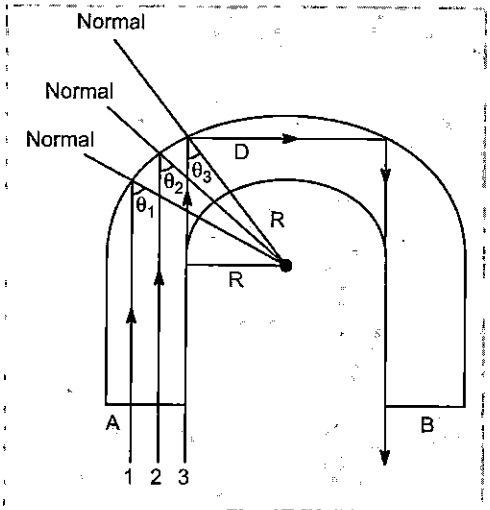


Fig. 1E.70 (b)

Hence the required condition is

$$\theta_3 \geq \theta_{\text{critical}}$$

As $\sin\theta_{\text{critical}} = \dfrac{1}{\mu} = \dfrac{2}{3}$,

from geometry of figure, we have

$$\sin\theta_3 = \frac{R}{d+R}$$

So $\sin n\theta_3 \geq \sin\theta_{\text{critical}}$

$$\frac{R}{d+R} \geq \frac{2}{3}$$

or $$\frac{d}{R} \leq \frac{1}{2}$$

Therefore $\left(\dfrac{d}{R}\right)_{\text{maximum}} = \dfrac{1}{2}$

Example 71

Rays of light fall on the plane surface of a semicylinder of refractive index $n = \sqrt{2}$, at angle 45° in the plane normal to the axis of cylinder. Discuss the condition that the rays do not suffer total internal reflection.

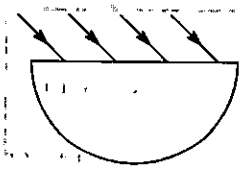


Fig. 1E.71 (a)

Solution: First we consider a ray incident at A. From Snell's law,

$$1\sin 45° = \sqrt{2}\sin\theta_2$$

$$\sin\theta_2 = \frac{1}{2}$$

$$\theta_2 = 30°$$

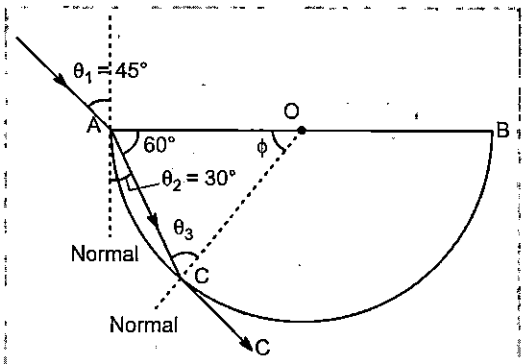


Fig. 1E.71 (b)

Let the angle $\phi = \angle AOC$ denote the position of the point C on the curved surface.

$$\angle CAO = 60°$$

The critical angle for glass to air interface can be determined from Snell's law.

$$n\sin C = 1\sin 90°$$

$$\sin C = \frac{1}{n} = \frac{1}{\sqrt{2}}$$

$$C = 45°$$

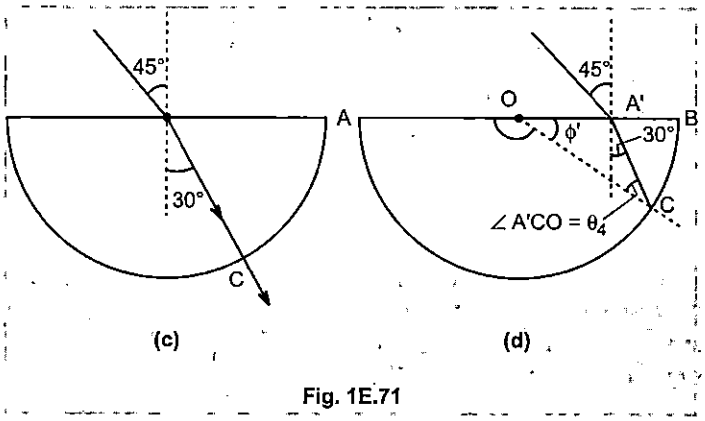


Fig. 1E.71

If total internal reflection has to take place at the curved surface, angle θ_3 must be greater than the critical angle, $C = 45°$.

As $\theta_3 = 180° - \phi - 60°$, therefore, for no total internal reflection,

$$180° - \phi - 60° < 45°$$

or $$\phi > 75°$$

When the ray falls at O, the refracted ray will move radially out, without deviation. The normal rays do not suffer deviation. Next we consider a ray to the right of O.

For no total internal reflection,

$$\angle A'CO < 45°$$

In $\Delta OA'C$, $\angle OA'C + \angle A'CO + \angle COA' = 180°$

$$120° + \theta_4 + (180 - \phi) = 180°$$

$$\theta_4 = \phi - 120°$$

Thus, $$\phi - 120° < 45°$$

$$\phi < 165°$$

Hence for rays to transmit through curved surface,

$$75° < \phi < 165°$$

Example 72

A fish is at a depth d under water, refractive index n. Show that when the fish is viewed at an angle θ, the apparent depth d′ of the fish is, $d' = \dfrac{n^2 h\cos^3\theta}{(n^2 - \sin^2\theta)^{3/2}}$

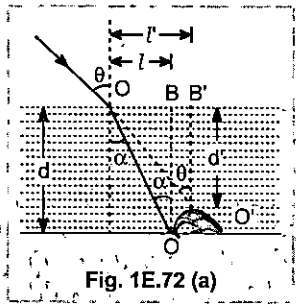


Fig. 1E.72 (a)

Solution: We cannot use formula for apparent depth, that is applicable for small angles of vision. The object appears to be shifted not only upwards but also laterally.

From figure, $AB = d\tan\alpha$

$$AB' = d'\tan\theta$$

So, $$l = AB' - AB$$

$$= d'\tan\theta - d\tan\alpha \quad ...(1)$$

As $$1\sin\theta = n\sin\alpha$$

$$\sin\alpha = \frac{\sin\theta}{n}$$

$$\cos\alpha = \sqrt{1 - \left(\frac{\sin\theta}{n}\right)^2}$$

$$\tan\alpha = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}} \quad ...(2)$$

$$l = d'\tan\theta - \frac{d\sin\theta}{\sqrt{n^2 - \sin^2\theta}} \quad ...(3)$$

Let $$x(\theta) = \tan\theta \quad ...(4)$$

and $$y(\theta) = \frac{d\sin\theta}{\sqrt{n^2 - \sin^2\theta}} \quad ...(5)$$

Thus $$l = x(\theta)d' - y(\theta) \quad ...(6)$$

The two rays incident at angles θ and $\theta + d\theta$ reach the image point S'. Hence for the other ray

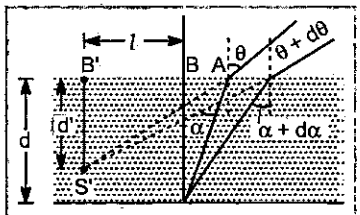


Fig. 1E.72 (b)

at an angle $\theta + d\theta$, we have

$$l = x(\theta + d\theta)d' - y(\theta + d\theta) \quad \ldots (7)$$

From eqns. (1) and (2), wet get

$$x(\theta + d\theta)d' - x(\theta)d' = y(\theta + d\theta) - y(\theta)$$

or $$d' = \frac{y(\theta + d\theta) - y(\theta)}{x(\theta + d\theta) - x(\theta)} \quad \ldots (8)$$

If we take limit $\Delta\theta \to 0$, we will get the desired result

$$d' = \lim_{\Delta\theta \to 0} \left\{ \frac{y(\theta + d\theta) - y(\theta)}{d\theta} \right\} \Big/ \left\{ \frac{x(\theta + d\theta) - x(\theta)}{d\theta} \right\}$$

$$= \frac{dy(\theta)}{d\theta} \Big/ \frac{dx(\theta)}{d\theta} \quad \ldots (9)$$

From eqn. (5), $\frac{dy(\theta)}{d\theta} = \frac{n^2 d\cos\theta}{(n^2 - \sin^2\theta)^{3/2}}$

From eqn. (4), $\frac{dx(\theta)}{d\theta} = \sec^2\theta$

Hence $$d' = \frac{n^2 d\cos^3\theta}{(n^2 - \sin^2\theta)^{3/2}}$$

Also $$l = d'\tan\theta - \frac{d\sin\theta}{(n^2 - \sin^2\theta)^{3/2}}$$

$$= \frac{-(n^2 - 1)d\sin^3\theta}{(n^2 - \sin^2\theta)^{3/2}}$$

❐ When the object is viewed at grazing angle,

$$\theta \simeq \pi/2, d' = 0 \text{ and } l = \frac{d}{\sqrt{n^2 - 1}}$$

the object appears almost at the surface but laterally displaced by l.

Example 73

The primary reflecting surface of a bathroom mirror lies behind a thin sheet of glass. This sheet is used to protect the reflecting surface from tarnishing. Consider a bathroom mirror with a glass sheet of thickness d and index of refraction n. Your face is located at a distance h from the front surface of the glass sheet.

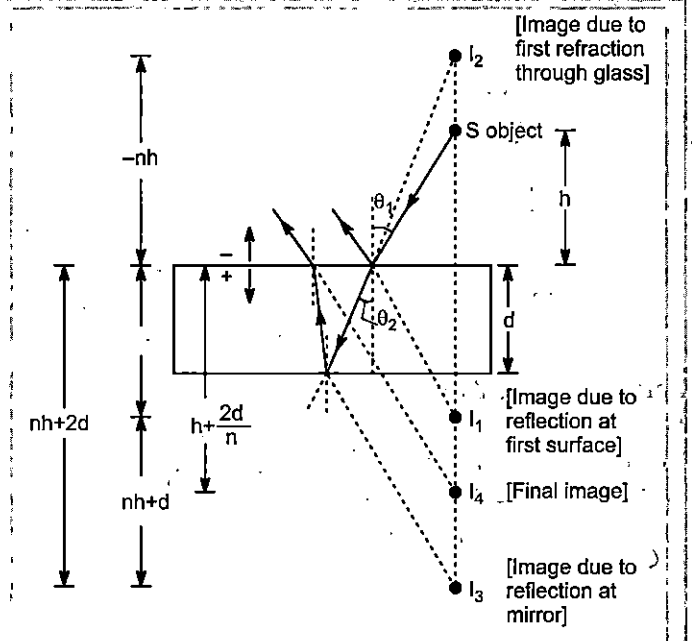


Fig. 1E.73

(a) Find the position of the image of your face that you see due to reflection from the front surface of the glass. (b) Find the position of the image of your face that you see due to reflection from the mirror surface behind the glass. Take into account the effects of refraction at the glass-air interface. (c) How far apart are the images in parts (a) and (b) ?

Solution: (a) First image I_1 is formed due to reflection at upper surface formed at h distance below upper surface.

(b) Second image I_2 is due to refraction at air-glass interface. From single surface refraction equation, we have

$$\frac{n}{v} - \frac{1}{(-h)} = 0 \quad \text{or} \quad v = -nh \quad \ldots (1)$$

Image is virtual, formed a distance nh above the upper surface. Image I_2 is an object for reflection at the mirror. The image I_3 is formed a distance $(nh + d)$ below it. Image I_3 acts as object for final refraction at glass-air interface, when the rays come out of glass. From single surface refraction equation, we have

$$\frac{1}{v'} - \frac{n}{(nh + 2d)} = 0$$

$$v' = \left(h + \frac{2d}{n}\right)$$

VARIABLE REFRACTIVE INDEX

If refractive index of medium is variable, then ray travells along curved path.

Consider a case where refractive index is function of y

If $$\mu = f(y)$$

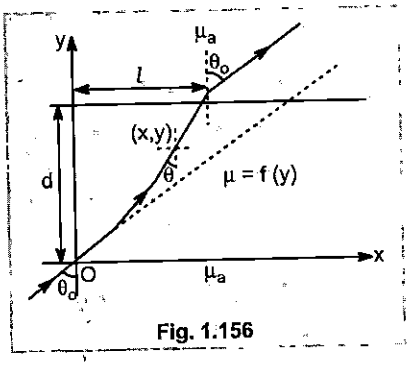


Fig. 1.156

Normal is along Y-axis and boundary is parallel to X-Z plane.

Tangent drawn at a point (x, y) to the trajectory followed by ray gives direction of incident ray at that point.

$$\mu_a \sin\theta_0 = \mu \sin\theta$$

$$\mu_a \sin\theta_0 = f(y)\sin\theta$$

$$\sin\theta = \frac{k}{f(y)} \qquad ...(1)$$

$$k = \mu_a \sin\theta_0 \quad \text{(say)}$$

If equation of trajectory followed by ray is given by,

$$y = f_1(x)$$

$$\frac{dx}{dy} = \tan\theta$$

$$\sin\theta = \frac{(dx/dy)}{\sqrt{1+\left(\frac{dx}{dy}\right)^2}} \qquad ...(2)$$

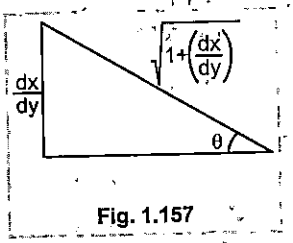


Fig. 1.157

Equating eqns. (1) and (2) we get,

$$\frac{(dx/dy)}{\sqrt{1+\left(\frac{dx}{dy}\right)^2}} = \frac{k}{f(y)}$$

$$\left(\frac{dx}{dy}\right)^2 \{f(y)\}^2 = k^2 + k^2\left(\frac{dx}{dy}\right)^2;$$

$$\left(\frac{dx}{dy}\right)^2 [\{f(y)\}^2 - k^2] = k^2$$

$$\frac{dx}{dy} = \frac{k}{\sqrt{\{f(y)\}^2 - k^2}};$$

$$\int_0^x dx = k\int_0^y \frac{dy}{\sqrt{\{f(y)^2\} - k^2}}$$

The integral yields required equation of trajectory.

$$y = f_1(x)$$

Secondly refractive index may vary as function of x

If *i. e.*, $\mu = f(x)$

$$\sin\theta_0 = \frac{\mu_a \sin\theta}{\mu_0}$$

$$\mu_0 \sin(90°-\theta_0) = f(x)\sin\theta$$

$$\sin\theta = \frac{k}{f(x)} \qquad ...(3)$$

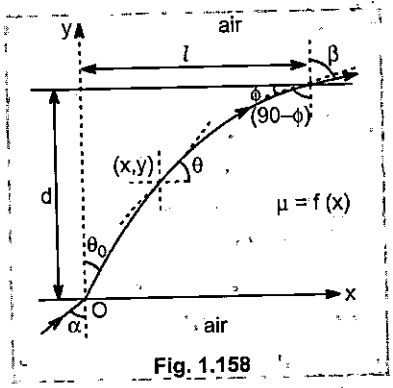


Fig. 1.158

Normal inside medium is along or parallel to x-axis and boundary is along or parallel to y-z plane.

$$\frac{dy}{dx} = \tan\theta$$

$$\sin\theta = \frac{(dy/dx)}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} \qquad ...(4)$$

Equating eqns. (3) and (4) we get,

$$\frac{(dy/dx)}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = \frac{k}{f(x)}$$

$$\left(\frac{dy}{dx}\right)^2 \{f(x)\}^2 = k^2 + k^2\left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{dy}{dx}\right)^2 [\{f(x)\}^2 - k^2] = k^2$$

$$\frac{dy}{dx} = \frac{k}{\sqrt{\{f(x)\}^2 - k^2}}$$

$$\int_0^y dy = k\int_0^x \frac{dx}{\sqrt{\{f(x)\}^2 - k^2}}$$

The integral yields required equation of trajectory.

$$y = f(x)$$

$$\frac{dy}{dx} = f(x) = \tan\phi$$

$$\phi = \tan^{-1}\{f'(x)\}$$

If angle $(90°-\phi)$ is greater than critical angle at the uppermost boundary then total internal reflection takes place, otherwise ray immerges at an angle β to normal.

$$\frac{\sin(90°-\phi)}{\sin\beta} = \frac{\mu_a}{f(l)}$$

From above formula you can find β.

Example 74

A ray of light travelling in air is incident at grazing angle (angle of incidence $\simeq 90°$) on a long rectangular slab of a transparent medium of thickness $t = 1.0m$. The point of incidence is the origin A(0,0). The medium has a variable index of refraction given by

$$n(y) = (ky^{3/2} + 1)^{1/2}$$

where $k = 1.0\,(m)^{-3/2}$. The refractive index of air is 1. (a) Obtain a relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incidence angle at that point.

(b) Obtain an equation for the trajectory $y(x)$ of the ray in the medium. (c) Determine the coordinates (x_1, y_1) of the point P where the ray intersects the upper surface of the slab-air boundary. (d) Indicate the path of the ray subsequently.

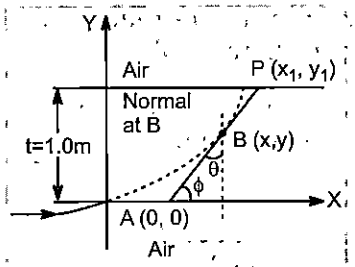


Fig. 1E.74

Solution: (a) The trajectory of the ray is shown by dotted curve. The slope of tangent at point B is

$$\tan\phi = \frac{dy}{dx}$$

The angle of incidence at B is $\theta = 90°-\phi$.

Hence $$\tan(90°-\theta) = \frac{dy}{dx}$$

or $$\cot\theta = \frac{dy}{dx} \quad \ldots (1)$$

(b) From Snell's law at A and B, we have

$$1\sin 90° = n(y)\sin\theta \quad \ldots (2)$$

$$\sin\theta = \frac{1}{n(y)} = \frac{1}{(ky^{3/2}+1)^{1/2}} \quad \ldots (3)$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{[ky^{3/2}+1]}}$$

$$\cot\theta = \frac{\sqrt{1-[1/ky^{3/2}+1]}}{1/[ky^{3/2}+1]^{1/2}} = k^{1/2}y^{3/4}$$

From eqn. (3),

$$\sin\theta = \frac{1}{n(y)} = \cos\phi$$

$$= \frac{1}{\sqrt{1+\tan^2\phi}} = \frac{1}{\sqrt{1+(dy/dx)^2}}$$

$$n(y) = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2}$$

$$[1+ky^{3/2}] = 1+\left(\frac{dy}{dx}\right)^2$$

$$\frac{dy}{dx} = \sqrt{k}y^{3/4}$$

$$4y^{1/4} = \sqrt{k}x + C \quad \ldots (4)$$

Now we substitute boundary conditions in eqn. (4), $x = 0$, $y = 0$, hence $C = 0$. The required trajectory is

$$y = k^2\left(\frac{x}{4}\right)^4$$

(c) At point P, $y = 1m$, $k = 1.0(m)^{3/2}$, we get

$$x = 4m$$

The coordinates of P are (4,1).

(d) From Snell's law,

$$n_A \sin i_A = n_P \sin i_P$$

As $$n_A = n_P = 1$$

so $$i_P = i_A = 90°$$

The ray will emerge parallel to boundary.

Example 75

Due to a vertical temperature gradient in the atmosphere the index of refraction varies. Suppose index of refraction varies as $n = n_0\sqrt{1+ay}$ where n_0 is the index of refraction at the surface and $a = 2.0\times10^{-6}\,m^{-1}$. A person of height $h = 2.0m$ stands on a level surface. Beyond what distance he cannot see the runway ?

Solution: Let O be the distant object just visible to the man. Let P be a point on the trajectory of the ray. From Fig. 1E.75, $\theta = 90 - i$.

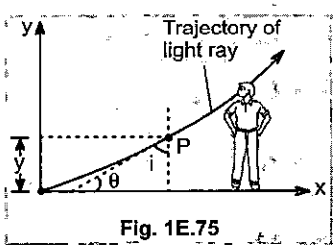


Fig. 1E.75

The slope of tangent at point P is $\tan\theta = dy/dx = \cot i$.

From Snell's law, $n\sin i$ = constant

At the surface $n = n_0$ and $i = 90°$

$$n_0 \sin 90° = n \sin i = (n_0\sqrt{1+ay}) \sin i$$

$$\sin i = \frac{1}{\sqrt{1+ay}}$$

$$\cot i = \frac{dy}{dx} = \sqrt{ay}$$

$$\int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx$$

$$x = 2\sqrt{\frac{y}{a}}$$

On substituting $y = 2.0\,\text{m}$ and $a = 2\times10^{-6}\,\text{m}^{-1}$, we have

$$x_{max} = 2\sqrt{\frac{2}{2\times10^{-6}}} = 2000\,\text{m}$$

Example 76

Fig. 1E.76 shows the cross-section of a thin slice of atmosphere in the vertical plane, the thickness of the layer is d. The refractive index of air varies as $n = \frac{n_0}{1 - x/a}$ *with the ray entering at origin; n_0 and a are constant. (a) Obtain an equation for the trajectory y(x) of the ray in the medium. (b) Find coordinates of the point A where it emerges. (c) What is index of refraction at A if it emerges at an angle α with the vertical ?*

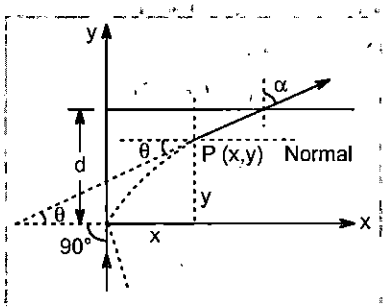


Fig. 1E.76 (a)

Solution: The refractive index varies along x-axis. The normal at any point $P(x, y)$ is parallel to the x-axis.

Slope of tangent at point $P = \tan\theta = \frac{dy}{dx}$

From Snell's law,

$$n_0 \sin 90° = n \sin\theta$$

$$n_0 = \frac{n_0}{1-(x/a)} \sin\theta$$

$$\sin\theta = \frac{a-x}{a},$$

$$\tan\theta = \frac{a-x}{\sqrt{a^2-(a-x)^2}}$$

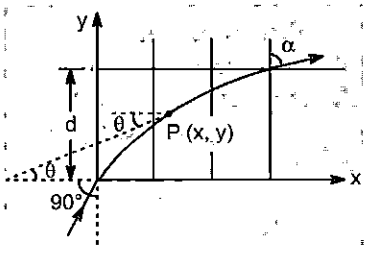


Fig. 1E.76 (b)

$$\frac{dy}{dx} = \frac{a-x}{\sqrt{a^2-(a-x)^2}}$$

$$y = \int dy = \int \frac{a-x}{\sqrt{a^2-(a-x)^2}}\,dx$$

To integrate the above expression we substitute

$$t^2 = a^2 - (a-x)^2$$

$$2t\,dt = -2(a-x)(-dx)$$

$$t\,dt = (a-x)\,dx$$

$$y = \int \frac{t\,dt}{t} = t = \sqrt{a^2-(a-x)^2}$$

Thus $y^2 = a^2 - (a-x)^2$

$$y^2 + (x-a)^2 = a^2$$

This is the equation of a circle of radius a with centre at $(a, 0)$

As y-coordinate of point A is d, hence

$$d^2 + (x-a)^2 = a^2$$

$$x = a \pm \sqrt{a^2-d^2}$$

As x cannot be greater than radius of circle at $x = a$, the ray grazes the upper surface.

Example 77

A thick plate is made of a transparent material whose refraction index changes from n_1 on its upper edge to n_2 on its lower edge. A beam enters the plate at the angle α. At what angle will the beam leave the plate ?

Solution: Let us divide the plate into many plates so thin that their refraction index can be assumed constant within the limits of each plate [Fig. 1E.77 (a)].

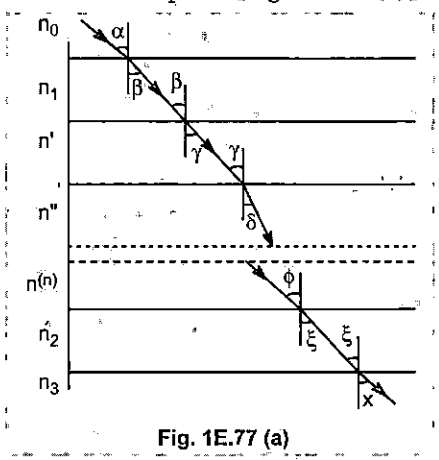


Fig. 1E.77 (a)

Assume that the beam enters the plate from a medium with a refraction index of n_0 and leaves it for a medium with a refraction index of n_2.

Then, according to the law of refraction,

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_1}{n_0}$$

$$\frac{\sin\beta}{\sin\lambda}=\frac{n'}{n_1}$$

$$\frac{\sin\gamma}{\sin\delta}=\frac{n''}{n'}$$

$$----$$

$$----$$

$$----$$

$$\frac{\sin\phi}{\sin\xi}=\frac{n_2}{n^{(n)}}$$

$$\frac{\sin\xi}{\sin x}=\frac{n_3}{n_2}$$

Upon multiplying these equations, we get

$$\frac{\sin\alpha}{\sin x}=\frac{n_3}{n_0}$$

Hence, the angle at which the beam leaves the plate, $x = arc\sin\left(\frac{n_0}{n_3}\sin\alpha\right)$ depends only on the angle of incidence of the beam on the plate and on the refraction indices of the media on both sides of the plate. In particular, if $n_3 = n_0$, then $x = \alpha$.

Note that the angle θ at which the beam is inclined to the vertical is related to the refraction index n at any point on the plate by the ratio $n\sin\theta$= constant $= n_0\sin\alpha$. If the refraction index reaches a value of $n = n_0\sin\alpha$ anywhere inside the plate, full internal reflection will take place. In this case the beam will leave the plate for the medium at the same angle α at which it entered the plate [Fig. 1E.77(b)]

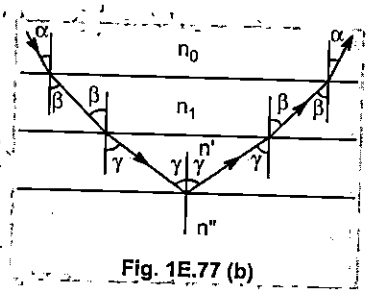


Fig. 1E.77 (b)

Example 78

The refractive index of the medium within a certain region, $x > 0$, $y > 0$, changes with y. A thin light ray travelling in the x-direction strikes the medium at right angles and moves through the medium along a circular arc.

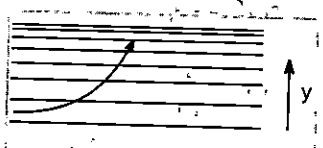


Fig. 1E.78 (a)

How does the refractive index depend on y ? What is the maximum possible angular size of the arc ?

Solution: Fig. 1E.78 (b) shows a light ray passing through successive planoparallel plates of different refractive indices.

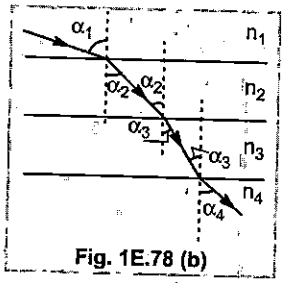


Fig. 1E.78 (b)

According to Snell's law,

$$\frac{\sin\alpha_2}{\sin\alpha_1}=\frac{n_1}{n_2},\ \frac{\sin\alpha_3}{\sin\alpha_2}=\frac{n_2}{n_3},\ldots.$$

The product of the sine of the angle of incidence and the absolute refractive index has the same value at all interfaces, *i.e.*, $n\sin\alpha$ is constant along the light ray's trajectory.

This relationship is also valid for a medium whose refractive index continuously changes in one direction, since the medium can be considered as consisting of thin planoparallel plates. Place the origin of the coordinate system at the point where the light ray enters the medium. In this case, the angle of incidence for the first 'plate' starting at $y = 0$ is 90° and the refractive index is n_0, which gives the above constant as $n(y)\sin\alpha = n_0$.

The light travels along a circular arc of radius R and we first examine its relationship to coordinate y. From Fig. 1E.78 (c), it is clear that

$$n_0 = n\sin\alpha = n(y)\frac{R-y}{R}$$

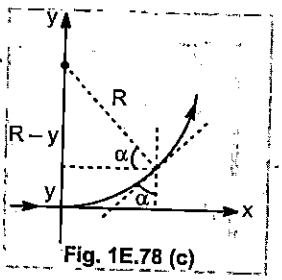


Fig. 1E.78 (c)

This gives the space-dependence of the refractive index as

$$n(y)=\frac{R}{R-y}n_0$$

The material with the greatest known refractive index is diamond, but even the refractive index of this material does not reach the value $n_{max} = 2.5$. It is this limit that sets the maximum angular size of the arc the light ray can cover. If the refractive index changes from $n_0 = 1$ to $n_{max} = 2.5$, then the maximum value of y is $\frac{3}{5}R$, corresponding to an arc of angular size 66.4°.

Example 79

A portion of a straight glass rod of diameter 4 cm and refractive index 1.5 is bent into an arc of a circle of radius R cm and a parallel beam of light is incident on it as shown in Fig. 1E.79(a). Find the smallest R which permits all the light to pass around the arc.

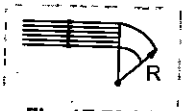


Fig. 1E.79 (a)

Solution: The necessary and sufficient condition for all the rays to pass around the arc is that the ray with least angle of incidence should get internally reflected.

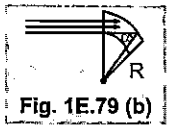


Fig. 1E.79 (b)

From the figure,it becomes obvious that the ray with least angle of incidence is the one which is incident almost grazingly with the inner wall.

For this ray, if 'α' be the angle of incidence, $\sin\alpha = \frac{R-d}{R}$, where d is the diameter of the tube.

But $\alpha \geq C \Rightarrow \sin\alpha \geq \sin C \Rightarrow \frac{R-d}{R} \geq \frac{1}{\mu}$

$\Rightarrow 1-\frac{d}{R} \geq \frac{1}{\mu} \Rightarrow \frac{d}{R} \leq 1-\frac{1}{\mu} \Rightarrow R \geq \frac{d\mu}{\mu-1}$

$\Rightarrow R \geq \frac{4(3/2)}{(1/2)} \Rightarrow R \geq 12\,\text{cm}$

Hence the least radius required is 12 cm.

Example 80

A spider and a fly are on the surface of a glass sphere. Where must the fly be for the spider to be able to see it ? Assume that the radius of the sphere is much larger than the size of the spider and the fly. The refractive index of glass is μ = 1.5.

Solution: Let us consider the spider to be at a height h above the sphere. We will let $h \to 0$ to make it an object on the sphere. In Fig. 1E.80, A is the position of the spider. Draw tangential rays from A to the sphere. Then these rays are incident at 90°. The corresponding angle of refraction is C (critical angle). Let the refracted rays cut the sphere at D and D'. Then D and D' define the extreme positions of the fly from where it is visible to the spider because by the principle of reversibility of the path of light rays, any ray from the region between D and D' will reach the spider.

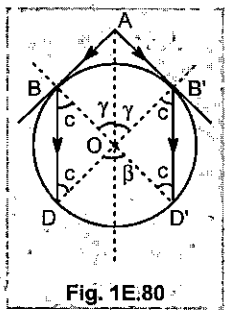


Fig. 1E.80

Let $\angle DOD' = \beta$. Then

$\angle BOB' = 2\gamma$ (say) $= 2\pi - \beta - 2(\pi - 2C)$

$\Rightarrow \gamma = 2C - \beta/2$

Now $\cos\gamma = \frac{R}{R+h}$

As $h \to 0$, $\cos\gamma \to 1 \Rightarrow \gamma \to 0 \Rightarrow \beta = 4C$

Hence the fly must be within $4C$ in the opposite part.

Now $4C = 4\sin^{-1}\frac{1}{1.5} = 4 \times 41.8 = 167°$

So the fly must be within 167° in the opposite hemisphere.

Example 81

A circular disc of diameter 7 cm lies horizontally inside a metallic hemispherical bowl of diameter 25 cm. The disc is just visible to any eye looking over the edge. The bowl is now filled with a liquid. It is observed that the whole of the disc is just visible to the eye in the same position. Find the refractive index μ of the liquid.

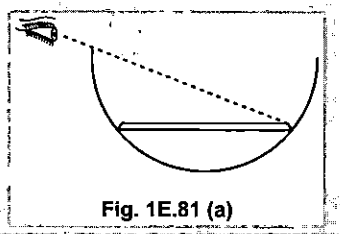


Fig. 1E.81 (a)

Solution: In the figure, AB is the disc and O, the centre of bowl.

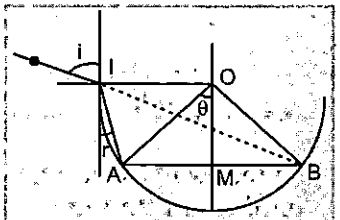


Fig. 1E.81 (b)

Let $\angle AOM = \theta$

Then $\angle AIB = \theta$

Now from geometry of figure $\angle AOI = 2\angle IBA$

i.e., $\angle IBA = \frac{1}{2}(90°-\theta)$

$\angle r = \angle IBA \frac{90°-\theta}{2}$

Also $\angle i = \frac{90°-\theta}{2} + \theta = \frac{90°+\theta}{2}$

Now, Snell's law gives $\frac{\sin\left(\frac{90°+\theta}{2}\right)}{\sin\left(\frac{90°-\theta}{2}\right)} = \mu$

or $\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)^2 = \mu^2$

or $\frac{1+\sin\theta}{1-\sin\theta} = \mu^2$

or $\sin\theta = \frac{\mu^2-1}{\mu^2+1}$

But $\sin\theta = \frac{d}{2a}$

or $d = 2a\left(\frac{\mu^2-1}{\mu^2+1}\right)$

For $d = 7$ and $2a = 25$, we get $\mu = 4/3$

Example 82

A man standing symmetrically infront of a plane mirror with bevelled edges can see three images of his eyes when he is 3 ft. from the mirror [see Fig. 1E.82 (a)]. The mirror is silvered on the back, is 2 ft. 6 in. wide, and is made of glass of refractive index 1.54. What is the angle of bevel of the edges ?

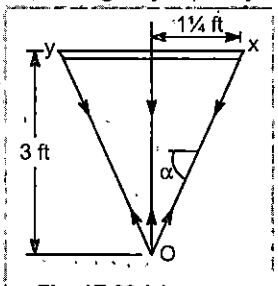


Fig. 1E.82 (a)

Solution: The man can only see an image of his eyes if light leaves them, strikes the mirror and is reflected back along the same path. The central image is thus formed by light traversing the perpendicular from his eyes to the mirror. The outer images are formed by light striking the bevelled edges at the point A [see Fig. 1E.82(b)], at an angle of incidence ϕ such that the angle of refraction ϕ' makes the refracted ray strike the silvered surface normally.

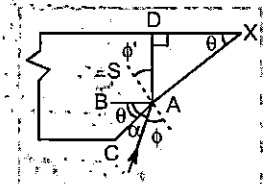


Fig. 1E.82 (b)

This must be the case if the ray of light it to leave the bevelled edge the same path with which it arrived. The angle ϕ' lies between the normal to the bevelled adge and the normal to the back surface. Since $\angle SAX = 90°$ [see Fig. 1E.82 (b)]

$$\phi' + \angle DAX = 90°$$

But $\angle DAX = 90° - \theta$

Hence $\phi' = 90° - 90° + \theta = \theta$

Draw BA, a construction line at A parallel to the back of the mirror. Angle BAC is also equal to θ.

But by Snell's law, $n_1 \sin\phi = n \sin\theta'$, where n_1 is the refractive index of air ($n_1 = 1$) and n is that of glass. Then $\sin\phi = n \sin\phi' = n \sin\theta$. Also $\alpha = \theta + (90° - \phi)$ [see Fig. 1E.82(b)]

$$\sin[90° - (\alpha - \theta)] = n \sin\theta$$

But $\sin(90° - \psi) = \cos\psi$

and $\cos(\alpha - \theta) = n \sin\theta$

By the trigonometric relation for double angles,

$$\cos\alpha\cos\theta + \sin\alpha\sin\theta = n\sin\theta$$

$$\cos\alpha + \sin\alpha\tan\theta = n\tan\theta$$

$$\cos\alpha = \tan\theta[n - \sin\alpha]$$

$$\tan\theta = \frac{\cos\alpha}{n - \sin\alpha}$$

Looking at Fig. 1E.82 (a),

$$\cos\alpha = \frac{\frac{5}{4}}{\sqrt{\left(\frac{5}{4}\right)^2 + (3)^2}} = \frac{5}{13}$$

$$\sin\alpha = \frac{3}{\sqrt{\left(\frac{5}{4}\right)^2 + (3)^2}} = \frac{12}{13}$$

hence $\tan\theta = \dfrac{5/13}{1.54 - (12/13)} = 0.625$

$\therefore \quad \theta = 32°$

Example 83

A ray of light is propagating in a horizontal direction close to the Earth's surface where the gradient of the refractive index is $\frac{d\mu}{dz} = 3 \times 10^{-8} m^{-1}$ (z = height above the Earth's surface). The refractive index of air = 1.0 at the surface and radius of earth = 6400 km.

(i) Find the radius of curvature of the ray of light.

(ii) For what value of the gradient would the ray of light propagate all the way round the earth ?

Solution: Let us consider a vertical wavefront whose lower end is at a height z and the upper end is at a height $z + \delta z$. In a short time δt, the upper end will move by $(v + dv)\delta t$ and lower end by $v\delta t$.

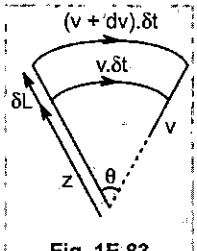


Fig. 1E.83

So the angle through which the wavefront has turned is

$$\theta = \frac{(\text{Change in arc length})}{(\text{Change in radius})}$$

$$= \frac{(v + \delta v)\delta t - v\delta t}{\delta z}$$

$$\theta = \frac{\delta v}{\delta z}\delta t$$

Now R = radius of curvature $= \dfrac{v\delta t}{\theta} = \dfrac{vdt}{\left(\frac{\delta v}{\delta z}\delta t\right)}$

$$\Rightarrow \quad R = \frac{v}{\left(\frac{\delta v}{dz}\right)} = \left|\frac{-\mu}{d\mu/dz}\right| = \frac{1}{3} \times 10^8 \text{ m}$$

If radius of curvature = R_{Earth}, the ray would propagate all around the Earth.

$$\frac{\mu}{d\mu/dz} = 6.4\times10^{6}$$

$$\Rightarrow \quad \frac{d\mu}{dz} = \frac{1}{6.4\times10^{6}} = 1.56\times10^{-7}\ \text{m}^{-1}$$

Example 84

A fixed cylindrical tank of height H = 4m and radius R = 3m is filled up with a liquid. An observer observes through a telescope fitted at the top of the wall of the tank and inclined at θ = 45° with the vertical. When the tank is completely filled with liquid, he notices an insect which is at the centre of the bottom of the tank. At t = 0, he opens a cork of radius = 3 cm at the bottom of the tank. The insect moves in such a way that it is visible for a certain time. Determine

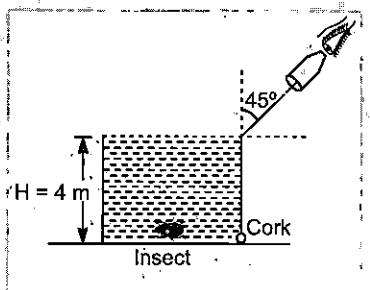


Fig. 1E:84 (a)

(a) The refractive index of the liquid,

(b) The velocity of the insect as a function of time.

Solution: From Snell's law, we have

$$n_1 \sin i = n_2 \sin r$$

$$n_1 \sin i = 1\cdot\sin 45°$$

as $$\sin i = \frac{3}{5}$$

therefore $$n_1 = \frac{5\sqrt{2}}{6}$$

When we open the cork, the water level decreases in the tank. Let after the time t the water level be h and the insect be at point P.

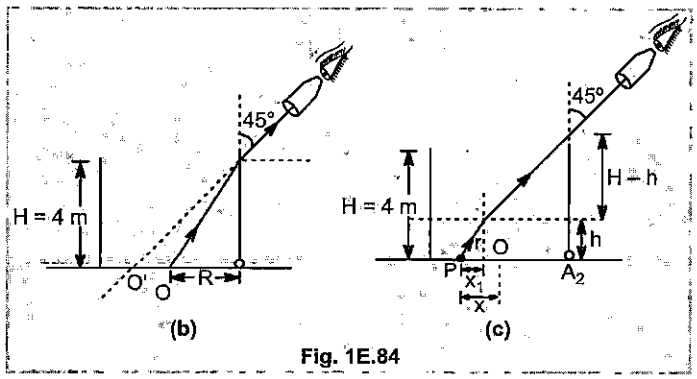


Fig. 1E.84

$$\frac{x_1}{h} = \frac{3}{4} \Rightarrow x_1 = \frac{3}{4}h$$

Distance of insect from the centre,

$$x = (H - h + x_1) - 3$$

$$= x_1 - \frac{4}{3}x_1 + 1$$

$$= \frac{-x_1}{3} + 1$$

$$= \frac{-h}{4} + 1 \qquad \ldots (1)$$

From the equation of continuity,

$$-A_1\frac{dh}{dt} = A_2\sqrt{2gh}$$

From eqn. (1),

$$+4A_1\frac{dx}{dt} = A_2\sqrt{2g\times4(1-x)} \qquad \ldots (2)$$

or $$\int_0^x \frac{dx}{\sqrt{1-x}} = \frac{A_2}{4A_1}\sqrt{8g}\int_0^t dt$$

or $$-[2(1-x)^{1/2}]_0^x = \frac{A_2}{4A_1}\times\sqrt{8g}\cdot t$$

or $$\sqrt{1-x} = 1 - \frac{A_2}{8A_1}\sqrt{8g}\cdot t \qquad \ldots (3)$$

From equations (2) and (3), we get

$$4A_1\times\frac{dx}{dt} = A_2\times\sqrt{8g}\times\left[1 - \frac{A_2}{8A_1}\sqrt{8g}\cdot t\right]$$

or $$\frac{dx}{dt} = \frac{A_2}{4A_1}\sqrt{8g} - \frac{A_2^2}{32A_1^2}\cdot8g\cdot t$$

or $$v = \frac{A_2}{4A_1}\sqrt{8g} - \frac{A_2^2}{32A_1^2}\cdot8g\cdot t$$

$$\frac{(9\times10^{-4})}{4\times9}\sqrt{80} - \frac{(9\times10^{-9})^2\times80\times t}{32\times9\times9}$$

$$= [2.24 - 2.5\times10^{-3}t]\times10^{-4}\ \text{m/s}$$

Example 85

An opaque sphere of radius R lies on a horizontal plane. On the perpendicular through the point of contact there is a point source of light a distance R above the sphere.

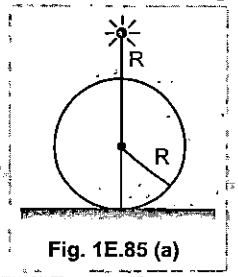


Fig. 1E.85 (a)

(a) Show that the area of the shadow on the plane is $3\pi R^2$.
(b) A transparent liquid of refractive index $\sqrt{3}$ is filled above the plane such that the sphere is just covered with the liquid. Show that the area of shadow now becomes $2\pi R^2$.

Solution: (a) Radius of shadow on the ground is MP

Let $\angle MSP = \theta \Rightarrow \angle SPM = 90°-\theta$

In ΔOSQ, $\angle SOQ = 90°-\theta$

Triangles OSQ and PSM are similar

hence $$\frac{r}{3R} = \frac{R}{\sqrt{(2R)^2 - R^2}}$$

or $$r = \sqrt{3}R$$

Area of the shadow $= \pi(\sqrt{3}R)^2 = 3\pi R^2$

(b) From the geometry of figure, we have

$$P'N = P'Q' + NQ'$$
$$= R' + NT$$
$$= R' + R\tan i$$
$$\cos r = \frac{NN'}{P'N} = \frac{2R}{R' + R\tan i}$$

or $$R' = 2R\sec r - R\tan i \quad ...(1)$$

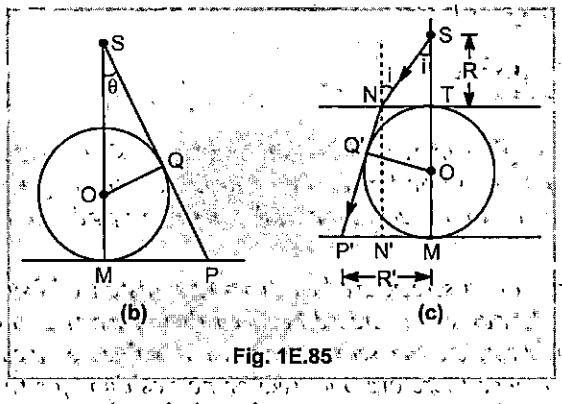


Fig. 1E.85

But $$R' = P'N' + N'M = 2R\tan r + R\tan i \quad ...(2)$$

From eqns. (1) and (2),

$$2\sec r - \tan i = 2\tan r + \tan i$$
$$\sec r - \tan r = \tan i$$

$\Rightarrow$ $$\tan i = \tan\left(\frac{\pi}{4} - \frac{r}{2}\right) \text{ (using trigonometry)}$$

$$2i = \frac{\pi}{2} - \frac{r}{2} \quad ...(3)$$

Also from Snell's law,

$$\sin i = \sqrt{3}\sin r$$

From eqns. (3) and (4), we get

$$\sin i = \frac{1}{\sqrt{3}}\sin r = \frac{1}{3} \quad ...(4)$$

$\Rightarrow$ $$\tan i = \frac{1}{\sqrt{2}}, \quad \tan r = \frac{1}{2\sqrt{2}}$$

Putting in eqn. (2), we get $R' = \sqrt{2}R$

Area of the shadow $= \pi R'^2 = 2\pi R^2$

Example 86

A light beam of diameter $\sqrt{3}R$ is incident symmetrically on a glass hemisphere of radius R and of refractive index $n = \sqrt{3}$. Find the radius of the beam at the base of hemisphere.

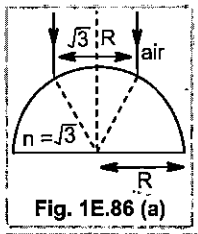


Fig. 1E.86 (a)

Solution: $\mu_1 \sin i = \mu_2 \sin r$

$$1 \times \sin 60° = \sqrt{3} \times \sin r$$
$$r = 30°$$

ΔMNP

$$\tan 30° = x/(R/2)$$
$$x = \frac{R}{2\sqrt{3}}$$
$$r = \frac{\sqrt{3}}{2}R - x = \frac{\sqrt{3}}{2}R - \frac{R}{2\sqrt{3}}$$
$$= \left(\frac{3-1}{2\sqrt{3}}\right)R = \frac{R}{\sqrt{3}}$$

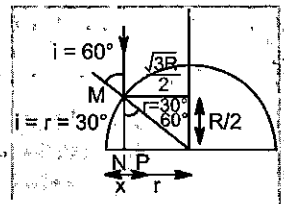


Fig. 1E.86 (b)

Example 87

A ray of light travelling in air is incident at angle of incident 30° on one surface of slab in which refractive index varies with y. The light travels along the curve $y = 4x^2$ (y and x are in metre) in the slab. Find out the refractive index of the slab at $y = 1/2$ m in the slab.

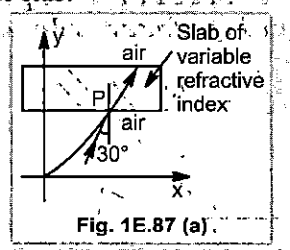


Fig. 1E.87 (a)

Solution: Let R.I. at $y = y$ is μ and corresponding angle of refraction is θ.

$$\mu \sin\theta = 1 \sin 30° \quad ...(1)$$

and $$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{dy}{dx} \Rightarrow \cot\theta = 8x$$

$$\Rightarrow \quad \cot\theta = \frac{8y^{1/2}}{2}$$

$$\cot\theta = 4y^{1/2}$$

at $y = 1/2 \Rightarrow \cot\theta = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

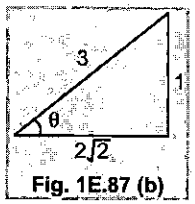


Fig. 1E.87 (b)

$$\Rightarrow \quad \sin\theta = \frac{1}{3}$$

$$\Rightarrow \quad \mu \times \frac{1}{3} = \frac{1}{2}$$

$$\mu = \frac{3}{2}$$

Example 88

A vessel, whose bottom is flat and perfectly reflecting, is filled with water (index = 4/3) upto a height = 40 cm. A point object in air above is moving towards the water surface with a constant speed = 4 m/s. What is the relative speed of its final image (in m/s), as seen by the object itself, at a moment when the object is 30 cm above the water surface ?

Solution: $v_O = -4$ m/s

$$v_{I/M} = \mu v_O$$

$$v_{I/M} = 4/3 \times 4$$

$$v_{I/M} = 16/3$$

$$v_{I/G} = \frac{v_{I/m}}{\mu}$$

$$v_{I/G} = \frac{+16/3}{4/3} = 4$$

$v_{I/O} = 4 - (-4) = 8$ m/s

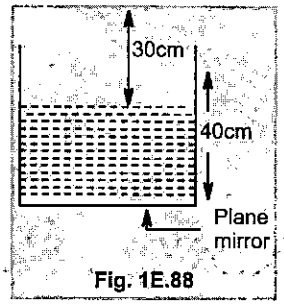


Fig. 1E.88

Example 89

Fig. 1E.89 (a) shows a transparent hemisphere of radius 3.0 cm made of a material of refractive index 2.0 :

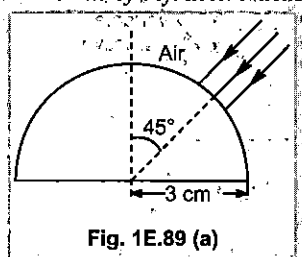


Fig. 1E.89 (a)

(a) A narrow beam of parallel rays is incident of the hemisphere as shown in figure. Are the rays totally reflected at plane surface ?

(b) Find the image formed by refraction at the first surface.

(c) Find the image formed by the reflection or by refraction at the plane surface.

Solution: (a) The critical angle for material-air interface

$$\sin C = \frac{1}{\mu} = \frac{1}{2}$$

$$\therefore \quad C = 30°$$

The rays are incident normally on the spherical surface, so they pass undeviated and then incident of plane face at an angle 45°. As the angle of incidence is greater than critical angle (30°), so rays get totally reflected.

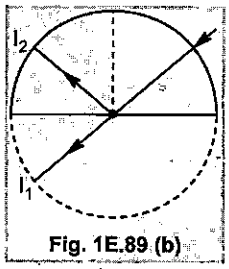


Fig. 1E.89 (b)

(b) For spherical surface :

$$u = \infty$$

We have $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

or $\frac{2}{v} - \frac{1}{\infty} = \frac{2-1}{R}$

$$\therefore \quad v = 2R.$$

Thus the image will form on diametrically opposite point.

(c) Some of the rays get totally reflected and so they will form the image at I_2.

Example 90

(a) A ray of light suffers an internal reflection inside a water drop. Find the condition for minimum deviation, the angle of incidence at minimum deviation and the value of minimum deviation.

(b) A source and a screen are held fixed at a distance l from each other. A thin lens is placed between them such that the source is focused on the screen. For, what values of focal length of the lens there are one, two or no positions for the lens ?

Solution: (a)

$$D(\text{deviation}) = (i - \theta) + (\pi - 2\theta) + (i - \theta) = \pi + 2i - 4\theta$$

$$\frac{dD}{di} = 2 - 4\frac{d\theta}{di}$$

By Snell's law, $\mu \sin\theta = \sin I$

Differentiating w.r.t. 'i' we get,

$$\mu \cos\theta \frac{d\theta}{di} = \cos i$$

$$\Rightarrow \quad \frac{dD}{di} = 2 - 4\left[\frac{\cos i}{\mu \cos\theta}\right]$$

When D is minimum $\frac{dD}{di} = 0$

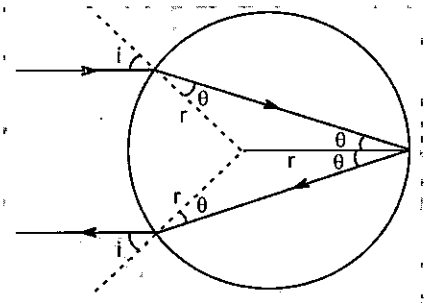


Fig. 1E.90

$$\Rightarrow \quad 2-\frac{4\cos i}{\mu\cos\theta}=0$$

$$2=\frac{4\cos i}{\mu\cos\theta}$$

$$\mu\cos\theta=2\cos i$$

$$\mu\cdot\sqrt{1-\sin^2\theta}=2\cos i$$

$$\mu\cdot\sqrt{1-\frac{\sin^2 i}{\mu^2}}=2\cos i$$

$$\Rightarrow \quad \mu^2-\sin^2 i=4\cos^2 i=4[1-\sin^2 i]$$

$$\Rightarrow \quad \mu^2=4-3\sin^2 i$$

$$\sin i=\sqrt{\frac{4-\mu^2}{3}}$$

$$\Rightarrow \quad i=\sin^{-1}\sqrt{\frac{4-\mu^2}{3}}$$

$$D_{\min}=\pi+2\sin^{-1}\sqrt{\frac{4-\mu^2}{3}}-4\sin^{-1}\frac{1}{\mu}\sqrt{\frac{4-\mu^2}{3}}$$

Example 91

A beam of light is incident vertically on a glass hemisphere of radius R lying with its plane side of table. The axis of the beam coincides with the vertical axis passing through the centre of the base of the hemisphere, and the radius r_0 of the cross-section of the beam is smaller than R. Find the radius of the luminous spot formed on the table.

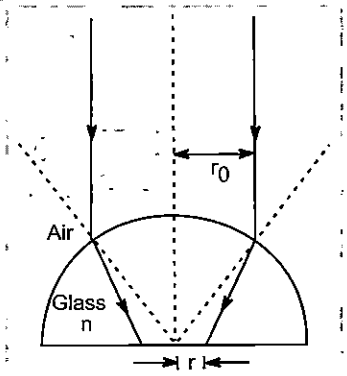


Fig. 1E.91 (a)

Solution: In the triangle *OAB*

$$\frac{r}{\sin\theta_2}=\frac{R}{\sin(90+\theta_1-\theta_2)}$$

$$r=\frac{\sin\theta_2}{\cos(\theta_1-\theta_2)} \qquad ...(1)$$

Also, from the figure

$$r_0=R\sin\theta_1 \qquad ...(2)$$

From Snell's rule

$$\sin\theta_1=n\sin\theta_2 \qquad ...(3)$$

Eliminating $\sin\theta_1$ and $\sin\theta_2$ from equations (1), (2) and (3)

$$r=\frac{r_0/n}{\cos(\theta_1-\theta_2)}$$

since, $\cos(\theta_1-\theta_2)=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2$

and $$\cos\theta_1=\sqrt{1-\sin^2\theta_2}=\sqrt{1-\left(\frac{r_0}{R}\right)^2}$$

$$\cos\theta_2=\sqrt{1-\sin^2\theta_2}=\sqrt{1-\frac{\sin^2\theta_1}{n^2}}$$

$$=\frac{1}{n}\sqrt{n^2-\left(\frac{r_0}{R}\right)^2}$$

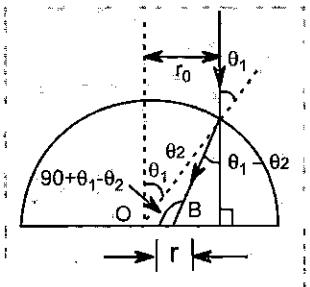


Fig. 1E.91 (b)

$$\therefore \quad r=\frac{r_0/n}{\sqrt{1-\left(\frac{r_0}{R}\right)^2}\,\frac{1}{n}\sqrt{n^2-\left(\frac{r_0}{R}\right)^2}+\frac{1}{n}\left(\frac{r_0}{R}\right)^2}$$

Example 92

Consider the situation sketched. An illuminated object lies 40 cm under the surface of water ($n = 4/3$), 80 cm above a concave spherical mirror whose radius of curvature is 120 cm. An observer looking down from above in the air ($n = 1.00$) sees two images. Find distance (in meters) between these two images.

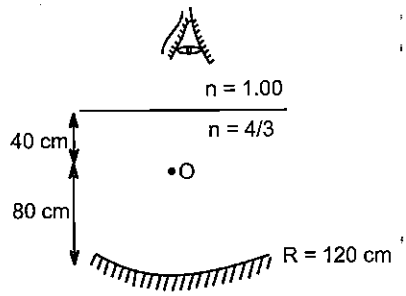


Fig. 1E.92 (a)

Solution: Position of object seen directly

$$\frac{40}{4/3} \times 1 = 30\,\text{cm below}$$

Position of image seen in mirror there are two events Reflection and refraction

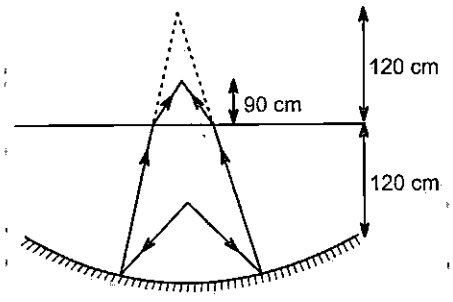


Fig. 1E.92 (b)

Mirror image is at v

$$v = \frac{(-80)(-60)}{(-80)-(-60)} = \frac{80 \times 60}{-20} = -240\,\text{cm}$$

Real image mirror is at water surface 120 cm above

Thus for refraction at mirror image is formed at

$$\frac{120}{4/3} = 90\,\text{cm}$$

Example 93

A particle lies on the bottom of a tank T filled with water upto a height of 90cm. The medium above the surface of water is of R.I.=1.2 above which there is mirror M. Beyond the mirror M the region contains air ($\mu = 1$). The distance of the image formed by the mirror after reflection of the rays coming from P is :

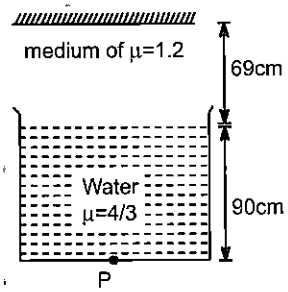


Fig. 1E.93 (a)

Solution: Distance of first image from water surface

$$= \frac{90\mu_m}{\mu_w} = \frac{90 \times 6/5}{4/3} = 81\,\text{cm}$$

Distance of I_1 from mirror = 69 + 81 = 150 cm

$\Rightarrow$ Distance of final image from mirror = 150 cm

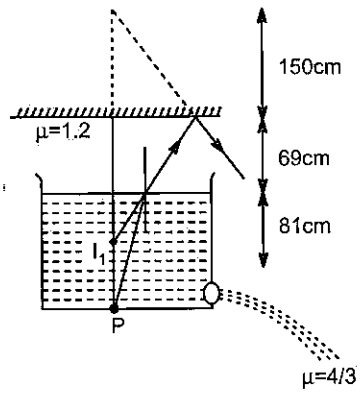


Fig. 1E.93 (b)

Example 94

A person on a bridge spanning a pool looks over the side and sees a small object vertically below. He moves along a distance x till his line of vision makes an angle of 45° with the surface of the water. If h is the height of eye level above the water surface, then find the depth d of the pool taking μ as the refractive index of water.

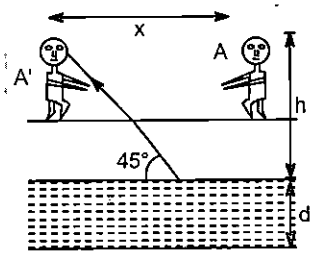


Fig. 1E.94 (a)

Solution: Using Snell's law

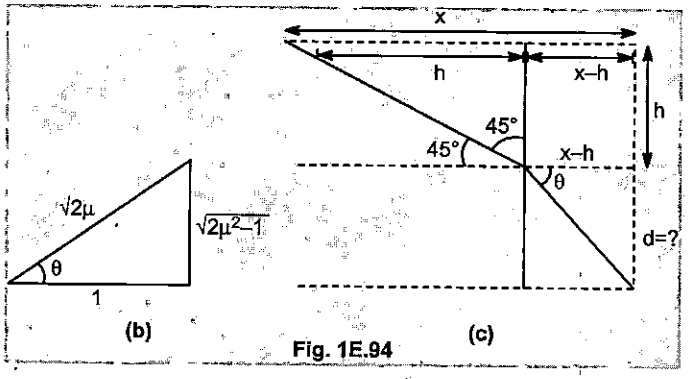


Fig. 1E.94

$$\frac{\sin 45°}{\sin\left(\frac{\pi}{2}-\theta\right)}=\mu$$

$$\cos\theta=\frac{1}{\sqrt{2}\mu}$$

$$\tan\theta=\frac{d}{x}\Rightarrow d=(x-h)\tan\theta=(x-h)\sqrt{2\mu^2-1}$$

Example 95

Water level in the tank is decreasing at a constant rate of 1 cm/s. *A small metal sphere is moving downwards with a constant velocity* 5 cm/s. *Base of the tank is a concave mirror of radius 40 cm. Find the velocity of the image seen* [Take $_a\mu_w = 4/3$]

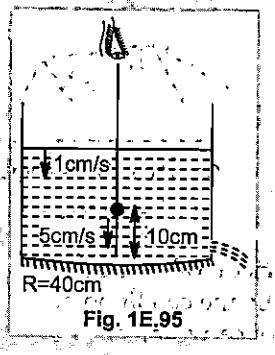


Fig. 1E.95

(a) Directly

(b) After reflection at the mirror.

Solution: (a) Here we calculate the velocity with respect to surface and then convert it to ground frame.

$$\text{Directly } v_{i/s}=\frac{v_{os}}{\mu}$$

$$v_{i/s}=(+1)=\frac{3}{4}(5-1)$$

$$v_i = 4\,\text{cm/s downwards}$$

(b) $$m=\frac{f}{f-u}=\frac{-20}{-20-(-10)}=2$$

Object is placed at $u = -10$ cm its image is virtual and magnified

$$v_{i/m}=-m^2 v_{O/M}$$

Finally after refraction

$$v_{i/s}=\frac{u_{O/s}}{M}$$

or $$v_i - v_s = \frac{3}{4}(u_O - u_s)$$

$$v_i-(1)=\frac{3}{4}(-20-1)$$

$$v_i=-\frac{63}{4}+1$$

$$=\frac{59}{4}\text{ cm/s upwards}$$

DISPERSION

For most transparent dielectrics the index of refraction gradually decreases as the wavelength changes from shorter visible wavelengths (blue light) to the longer visible wavelengths (red light). So white light, composed of a combination of wavelengths, when refracts through an air-glass boundary, the blue light is bent more toward the normal than the red light. A **prism** can be used to separate the wavelengths into a spectrum in which the colours are arranged according to wavelength. When an index of refraction n decreases with increasing wavelength, the phenomenon is known as **normal dispersion.**

Prism Geometry

Angle of deviation δ is the angle between the incident ray and the emergent ray.

$$\delta=\delta_{\text{first surface}}+\delta_{\text{second surface}}$$

$$=(i-r_1)+(e-r_2)$$

$$=(i+e)-(r_1+r_2) \quad \ldots(1)$$

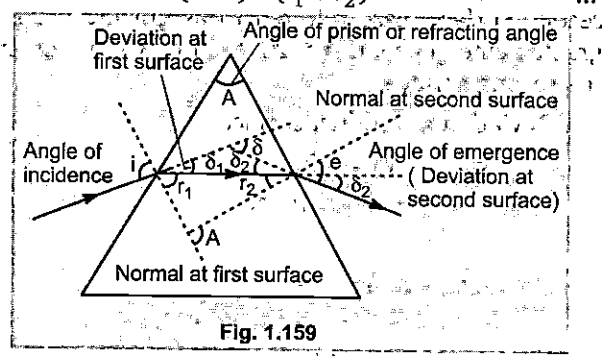


Fig. 1.159

From figure, $A = r_1 + r_2$ ... (2)

Snell's law is applicable at both refracting surfaces.

$$1\sin i = n\sin r_1 \quad \ldots(3)$$

$$n\sin r_1 = 1\sin e \quad \ldots(4)$$

❒ **Normal Incidence :** In this case, $i = 0, r_1 = 0$.

From geometry we can see that $r_2 = A$.

The ray of light will emerge at the second surface only if

$$r_2 < C \quad \text{or} \quad A < C$$

where C is the critical angle for glass-air interface.

Deviation at the first surface,

$$\delta_1 = 0$$

Deviation at the second surface,

$$\delta_2 = e - r_2 = e - A$$

Net deviation, $\delta = \delta_1 + \delta_2 = e - A$

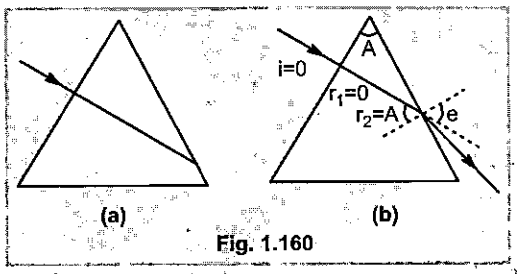


Fig. 1.160

In this position, deviation is maximum.

From Snell's law, we have

$$\sin e = n \sin A$$

or $$e = \sin^{-1}(n \sin A)$$

❒ **Grazing Incidence :** In this case, $i = 90°$, $r_1 = C$

$$C + r_2 = A$$

or $$r_2 = A - C$$

If the ray of light is to emerge from the second surface,

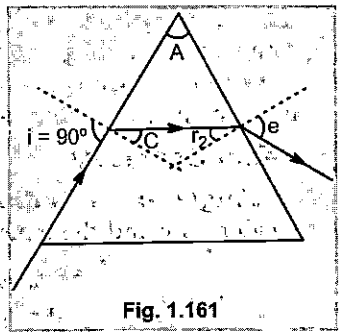


Fig. 1.161

$$r_2 < C$$

or $$A - C < C$$

or $$A < 2C$$

If $r_2 = C$, the ray of light will emerge grazing from the second surface, case referred to as grazing incidence and grazing emergence. If $A > 2C$, the ray will not emerge from the second surface.

Deviation at the first surface,

$$\delta_1 = 90 - C$$

Deviation at the second surface

$$\delta_2 = e - r_2 = e - (A - C)$$

Net deviation $\delta = \delta_1 + \delta_2$

$$= (90 - C) + (e - A + C)$$

$$= 90 - A + e$$

From Snell's law, we have

$$\sin e = n \sin(A - C)$$

or $$e = \sin^{-1}[n \sin(A - C)]$$

❒ **Prisms of small angle :** Consider a small angle prism shown in Fig. 1.162.

The deviation,

$$\delta = (i - r_1) + (e - r_2)$$

and $$A = r_1 + r_2$$

If the refracting angle (A) is small, then i, e, r_1 and r_2 are small, and so

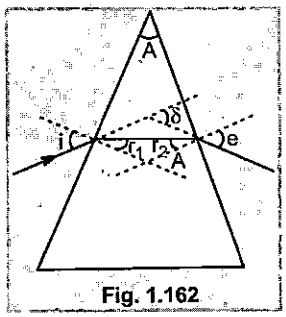


Fig. 1.162

$$i = nr_1 \text{ and } e = nr_2$$

$$\therefore \quad \delta = nr_1 + nr_2 - A$$

$$= n(r_1 + r_2) - A$$

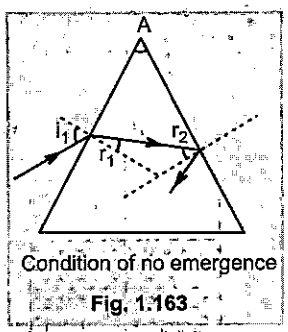
$$= nA - A$$

$$= (n - 1)A$$

The deviation is therefore independent of the angle of incidence and depends only on the refracting angle of the prism and the refractive index of light.

1. Condition of no Emergence: A ray of light incident on a prism of angle A and refractive index μ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where θ_c is the critical angle.

i.e., $$\mu > \frac{1}{\sin(A/2)}$$

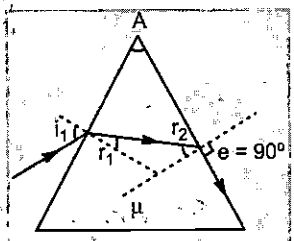


Condition of no emergence

Fig. 1.163

2. Condition of Grazing Emergence: By the condition of grazing emergence we mean the angle of incidence i at which the angle of emergence becomes $e = 90°$.

$$i = \sin^{-1}\left[\sqrt{\mu^2 - 1}\sin A - \cos A\right]$$



Grazing emerging ray in a prism

Fig. 1.164

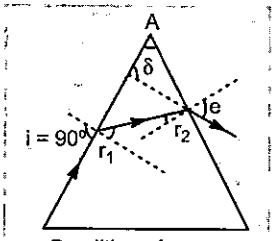
Note: The light will emerge out of a given prism only if the angle of incidence is greater than the condition of grazing emergence.

3. Condition of Maximum Deviation: Maximum deviation occurs when the angle of incidence is 90°.

$$\delta_{max} = 90° + e - A$$

where

$$e = \sin^{-1}[\mu \sin(A - \theta_c)]$$



Condition of maximum derivation

Fig. 1.165

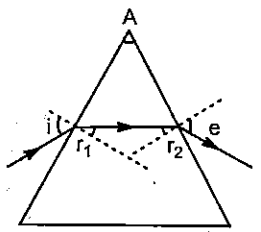
4. Condition of Minimum Deviation: The minimum deviation occurs when the angle of incidence is equal to the angle of emergence, *i.e.*,

$$i = e$$

$$\delta_{min} = 2i - A$$

Using Snell's law

$$\mu = \frac{\sin\left[\frac{\delta_{min} + A}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$



Light ray passes through a prism symmetrically in the condition of the minimum deviation

Fig. 1.166

Note: In the condition of minimum deviation the light ray passes through the prism symmetrically, *i.e.*, the light ray in the prism becomes parallel to its base.

Characteristic of a Prism

(a) Variation of δ *versus* i (shown in diagram).

For each value of δ there are two values of angle of incidence except for angle of minimum deviation.

If i and e are interchanged then we get the same value of δ because of reversibility principle of light.

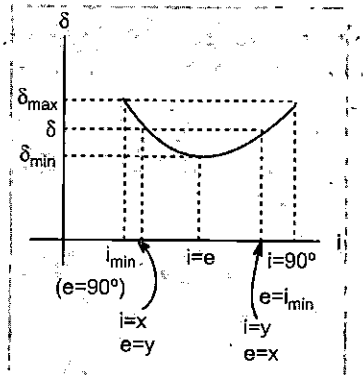


Fig. 1.167

(b) There is one and only one angle of incidence for which the angle of deviation is minimum.

(c) When $\delta = \delta_{min}$, the angle of minimum deviation, then $i = e$ and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show that $\delta_{min} = 2i_{min} - A$ where i_{min} = angle of incidence for minimum deviation, and $r = A/2$

$$\therefore \quad n_{rel} = \frac{\sin\left[\frac{A + \delta_{min}}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$

where $\quad n_{rel} = \dfrac{n_{prism}}{n_{surrounding}}$

Also $\quad \delta_{min} = (n - 1)\,A$ (for small values of $\angle A$)

Illustration 12: A prism with angle $A = 60°$ produces a minimum deviation of 30°. Find the refractive index of the material.

Solution: We know that

$$\mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Here $\quad A = 60°,\ \delta_{min} = 30°$

$$\therefore \quad \mu = \frac{\sin\left(\frac{60° + 30°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

Example 96

Find the co-ordinates of image of the point object 'O' formed after reflection from concave mirror as shown in Fig. 1E.96 (a) assumming prism to be thin and small in size of prism angle 2°. Refractive index of prism material is 3/2.

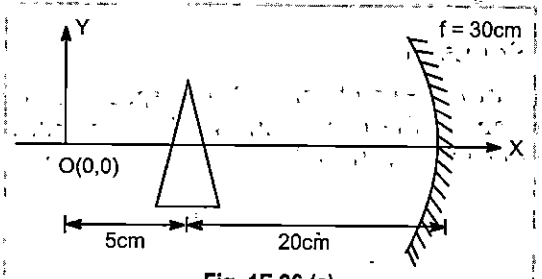


Fig. 1E.96 (a)

Solution: Consider image formation through prism. All incident rays will be deviated by

$$\delta = (\mu - 1)A = \left(\frac{3}{2} - 1\right)2° = 1° = \frac{\pi}{180} \text{ rad}$$

Now as prism is thin so object and image will be in same plane as shown in Fig. 1E.96 (b).

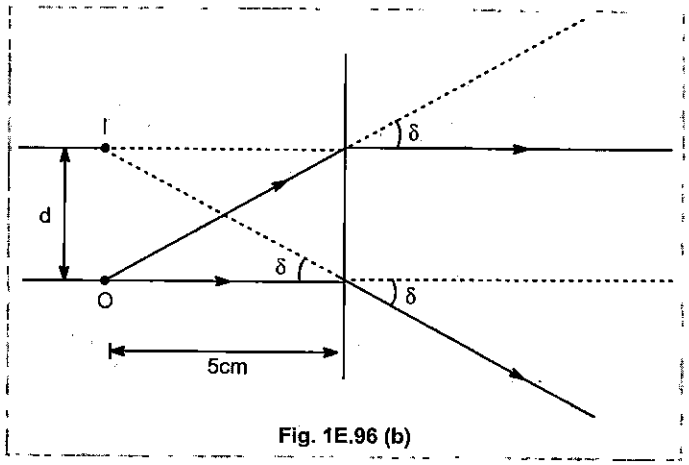


Fig. 1E.96 (b)

It is clear $\frac{d}{5} = \tan\delta \approx \delta$ ($\because \delta$ is very small)

or $d = \frac{\pi}{36}$ cm

Now this image will act as an object for concave mirror.

$u = -25\,\text{cm}, f = -30\,\text{cm}$

$\therefore \quad v = \frac{uf}{u-f} = 150$ cm

Also, $m = \frac{-v}{u} = +6$

$\therefore$ Distance of image from principal axis

$= \frac{\pi}{36} \times 6 = \frac{\pi}{6}$ cm

Hence, co-ordinates of image formed after reflection from concave mirror are $\left(175\text{cm}, \frac{\pi}{6}\text{cm}\right)$.

The two faces of the prism through which rays enter and emerge are termed *refraction faces*, the term for the angle ϕ between these faces being *refraction angle* of the prism (Fig. 1.168).

Important Points

1. Let a narrow beam of light AO_1 of a specified colour fall from the air on the prism with a refraction index n. Inside the prism it follows the path O_1O_2. Emerging from the prism it moves away from the normal to the face and follows the path O_2B. Hence the effect of the prism is to deflect the beam towards its base. Since the direction of the light before it entered the prism was AO_1 and after it left it became O_2B, the beam was deflected by the angle δ [see Fig. 1.168 (a)], termed *the angle of deflection*. This angle is the greater refractive index of the material of the prism or its refraction angle ϕ.

2. If a triangular prism is placed in a medium for which its relative refractive index is below unity, the ray AO_1 [see Fig. 1.168(b)], after passing through the prism, will be deflected by the angle δ not towards the base but towards the apex of the prism.

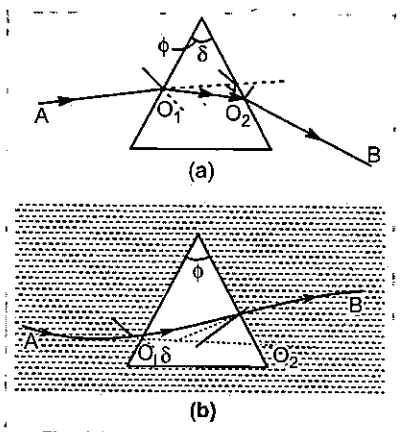


Fig. 1.168: Path of monochromatic ray through triangular prism made of material of (a) greater optical density than medium and (b) lesser optical density

3. Calculation shows the critical angle for glass to be about 42°. Therefore it is easy to obtain total reflection in a triangular glass prism with two angles of 45° Fig. 1.169 (a) shows a change in direction of light rays of90° in such a prism, while Fig. 1.169 (b) shows the reversal of an image by the same prism.

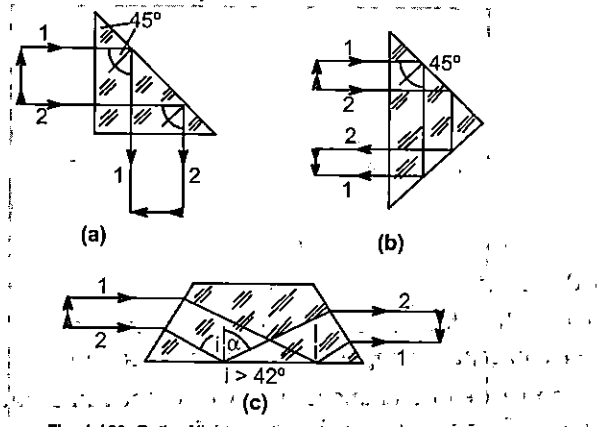


Fig. 1.169: Path of light rays in total reflection prism

Fig. 1.169 (c) shows a direct-vision prism and the path of rays in it. The upper and lower rays are seen to change places, but they continue to propagate in the original direction.

4. There is one more point to be noted. Since the index of refraction n depends on the wavelength λ, the deflection of the rays in a prism also depends on their colour. For instance, the angle of deflection δ, for red rays is less than for blue.

5. Prisms have the property to change the direction of light. Figs. 1.170 (a) to (e) present some of the useful applications.

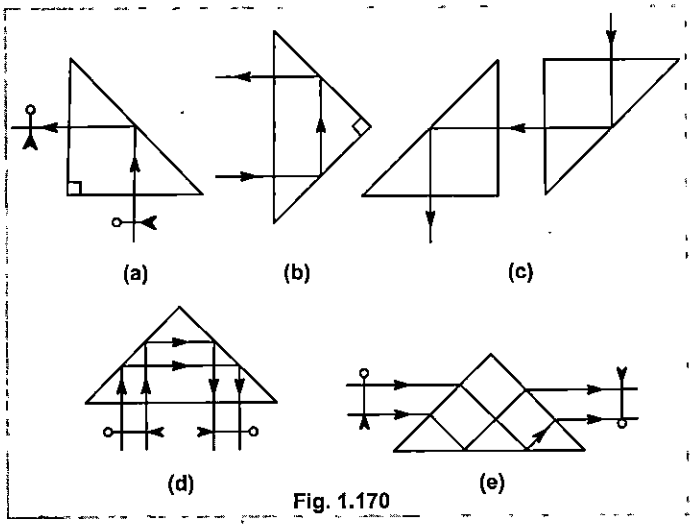


Fig. 1.170

6. The right-angled prism in Fig. 1.170 (a) turns the light through 90°. A poro prism shown in Fig. 1.170 (b) is a right- angled prism, it turns light through 180°. Fig 1.170 (c) shows two right-angled prisms used in binoculars. Figs. 1.170 (d) and (e) illustrate image formation with and without deviation; in both the cases the image is laterally inverted.

Dispersion

According to Cauchy's experimental relation, the refractive index of a material has dependence on the wavelength of light, and may be expressed as

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots$$

This is a polynomial in $1/\lambda^2$. Normally only the first two terms are taken. The constants A and B can be determined by plotting $n(\lambda)$ *vs.* $1/\lambda^2$; A is the intercept on the $n(\lambda)$ axis for $1/\lambda^2 = 0$ and B is the gradient. A graph of refractive index against wavelength is shown in Fig. 1.171. The refractive index for violet light is greater than for red light. The prism deviates violet light the most and red the least. When white light passes through a prism, due to different deviation of component colours, white light disperses into its components. This phenomenon is called **dispersion of light.**

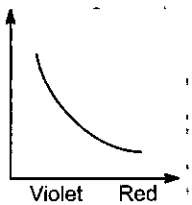


Fig. 1.171

Dispersive Power

The average deviation is measured from the deviation of yellow colour, referred to as **mean deviation** (δ_4). If δ_r and δ_v are the deviations of red and violet colours respectively, the emerging spectrum is spread in an angle $\delta_v - \delta_r$. This angle is called **angular dispersion.**

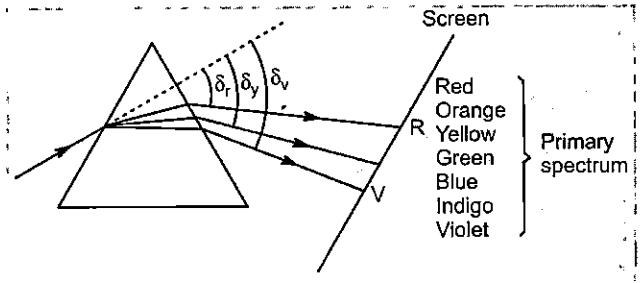


Fig. 1.172

For a small angle prism,

mean deviation, $\delta_y = (n_y - 1)A$

angular dispersion, $D = \delta_v - \delta_r$

$= (n_v - 1)A - (n_r - 1)A$

$= (n_v - n_r)A$

Dispersive power of a medium is defined as the ratio of angular dispersion and mean deviation.

$$\text{Dispersive power } (\omega) = \frac{D}{\delta_4} = \frac{(n_v - n_r)}{(n_y - 1)}$$

Dispersive power is a dimensionless number and depends on the material of the prism.

Combination of Prisms

Normally a prism deviates as well as disperses white light but with a combination of two prisms, deviation or angular dispersion can be nullified.

For the combination of two prisms,

net mean deviation, $\delta_y = \delta_{1y} + \delta_{2y}$

$= (n_y - 1)A + (n'_y - 1)A'$ ... (1)

net dispersive power, $D = D_1 + D_2$

$= (n_v - n_r)A + (n'_v - n'_r)A'$ ... (2)

We have two cases :

1. Dispersion without deviation : From eqn. (1) we have

$\delta_y = 0$

or $A' = -\dfrac{(n_y - 1)}{(n'_y - 1)}A$ ... (3)

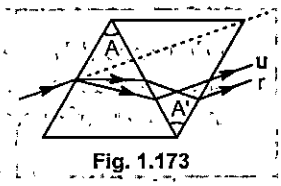


Fig. 1.173

In this case net dispersive power, from eqn. (2), is

$$D = (n_v - n_r)A + (n'_v - n'_r)\left\{-\frac{(n_y - 1)}{(n'_y - 1)}A\right\}$$

$$= (n_y - 1)A\left[\frac{n_v - n_r}{n_y - 1} - \frac{n'_v - n'_r}{(n'_y - 1)}\right]$$

$$= (n_y - 1)A(\omega - \omega')$$

$$= \delta_{1y}(\omega - \omega') \quad \ldots (4)$$

If $\omega' > \omega$, then $D < 0$, the dispersion is in opposite direction to deviation from first prism. Such a combination of prisms that produces dispersion without deviation is called a *direct vision prism.*

2. Deviation without dispersion : From eqn. (2) we have $D = 0$

$$\text{or} \quad A = -\frac{(n_v - n_r)}{(n'_v - n'_r)}A \qquad \ldots (5)$$

$$\text{As} \quad \omega = \left(\frac{n_v - n_r}{n_y - 1}\right)$$

$$\text{and} \quad \omega' = \left(\frac{n'_v - n'_r}{n'_y - 1}\right)$$

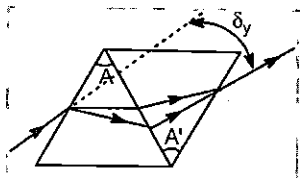


Fig. 1.174

$$n_v - n_r = \omega(n_y - 1)$$

$$\text{and } n'_v - n'_r = \omega'(n'_y - 1)$$

$$\text{Eqn. (5) reduced to } A' = -\frac{\omega(n_y - 1)}{\omega'(n'_y - 1)} \qquad \ldots (6)$$

In this case net deviation is

$$\delta_y = (n_y - 1)A + (n'_y - 1)A'$$

$$= (n_y - 1)A + (n'_y - 1)\left[-\frac{(n_v - n_r)}{(n'_v - n'_r)}A\right]$$

$$= (n_y - 1)A\left[1 - \frac{(n_v - n_r)(n'_y - 1)}{(n_y - 1)(n'_v - n'_r)}A\right]$$

$$= \delta_{1y}\left(1 - \frac{\omega}{\omega'}\right)$$

Normally $\omega > \omega'$. so net deviation (δ_y) is in the same direction as deviation by first prism (δ_{1y}). Such a combination of prisms is called an achromatic combination.

Example 97

What should be the minimum value of refractive index of a prism, refracting angle A, so that there is no emergent ray irrespective of angle of incidence ?

Solution: If the ray just emerges from face *AC*,

$$e = 90° \text{ and } r_2 = C \qquad \ldots (1)$$

From Snell's law at face *AB*, we have

$$1 \sin i = n \sin r_1 \qquad \ldots (2)$$

and

$$A = r_1 + r_2$$

$$= r_1 + C \qquad \ldots (3)$$

From eqn. (2) n is minimum when r_1 is maximum, *i.e.*, $r_1 = C$. In this case $i = 90°$.

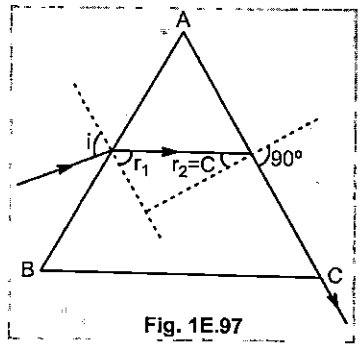


Fig. 1E.97

From eqn. (3), $A = 2C$ or $C = A/2$

As $\sin C = \frac{1}{n}$, $\sin\frac{A}{2} = \frac{1}{n}$

$$n = \text{cosec}\frac{A}{2}$$

Example 98

A ray of light PQ is incident at an angle i on face AB of a prism ABC and it emerges grazing face AC. If n is the refractive index and A the angle of the prism, show that

$$n = \left\{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2\right\}^{1/2}$$

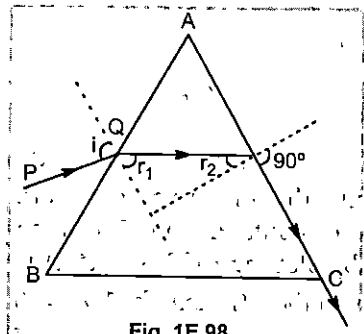


Fig. 1E.98

Solution: From Snell's law at face *AB*,

$$1 \sin i = n \sin r_1 \qquad \ldots (1)$$

$$n = \frac{\sin i}{\sin r_1} = \frac{\sin i}{\sin(A - r_2)} \qquad [\text{as } A = r_1 + r_2]$$

$$= \frac{\sin i}{(\sin A \cos r_2 - \cos A \sin r_2)} \qquad \ldots (2)$$

From Snell's law at face *AC*,

$$n \sin r_2 = 1 \sin 90°$$

$$\sin r_2 = \frac{1}{n} \qquad \ldots (3)$$

$$\cos r_2 = \frac{1}{n}(n^2 - 1)^{1/2} \qquad \ldots (4)$$

Now we substitute eqns. (3) and (4) in eqn. (2)

$$n = \frac{\sin i}{\sin A\left[\frac{1}{n}(n^2-1)^{1/2}\right] - \cos A(1/n)}$$

$$= \frac{n\sin i}{(n^2-1)^{1/2}\sin A - \cos A} \quad ...(5)$$

which on solving yields

$$(n^2-1)^{1/2} = \frac{\sin i + \cos A}{\sin A} \quad ...(6)$$

$$n^2 - 1 = \left(\frac{\sin i + \cos A}{\sin A}\right)^2$$

or $$n = \left[1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2\right]^{1/2}$$

Example 99

A ray of light suffers minimum deviation through a prism of refractive index $\sqrt{2}$. What is the angle of prism if the angle of incidence is double the angle of refraction within the prism ?

Solution: In the case of minimum deviation,

$$i_1 = i_2, \qquad r_1 = r_2$$

So $$\delta = 2i - A$$

and $$r = A/2$$

According to problem, $$i = 2r = A$$

So $$\delta_{min} = 2A - A = A$$

Thus $$n = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\cos\frac{A}{2} = \frac{n}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

or $$A = 90°$$

Example 100

A ray of light undergoes a deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism ?

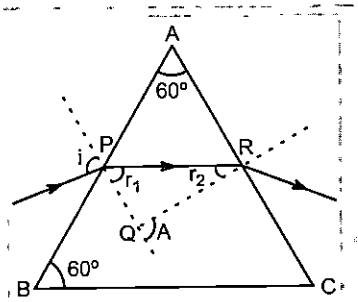


Fig. 1E.100 (a)

Solution: The given parameters are $\delta = 30°$ and $A = 60°$. Let us test whether the prism is in the position of minimum deviation.

$$n = \frac{\sin\left(\frac{30°+60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

As $n = \sqrt{2}$, the ray suffers minimum deviation through the prism. Thus

$$r_1 = r_2 = r = \frac{A}{2} = 30°$$

Inside the prism, the ray makes an angle 60° with the face AB, so it is parallel to base.

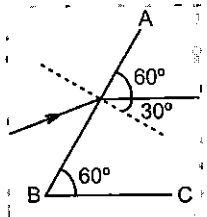


Fig. 1E.100 (b)

Example 101

Consider a prism with refracting angle A and refractive index n. What is the critical angle of incidence for which the emergent ray grazes out ?

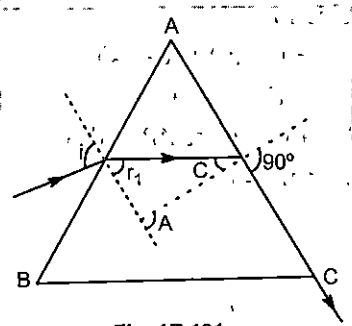


Fig. 1E.101

Solution: The ray grazes face AC

So, $$e = 90°, r_2 = C \quad ... (1)$$

and $$A = r_1 + r_2 = r_1 + C$$

Also $$\sin C = 1/n \quad ...(2)$$

From Snell's law at face AB,

$$1\sin i_1 = n\sin r_1 \quad \ldots (3)$$

$$\sin i_1 = n\sin(A-C)$$

$$= n[\sin A\cos C - \cos A\sin C]$$

$$= n[\sin A\sqrt{1-\sin^2 C} - \cos A\sin C$$

$$= \sqrt{n^2-1}\sin A - \cos A$$

$$i_1 = \sin^{-1}[\sqrt{n^2-1}\sin A - \cos A]$$

Example 102

A ray of light is incident at angle i on one face of a prism, refracting angle A. Show that angle of deviation δ is $\delta = \sin^{-1}(n\sin r_1) + \sin^{-1}[n\sin(A-r_1)] - A$

Solution: In ΔPQR,

$$r_1 + r_2 + \alpha = 180° \quad \ldots . (1)$$

In quadrilateral $PQRA$,

$$90°+\alpha + 90°+A = 360° \quad .. (2)$$

$$A + \alpha = 180° \quad \ldots (3)$$

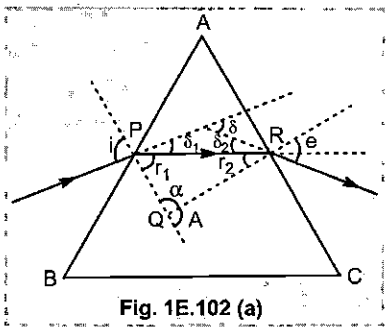


Fig. 1E.102 (a)

From eqns. (1) and (3), $r_1 + r_2 + \alpha = A + \alpha$

or $$r_1 + r_2 = A \quad \ldots (4)$$

Angle of deviation,

$$\delta = (i - r_1) + (e - r_2) \quad \ldots (5)$$

$$= (i + e) - A$$

From Snell's law at faces AB and AC,

$$1\sin i = n\sin r_1$$

$$i = \sin^{-1}(n\sin r_1) \quad \ldots (6)$$

$$n\sin r_2 = 1\sin e$$

$$\sin e = n\sin(A - r_1) \quad \ldots (7)$$

Now substituting eqns. (6) and (7) in (5), we have

$$\delta = \sin^{-1}(n\sin r_1) + \sin^{-1}[n\sin(A-r_1)] - A$$

❒ The angle of deviation is minimum when the ray travels symmetrically through the prism, *i.e.*,

$$i_1 = i_2 = i$$

$$r_1 = r_2 = r$$

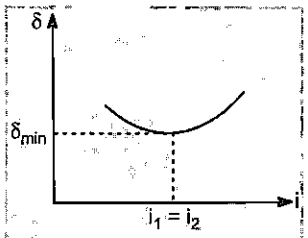


Fig. 1E.102 (b)

Eqn. (5) gives

$$\delta = 2i - A \quad \ldots (8)$$

and $$A = 2r \quad \ldots (9)$$

From eqn. (8), $i = \dfrac{A + \delta_{min}}{2}$

From eqn. (9), $r = A/2$

$$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

Example 103

An isosceles prism has one of the refracting surfaces silvered. A ray of light is incident normally on the refracting face AB. After two reflections the ray emerges from the base of the prism perpendicular to it. Find the angles of the prism.

Solution: The incident ray passes without deviation from face AB. It suffers reflections at P and Q. From Fig. 1E.103, incident ray and normal at Q are parallel; therefore

$$\alpha = 2A \quad \ldots (1)$$

Also

$$2\alpha + A = 180° \quad \ldots (2)$$

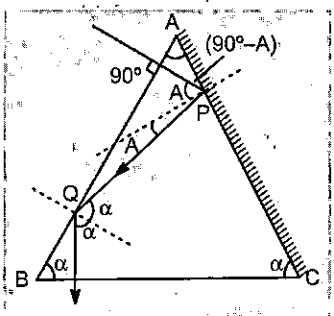


Fig. 1E.103

On solving eqns. (1) and (2), we get

$$A = 36°, \alpha = 72°$$

Example 104

Two identical thin isosceles prisms angle A and refractive index μ are placed with their bases touching each other. This system can act as a crude converging lens. Draw a neat diagram showing the path of parallel incident rays. Obtain the focal length of the system. The height of incident is h.

Solution: The deviation produced by thin prism

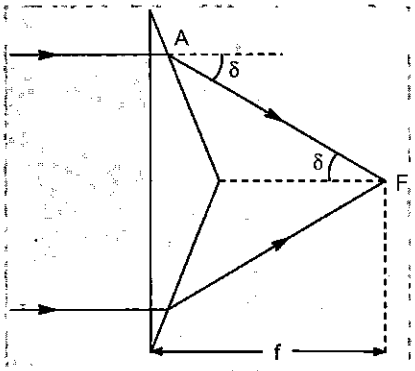


Fig. 1E.104

$$\delta = (\mu - 1)A$$

For small angled prism

$$\delta;\ \tan\delta = \frac{h}{f}$$

$$\therefore \quad \frac{h}{f} = (\mu - 1)A$$

or $$f = \frac{h}{(\mu - 1)A}$$

Example 105

An isosceles prism of angle 120° has a refractive index $\sqrt{2}$. Two parallel monochromatic rays enter the prism parallel to each other in air as shown in Fig. 1E.105. Find the angle between the emerging rays.

Solution: The angle of incidence on the inclined face of the prism is 30°. If r is the angle of refraction, then by Snell's law

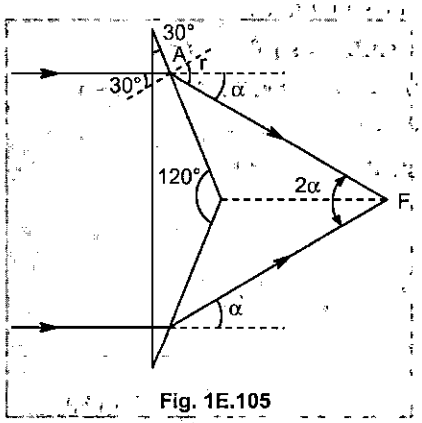


Fig. 1E.105

$$\frac{\sin r}{\sin 30°} = \sqrt{2}$$

or $$\sin r = \frac{1}{\sqrt{2}}$$

$$\therefore \quad r = 45°$$

Angle $\alpha = 45° - 30° = 15°$

The required angle $= 2\alpha$

$= 2 \times 15° = 30°$

Example 106

The refracting angle of a prism is 90°. If γ is the angle of minimum deviation and β is the deviation of ray which enters at grazing incidence, prove that $\sin\gamma = \sin^2\beta$ and $\cos\gamma = \mu\cos\beta$, μ is refractive index of material of the prism.

Solution:

$$\left.\begin{aligned} 1 \times \sin 90° &= \mu \sin\gamma_1 \\ 1 \times \sin\beta &= \mu\cos\gamma_1 \end{aligned}\right\} \Rightarrow 1 + \sin^2\beta = \mu^2 \qquad ....(1)$$

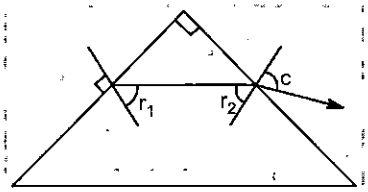


Fig. 1E.106

$$\mu = \frac{\sin\left(\frac{90°+\gamma}{2}\right)}{\sin 45°} \Rightarrow \mu = \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\left(\frac{\gamma}{2}\right) + \frac{1}{\sqrt{2}}\sin\left(\frac{\gamma}{2}\right)\right]$$

$$\mu = \cos\left(\frac{\gamma}{2}\right) + \sin\left(\frac{\gamma}{2}\right)$$

Squaring both sides

$$\mu^2 = 1 + \sin\gamma \qquad ....(2)$$

From eqns. (1) and (2),

$$\sin^2\beta = \sin\gamma$$ **Proved**

From eqn. (2),

$$1 - \cos^2\gamma = (\mu^2 - 1)^2 = \mu^4 + 1 - 2\mu^2$$

$$\therefore \quad \cos^2\gamma = \mu^2(2 - \mu^2) = \mu^2[2 - (1 + \sin^2\beta)]$$

$$= \mu^2[1 - \sin^2\beta] = \mu^2\cos^2\beta$$

$$\therefore \quad \cos\gamma = \mu\cos\beta$$ **Proved**

Example 107

The refractive indices of the crown glass for violet and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. A narrow beam of white light is incident at a small angle of incident on the shown combination of thin prisms.

Find the value of α for which the mean deviation of the incident beam is zero. Also calculate the magnitude of dispersion produced by the combination and state order of colours in emerging light from top to bottom.

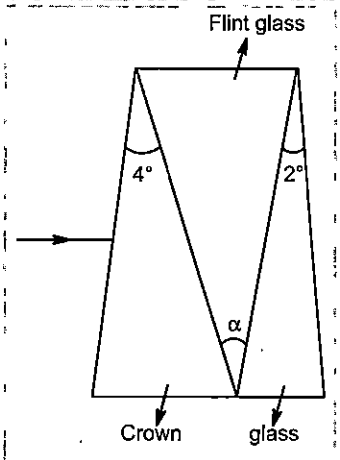


Fig. 1E.107 (a)

Solution: CG, $\mu_V = 1.51$; $\mu_R = 1.49$

FG, $\mu_V = 1.77$; $\mu_R = 1.73$

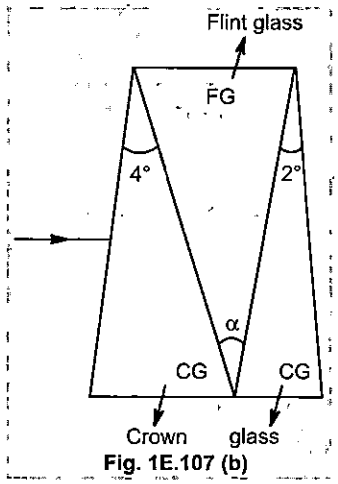


Fig. 1E.107 (b)

$$\theta = \theta_1 - \theta_2 + \theta_3 = 0$$

$$= (\mu_Y - 1)4 - (\mu_Y - 1)\alpha + (\mu_Y - 1)2 = 0$$

$$= (1.57)4 - (1.57 - 1)\alpha + (1.5 - 1)2 = 0$$

$$\Rightarrow \quad \alpha = 4^\circ$$

$$\delta_m = \delta_1 - \delta_2 + \delta_3 = 0$$

$$= (\mu_V - \mu_R)4 - (\mu_V - \mu_R)\alpha + (\mu_V - \mu_e)2$$

$$= 0.02 \times 4 - 0.04 \times 4 + 0.02 \times 2 = -0.04^\circ$$

Magnitude is 0.04° but (–) ve sign

Indicate spectrum inversed ⇒ top colour is violet and bottom is red.

Example 108

Consider a prism of refracting angle $A = 60^\circ$, refractive index $n = 1.60$ (a). What is the smallest angle of incidence θ for which a ray can enter the left face of prism and exit the right face? (b) What angle of incidence θ is required for the ray to exit the prism with an identical angle θ for its refraction?

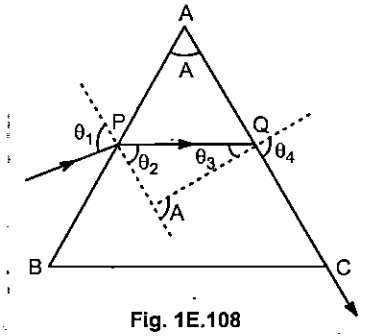


Fig. 1E.108

Solution: (a) From the geometry of the figure,

$$\theta_2 + \theta_3 = A \qquad \ldots (1)$$

Maximum value of θ_3 is critical angle, when emergent ray grazes surface *AC*.

From eqn. (1), θ_2 is minimum in this case. From Snell's law,

$$\sin\theta_1 = n \sin\theta_2 \qquad \ldots (2)$$

When θ_2 is minimum, θ_1 is also minimum.

When the incident ray is at the minimum angle for which the light ray emits the prism, the light emits along the second face.

From Snell's law, $\sin\theta_3 = \frac{1}{n} = \frac{1}{1.60} = 0.625$

$$\theta_3 = 38.68^\circ$$

As from eqn. (1),

$$\theta_2 = 60^\circ - 38.68^\circ = 21.32^\circ$$

from eqn. (2),

$$\sin\theta_1 = n \sin\theta_2 = 1.60 \sin 21.32^\circ = 0.5817$$

Thus $\theta_1 = 35.6^\circ$

(b) From Snell's law at *Q*,

$$n \sin\theta_3 = \sin\theta_4 = \sin\theta_1 \qquad \ldots (3)$$

$\theta_1 = \theta_4$, given in the problem

Also $\theta_2 + \theta_3 = A = 60^\circ \qquad \ldots (4)$

Thus $\sin\theta_1 = n \sin(60^\circ - \theta_2)$

$$= n \sin 60^\circ \cos\theta_2 - n \cos 60^\circ \sin\theta_2$$

From Snell's law at point *P*,

$$\sin\theta_1 = n \sin\theta_2$$

$$\sin\theta_2 = \frac{\sin\theta_1}{n} \qquad \ldots (5)$$

$$\cos\theta_2 = \sqrt{1 - \sin^2\theta_2} = \sqrt{1 - \left(\frac{\sin^2\theta_1}{n^2}\right)}$$

Hence $\sin\theta_1 = n \sin 60^\circ \sqrt{1 - \left(\frac{\sin^2\theta_1}{n^2}\right)} - \cos 60^\circ \sin\theta_1$

or $(1 + \cos 60^\circ)\sin\theta_1 = \sin 60^\circ \sqrt{n^2 - \sin^2\theta_1}$

On squaring both sides and solving for θ_1, we get

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}}$$

$$= \frac{1.60\sin 60°}{\sqrt{1+\cos 60°)^2 + \sin^2 60°}} = 0.80$$

Thus $\theta_1 = 53.1°$

Example 109

A prism of angle 60° deviates a ray of light through 40° for two angles of incidence which differ by 10°. What is the refractive index of the glass of the prism ?

Solution: In Fig. 1E.109 shown, incident ray *PQ* is deviated through 40°. In accordance with principle of reversibility, a ray incident along *SR* will also deviate through 40°.

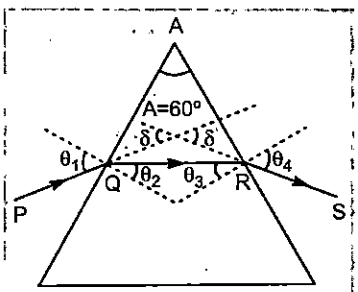


Fig. 1E.109

Hence, the two angles of incidence for which the ray has same deviation are θ_2 and θ_4. According to condition of the problem,

$$\theta_2 - \theta_4 = 11° \quad \ldots (1)$$

Also $\theta_2 + \theta_4 = A + \delta = 100°$ ... (2)

On solving eqns. (1) and (2), we get

$\theta_2 = 55°30'$ and $\theta_4 = 44°30'$

As $n = \dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{\sin\theta_4}{\sin\theta_3}$ ... (3)

where $\theta_3 = 60° - \theta_2$ ... (4)

hence $n = \dfrac{\sin 55°30'}{\sin\theta_2} = \dfrac{\sin 44°30'}{\sin(60°-\theta_2)}$

$$\frac{0.8241}{\sin r_1} = \frac{0.7009}{\frac{\sqrt{3}}{2}\cos r_1 - \frac{1}{2}\sin r_1}$$

Dividing both sides in denominator by $\sin r_1$, we get

$$0.8241 = \frac{0.7009}{\frac{\sqrt{3}}{2}\cot r_1 - \frac{1}{2}}$$

$$\frac{1}{2}(\sqrt{3}\cot r_1 - 1) = \frac{0.7009}{0.8241}$$

$$\cot r_1 = 1.559$$

$$r_1 = 32°40'$$

Thus $n = \dfrac{\sin 55°30'}{\sin 32°40'} = \dfrac{0.8241}{0.5400} = 1.526$

Example 110

Fig. 1E.110 shows a triangular prism of refracting angle 90°. A ray of light incident at face AB at an angle θ refracts at point Q with an angle of refraction 90°. (a) What is the refractive index of the prism in terms of θ ?

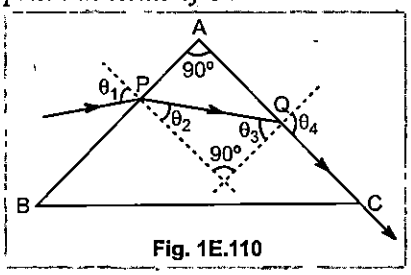


Fig. 1E.110

(b) What is the maximum value that the refractive index can have ? What happens to the light at Q if the incident angle at Q is (c) increased slightly and (d) decreased slightly ?

Solution: (a) Let the ray be incident at an angle θ_1 at face *AB*. It refracts at an angle θ_2 and is incident at an angle θ_3 at face *AC*. Finally the ray comes out at an angle $\theta_4 = 90°$.

From figure, the normals at faces *AB* and *AC* make an angle 90° with each other.

Hence $\theta_3 = 90° - \theta_2$

$$\sin\theta_3 = \sin(90°-\theta_2) = \cos\theta_2 = \sqrt{1-\sin^2\theta_2} \quad \ldots (1)$$

From Snell's law at face *AC*,

$$n\sin\theta_3 = 1$$

$$n\sqrt{1-\sin^2\theta_2} = 1 \quad \ldots (2)$$

From Snell's law at face *AB*,

$$1\sin\theta_1 = n\sin\theta_2$$

$$\sin\theta_2 = \frac{\sin\theta_1}{n} \quad \ldots (3)$$

From eqns. (2) and (3), we have

$$n\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = 1 \quad \ldots (4)$$

On squaring eqn. (4) and solving for *n*, we get

$$n = \sqrt{1+\sin^2\theta_1}$$

(b) The greatest possible value of $\sin^2\theta_1$ is 1, hence the greatest possible value of *n* is

$$n_{max} = \sqrt{2} = 1.41$$

(c) For a given *n*, if θ_1 is increased, the angle of refraction θ_2 increases. As $\theta_3 = 90° - \theta_2$, the angle θ_3 decreases, *i.e.*, the angle of incidence at face *AC* is less than the critical angle for total reflection; hence light emerges into air.

(d) If the angle of incidence is decreased, the angle of refraction θ_2 decreases. So the angle θ_3 increases. The angle

of incidence at the second surface is greater than the critical angle; so light is reflected at Q.

Example 111

There is a luminous point at the base of an equilateral prism of refractive index $\mu = 1.5$. Show that there is a direction which demarcates a brighter region from a darker region. Find this direction.

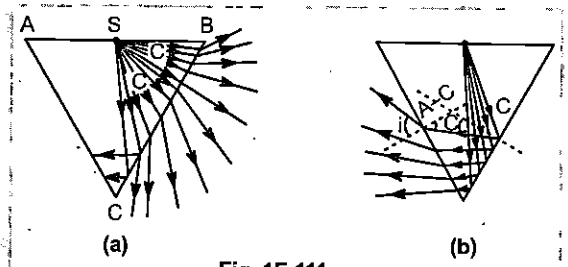


Fig. 1E.111

Solution: The rays diverging from S fall equally on faces AC and BC. Those rays which emerge into air produce uniform illumination [see Fig. 1E.111(a)]. The angles of incidence of these rays lie from 0 to C. Those rays which are incident at angles greater than C are totally reflected from BC to AC and *vice versa*. Those rays emerge into air and they produce greater illumination on one side and less illumination on the other [Fig. 1E.111(b)]. Let the direction make angle i with the normal. Then

$$\sin i = \mu \sin(A - C)$$

$$\Rightarrow \sin i = \mu(\sin A \cos C - \cos A \sin C)$$

$$= \mu\left(\sin A\sqrt{1-\frac{1}{\mu^2}} - \cos A\frac{1}{\mu}\right)$$

$$\Rightarrow \sin i = \sqrt{\mu^2 - 1}\sin A - \cos A$$

$$= \sqrt{1.5^2 - 1}\sin 60° - \cos 60°$$

$$\Rightarrow \sin i = 1.118 \times 0.8660 - 0.5 = 0.9682 - 0.5 = 0.4682$$

$$\Rightarrow i = 28°$$

Example 112

A prism with refracting angle 60° provides the least deviation angle 37° in air. Find the least deviation angle of that prism in water ($\mu = 4/3$)

Solution: We have

$${}_a\mu_g = \frac{\sin(60+37)/2}{\sin 60/2} = \frac{\sin 48.5}{\sin 30} = \frac{0.7489}{0.5000} = 1.498$$

Now ${}_w\mu_g = {}_w\mu_a \times {}_a\mu_g = \dfrac{1}{{}_a\mu_w} \times {}_a\mu_g = \dfrac{1.498}{1.333} = 1.1238$

$$\therefore \quad 1.1238 = \frac{\sin(60 + D_m)/2}{\sin 30}$$

$$\Rightarrow \quad \sin(60 + D_m)/2 = 0.5619$$

$$\Rightarrow \quad \frac{60 \times D_m}{2} = 34.2$$

$$\Rightarrow \quad D_m = 8.4°$$

Example 113

A parallel beam of light falls normally on the first face of a prism of small angle α. At the second face it is partly transmitted and partly reflected, the reflected beam striking the first face again and emerging from it in a direction making an angle of 6°30′ with the reversed direction of the incident beam. The refracted beam is found to have undergone a deviation of 1°15′ from the original direction. Calculate the refractive index of the glass and the angle of the prism.

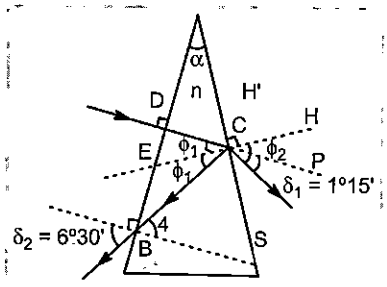


Fig. 1E.113

Solution: We must first solve for the refractive index, n, of the prism glass. Applying Snell's law to the refraction at point C (see Fig. 1E.113), we obtain

$$n \sin \phi_1 = n' \sin \phi_2$$

But n' is the refractive index of air, which is 1. Hence,

$$n \sin \phi_1 = \sin \phi_2 \qquad \ldots (1)$$

Similarly, applying Snell's law to the refraction at point B,

$$n \sin \psi = \sin \delta_2 \qquad \ldots (2)$$

Now, we must relate ϕ_1, ϕ_2 and ψ to known quantities.

Note that $\angle HCP = \angle DCE$, since they are vertical angles. Therefore,

$$\angle HCP = \angle DCE$$

$$\phi_2 - \delta_1 = \phi_1$$

or $$\delta_1 = \phi_2 - \phi_1 \qquad \ldots (3)$$

Noting that DC and BS are parallel,

$$\psi = 2\phi_1 \qquad \ldots (4)$$

We need one more equation relating any of O_1, O_2 and ψ to α.

$$\angle ACD = 90° - \alpha$$

But $$\angle ACD + \phi_1 = 90°$$

Hence $$90° - \alpha + \phi_1 = 90°$$

and $$\phi_1 = \alpha \qquad \ldots (5)$$

If all the angles $(\phi_1, \phi_2, \psi, \delta_1, \delta_2)$ are small, we may approximate the sine of an angle by the angle itself. Using eqns. (1) and (2),

$$n\phi_1 \approx \phi_2$$
$$n\psi \approx \delta_2 \quad \ldots (6)$$

Taking the ratio of each equation in (6),

$$\frac{\phi_1}{\psi} \approx \frac{\phi_2}{\delta_2}$$

Hence, $$\phi_1 \approx \frac{\psi\phi_2}{\delta_2}$$

Using (4) $$\phi_1 \approx \frac{2\phi_1\phi_2}{\delta_2}$$

or $$\delta_2 = 2\phi_2 \quad \ldots (7)$$

Hence, solving (3) for ϕ_1

$$\phi_1 = \phi_2 - \delta_1$$

Since $$\phi_2 = \frac{\delta_2}{2}$$

$$\phi_1 = \frac{\delta_2}{2} - \delta_1 \quad \ldots (8)$$

Solving the first equation of (6) for n

$$n \approx \frac{\phi_2}{\phi_1}$$

With eqns. (7) and (8),

$$n = \frac{\frac{\delta_2}{2}}{\frac{\delta_2}{2} - \delta_1} = \frac{1}{1 - \frac{2\delta_1}{\delta_2}}$$

Using the given data,

$$n = \frac{1}{1 - \frac{2(1°15')}{(6°30')}}$$

$$n = \frac{1}{1 - \frac{2(1\frac{1°}{4})}{(6\frac{1°}{2})}}$$

$$n = \frac{1}{1 - \frac{5/2}{13/2}} = \frac{1}{1 - 5/13}$$

$$n = \frac{13}{13-5} = \frac{13}{8} = 1.625$$

Furthermore, from eqn. (5),

$$\alpha = \phi_1$$

But, equation (8) tells us that

$$\alpha = \phi_1 = \frac{\delta_2}{2} - \delta_1 = \frac{6°30'}{2} - 1°15'$$

Hence $$\alpha = 3°15' - 1°15' = 2°$$

Example 114

The easiest method of measuring the refracting angle of a prism is to direct a parallel beam of light on to the angle (vertex A in figure) and measure the angular separation of the beams reflected from the two sides of the prism containing the refracting angle. Show that this angular separation is twice the angle of the prism.

Solution: Consider three incoming rays, all parallel and striking the prism at points *A*, *B* and *C*. Erect normals to *AB* at *A* and *B* and to *AC* at *A* and *C*. Designate the angles as in the diagram.

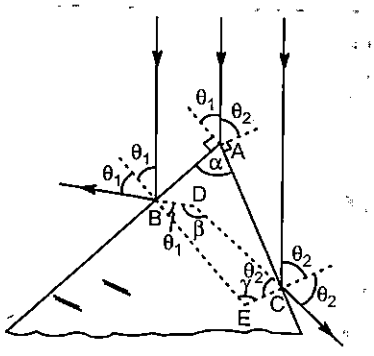


Fig. 1E.114

The rays striking at *B* and *C* are reflected according to the laws of optics, as shown, and the angle between the reflected rays is β. *E* is the point at which the normals at *B* and *C* meet.

The sum of the angles of quadrilateral *ABEC* is 360°. Since ∠*ABE* and ∠*ACE* are each 90°,

$$\alpha + \gamma = 180° \quad \ldots (1)$$

In the quadrilateral *BDCE*,

$$\beta + \gamma + \theta_1 + \theta_2 = 360° \quad \ldots (2)$$

Since two of the angles surrounding *A* are right angles,

$$\alpha + \theta_1 + \theta_2 = 180° \quad \ldots (3)$$

Adding eqns. (1) and (3),

$$2\alpha + \gamma + \theta_1 + \theta_2 = 360° \quad \ldots (4)$$

Subtracting eqn. (2) from (4),

$$2\alpha - \beta = 0$$

or $$2\alpha = \beta$$

Thus, the angle between the reflected beams, β, is twice the refracting angle of the prism α.

Example 115

The path of a ray of light passing through an equilateral glass prism ABC is shown in Fig.1E.115. The ray of light is incident on face BC at the critical angle for just total internal reflection. The total angle of deviation after the refraction at face AC is 108°. Calculate the refractive index of the glass.

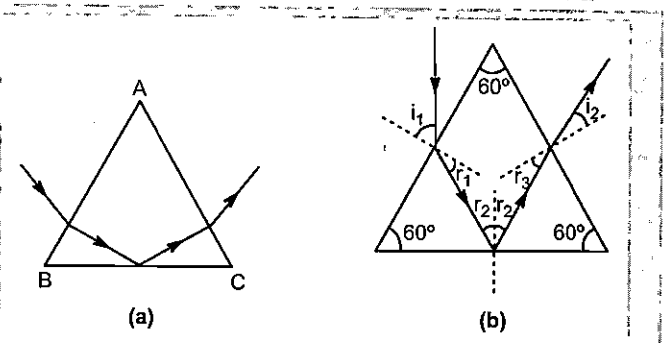


Fig. 1E.115

Solution: In the figure shown,

$r_2 = C$

$r_1 = 60 - r_2 = 60 - C$

$r_3 = 60 - r_2 = 60 - C$

$r_1 = r_3 = r$ (say)

$i_1 = i_2 = i$ (say)

Net deviation, $\delta = (i - r) + (180 - 2r_2) + (i - r) = 108°$

or $r_2 + r - i = 36°$

or $C + 60 - C - i = 36°$

or $i = 24°, \sin 24° \approx 0.40$

From Snell's law, we have

$\sin 24° = \mu \sin r$

or $0.4 = \mu \sin(60° - C)$

or $0.4 = \mu\left[\frac{\sqrt{3}}{2}\cos C - \frac{1}{2\mu}\right]$

or $0.8 = \sqrt{3}\mu\sqrt{1 - \frac{1}{\mu^2}} - 1$

or $\mu^2 - 1 = 1.08$

or $\mu = 1.447$

Example 116

The refractive index of the material of a prism has values μ_1, μ_2 and μ_3 respectively for light of three different wavelengths. If δ_1, δ_2 and δ_3 are the corresponding angles of minimum deviation for a given prism and these angles are in arithmetical progression, prove that

$$\frac{\mu_1 + \mu_3}{\mu_2} = \frac{\sin\frac{\delta_1}{2} + \sin\frac{\delta_3}{2}}{\sin\frac{\delta_2}{2}}$$

Solution: For minimum deviation,

$$\mu_1 = \frac{\sin\frac{A + \delta_1}{2}}{\sin\frac{A}{2}}$$

$$\mu_2 = \frac{\sin\frac{A + \delta_2}{2}}{\sin\frac{A}{2}}$$

$$\mu_3 = \frac{\sin\frac{A + \delta_3}{2}}{\sin\frac{A}{2}}$$

$$\therefore \quad \frac{\mu_1 + \mu_3}{\mu_2} = \frac{\sin\frac{A + \delta_1}{2} + \sin\frac{A + \delta_3}{2}}{\sin\frac{A + \delta_2}{2}}$$

$$= \frac{2\sin\frac{2A + \delta_1 + \delta_3}{4}\cos\frac{\delta_1 - \delta_3}{4}}{\sin\frac{A + \delta_2}{2}}$$

Since δ_1, δ_2 and δ_3 are in A.P.

$\therefore \quad 2\delta_2 = \delta_1 + \delta_3$

$$\therefore \quad \frac{\mu_1 + \mu_3}{\mu_2} = \frac{2\sin\frac{A + \delta_2}{2}\cos\frac{\delta_1 - \delta_3}{4}}{\sin\frac{A + \delta_2}{2}}$$

$$= 2\cos\frac{\delta_1 - \delta_3}{4}$$

Multiplying and dividing by $\sin\frac{\delta_1 + \delta_3}{4}$, we get

$$\frac{\mu_1 + \mu_3}{\mu_2} = \frac{2\sin\frac{\delta_1 + \delta_3}{4}\cos\frac{\delta_1 - \delta_3}{4}}{\sin\frac{\delta_1 + \delta_3}{4}} = \frac{\sin\frac{\delta_1}{2} + \sin\frac{\delta_3}{2}}{\sin\frac{\delta_2}{2}}$$

Example 117

In an isosceles prism of angle 45°, it is found that when the angle of incidence is same as the prism angle, the emergent ray grazes the emergent surface. Find the refractive index of the material of the prism. For what angle of incidence, the angle of deviation will be minimum ?

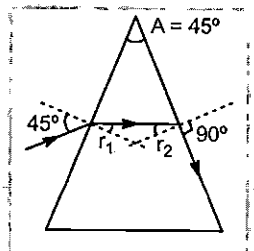


Fig. 1E.117

Solution: As the ray of light grazes the second surface, r_2 is the critical angle,

i.e., $\sin r_2 = \frac{1}{\mu}$

$r_2 = (45° - r_1)$

and $\sin r_1 = \frac{\sin 45°}{\mu} = \frac{1}{\sqrt{2}\mu}$,

$\sin r_2 = \sin(45° - r_1)$

$= \frac{1}{\sqrt{2}}[\cos r_1 - \sin r_1]$

$\frac{1}{\mu} = \frac{1}{\sqrt{2}}\left[\sqrt{1 - \frac{1}{2\mu^2}} - \frac{1}{\sqrt{2}\mu}\right]$

or $\frac{1}{\mu} = \frac{1}{\sqrt{2}}\left[\frac{\sqrt{2\mu^2 - 1}}{\sqrt{2}\mu} - \frac{1}{\sqrt{2}\mu}\right]$

or $2 = \sqrt{2\mu^2 - 1} - 1$

or $2\mu^2 - 1 = 9$

$2\mu^2 = 10$

$\mu^2 = 5 \quad \text{or } \mu = \sqrt{5}$

At minimum deviation,

$r_1 = r_2 = \frac{45°}{2} = 22.5°$

$\mu = \frac{\sin i_1}{\sin r_1} \Rightarrow \sin i_1 = (\sqrt{5})\sin(22.5°)$

$\therefore \quad i_1 = 58.8°$

Example 118

A biprism with an angle β that appreciably differs from 180° is placed into a vessel filled with a liquid having a refractive index n_1 or serves as one of the walls of this vessel. (see Fig. 1E.118). Calculate the angle δ of an equivalent biprism in air. The refractive index of the prism substance is n_2.

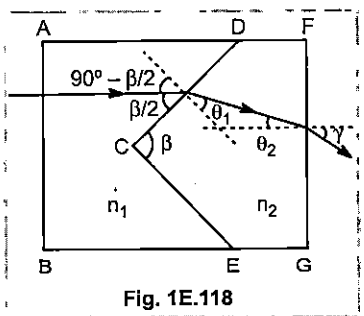


Fig. 1E.118

Solution: Consider a ray falling normally on *AB*, for refraction at *CD* and *RG*.

For refraction at *DC*,

$n_1\left(90 - \frac{\beta}{2}\right) = n_2\theta_1$

and for refraction at *EG*,

$n_2\left(90 - \frac{\beta}{2} - \theta\right) = 1 \times \gamma,$

Here, $\theta_2 = 90 - \frac{\beta}{2} - \theta_1$

or $r = n_2\left(90 - \frac{\beta}{2}\right) - n_1\left(90 - \frac{\beta}{2}\right)$ ... (1)

For a biprism of angle δ kept in air, similarly,

$r = n_2\left(90 - \frac{\beta}{2}\right) - 1\left(90 - \frac{\beta}{2}\right)$ ... (2)

From eqns. (1) and (2),

$\delta = \frac{\beta(n_2 - n_1)}{n_2 - 1} + 180°\left(\frac{n_1 - 1}{n_2 - 1}\right)$

Example 119

A thin biprism (see Fig. 1E.119) of obtuse angle $\alpha = 178°$ is placed at a distance $l = 20$ cm from a slit. How many images are formed and what is the separation between them ? Refractive index of the material $\mu = 1.6$.

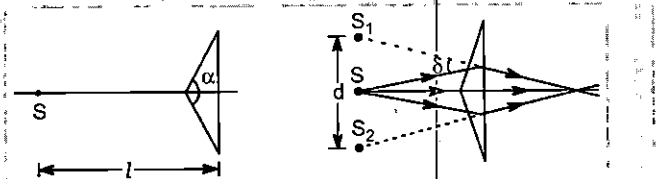


Fig. 1E.119

Solution: Two images are formed by the two thin prisms-one above the axis and the other below the axis by the same distance. The refracting angle of each thin prism $= \frac{\pi - \alpha}{2} = \frac{1}{2}(\pi - \alpha)$ where α is the obtuse angle in radian.

Then δ (deviation of a ray) $= (\mu - 1)\frac{1}{2}(\pi - \alpha)$

$\therefore \quad \frac{d}{2} = l\delta$

or $d = 2l(\mu - 1)\frac{1}{2}(\pi - \alpha)$

or $d = (\mu - 1)l(\pi - \alpha)$

Here $d = (1.6 - 1) \times 0.20\left(\pi - 178 \times \frac{\pi}{180}\right)$

$= 0.6 \times 0.20 \times \pi \times \frac{1}{90} = 0.004\,\text{m} = 4\,\text{mm}$

Example 120

A glass prism whose cross-section is an isosceles triangle, stands with its (horizontal) base in water, the angles that its two equal sides make with the base are each θ.

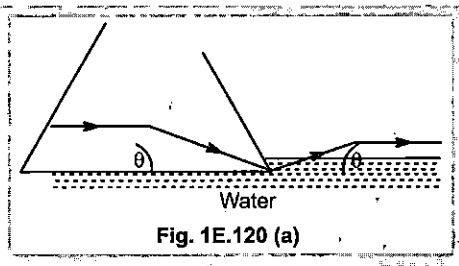


Fig. 1E.120 (a)

An incident ray of light, above and parallel to the water surface and perpendicular to the prism's axis, is internally reflected at the glass-water interface and subsequently re-emerges into the air. Taking the refractive indices of glass and water to be $\frac{3}{2}$ and $\frac{4}{3}$, respectively, show that θ must be at least 25.9°.

Solution: At the point where it enters the prism, angle of refraction is ϕ.

Snell's law applied to the initial entry into the glass gives

$$\sin\left(\frac{\pi}{2}-\theta\right) = n_g \sin\phi$$

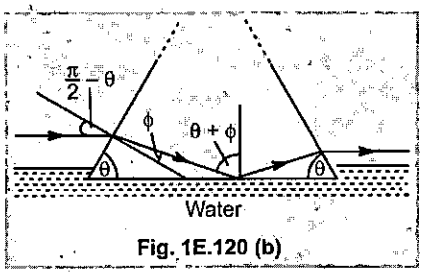


Fig. 1E.120 (b)

From geometry we determine the angle between the incident ray and the normal to the glass-water interface at the point where the ray meets the boundary to be $\theta + \phi$.

For total internal reflection to occur this must exceed the critical angle $C = \sin^{-1}(n_w/n_g)$. These two conditions can be combined using the formula $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$ to eliminate ϕ and obtain

$$n_g^2 - n_w^2 \geq \cos^2\theta(n_g^2 + 1 - 2n_w)$$

Substitution of the given values for the refractive indices yields the stated result.

Example 121

The faces of prism ABCD made of glass with a refraction index n form dihedral angles: ∠A = 90°, ∠B = 75°, ∠C = 135° and ∠D = 60° (the Abbe prism). A beam of light falls on face AB and after complete internal reflection from face BC escapes through face AD. Find the angle of incidence α of the beam onto face AB if a beam that has passed through the prism is perpendicular to the incident beam.

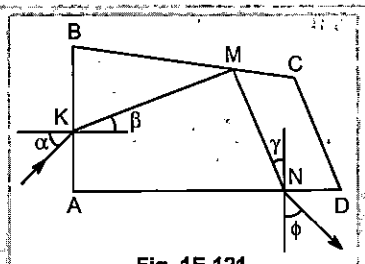


Fig. 1E.121

Solution: According to the initial condition, the incident beam and the beam that has passed through the prism are mutually perpendicular. Therefore, $\angle\phi = \angle\alpha$ and also $\angle\gamma = \angle\beta$ (Fig. 1E.121). The sum of the angles of the quadrangle *AKMN* is 360°. Therefore, $\angle KMN = 90°$ and beam *KM* is incident on to face *BC* at an angle of 45°. If we know the angles of triangle *KBM*, it is easy to find that $\beta = 30°$. In conformity with the law of refraction, $\frac{\sin\alpha}{\sin\beta} = n$

Hence, $\sin\alpha = 0.5n$ and $\arcsin 0.5n$

Since full internal reflection at an angle of 45° is observed only when $n \geq \sqrt{2}$, the angle α is within $45° \leq \alpha \leq 90°$.

Example 122

Three right-angled prisms of refractive indices μ_1, μ_2 and μ_3 are joined together so that the faces of the middle prism are in contact each with one of the outside prisms. If the ray passes through the composite block undeviated, show that

$$\mu_1^2 + \mu_3^2 - \mu_2^2 = 1$$

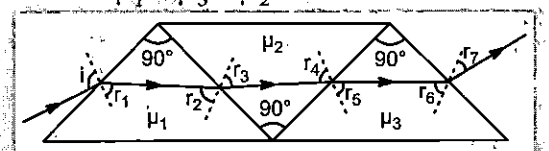


Fig. 1E.122

Solution: We have

$$r_1 + r_2 = 90° \quad \ldots(1)$$

$$r_3 + r_4 = 90° \quad \ldots(2)$$

and $$r_5 + r_6 = 90° \quad \ldots(3)$$

Since the emergent ray is parallel to incident ray, we have

$$r_7 = 90° - i \quad \ldots(4)$$

Writing the Snell's equations of refraction,

$$\sin i = \mu_1 \sin r_1$$

$$\mu_1 \sin r_2 = \mu_2 \sin r_3$$

$$\mu_2 \sin r_4 = \mu_3 \sin r_5$$

and $$\mu_3 \sin r_6 = \sin r_7$$

Using equation (1) to (4) in the above equations, we get

$$\sin i = \mu_1 \sin r_1 \quad \ldots(5)$$

$$\mu_1 \cos r_1 = \mu_2 \sin r_3 \quad \ldots (6)$$

$$\mu_2 \cos r_3 = \mu_3 \sin r_5 \quad \ldots (7)$$

$$\mu_3 \cos r_5 = \cos i \quad \ldots (8)$$

(equation 5)^2 + (equation 7)^2 – (equation 6)^2 – (equation 8)^2 gives $\sin^2 i + \mu_2^2 \cos^2 r_3 - \mu_1^2 \cos^2 r_1 - \mu_3^2 \cos^2 r_5$

$$= \mu_1^2 \sin^2 r_1 + \mu_3^2 \sin^2 r_5 - \mu_2^2 \sin^2 r_3 - \cos^2 i$$

or $\mu_2^2 + 1 = \mu_1^2 + \mu_3^2$

or $\mu_1^2 + \mu_3^2 - \mu_2^2 = 1$

SINGLE SURFACE REFRACTION EQUATION

Consider a thin transparent spherical shell filled with water, a solid crystal ball. Refraction through one side of this ball is an example of single surface refraction. The laws of refraction are applicable when rays pass through the interface.

The single surface refraction equation is

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

In Figs. 1.175 (a) to (f) the symbols have the following meaning :

$R \rightarrow$ Radius of curvature

$C \rightarrow$ Centre of curvature

$O \rightarrow$ Position of object

$I \rightarrow$ Position of image

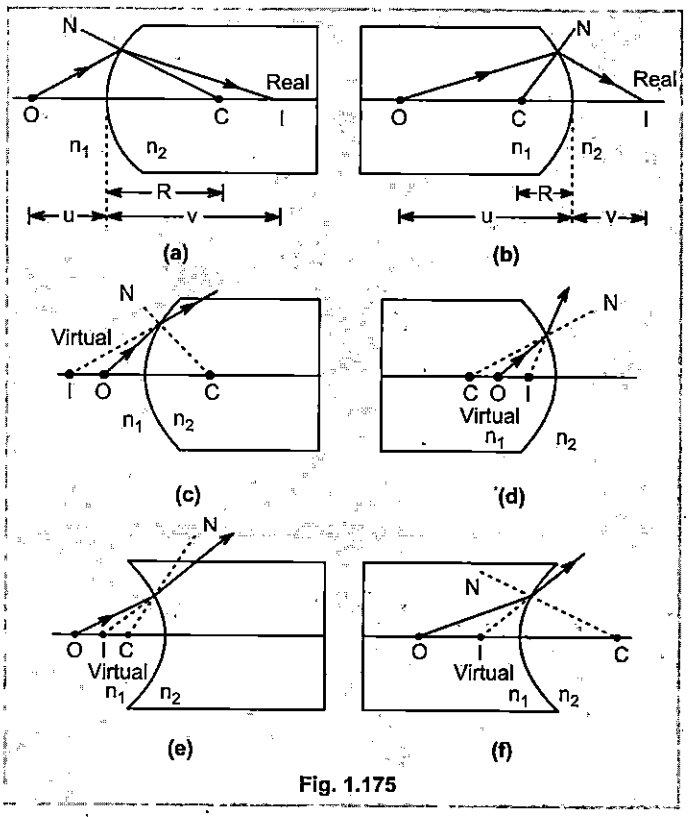


Fig. 1.175

$n_1 \rightarrow$ Refractive index of medium in which the object lies

$n_2 \rightarrow$ Refractive index of medium through which refraction takes place

$N \rightarrow$ Normal to refracting surface, a radial line through centre of curvature.

The shaded area shows a medium with higher refractive index. Due to refraction the ray bends toward the normal if it is entering in a medium of higher refractive index and away from normal if it is entering in a medium with lower refractive index. When refracted ray bends toward principal axis, it forms a real image on that axis.

- When more than one refracting surface is present the subsequent surfaces are taken in the order in which they are encountered by the light. The positions of images from successive refractive surfaces are obtained by using single surface refraction equation at each surface.
- While considering refraction or reflection of the light from an object by two or more surfaces in succession, the image formed by first surface becomes object for the second surface.
- Be careful while substituting for object and image distance in the equation. For every refraction the origin of the cartesian system is always taken at the particular refracting surface under consideration. **Distances are calculated from the origin of the particular surface under consideration and not from any previous surface.** The origin is shifted after each refraction ; so numerical values of distances (and their signs) must be adjusted according to new origin. If it is directed away from the principal axis, it cannot form a real image; however, backward extensions of it and other refracted rays can form a virtual image. Figs. (a) and (b) show real images and (c), (d), (e) and (f) show virtual images.
- The single surface refraction equation is valid for convex or concave and even a plane surface (in which case $R = \infty$). We use cartesian sign convention for all the distances in the equation.
- When the image is located to the right of the origin, the image is real since the refracted rays really exist in this region and converge to the image point. If the image is to the left of the origin (the refracting surface), the image is virtual. The refracted rays never intersect at a virtual image point; they appear to intersect there. A positive image distance indicates a real image and a negative image distance means a virtual image. The **magnification** for single surface refraction is

$$m = \frac{n_1 v}{n_2 u}$$

❐ The overall magnification of the entire system of surfaces is the product of the magnifications produced by individual surfaces.

Example 123

A very long glass rod of 2 cm diameter has one end ground to a convex spherical surface of radius 5 cm; its refractive index is 1.5. A point object is on the axis of the rod. The glass rod and object are immersed in water, refractive index 4/3. Characterize the image in each of the following cases: (a) object is on the axis of the rod 20 cm from the surface. (b) Object is 5 cm from the surface. (c) Object is very far from the surface. Draw a ray diagram for each case.

Solution: (a) From single surface refraction equation,

$$\frac{3}{2v}-\frac{(4/3)}{(-20)}=\frac{(3/2-4/3)}{(+5)}$$

where $n_1 = 4/3,\ n_2 = 3/2,\ r = +5,\ u = -20\,\text{cm},$ $v = -45\,\text{cm}$

Magnification, $m=\dfrac{n_1 v}{n_2 u}=\dfrac{(4/3)(-45)}{(3/2)(-20)}=2{\cdot}531$

The final image is formed 45 cm to the left of convex refracting surface, virtual, erect and magnified, see Fig. 1E.123.

(b) From single surface refraction equation,

$$\frac{3}{2v}-\frac{(4/3)}{(-5)}=\frac{(3/2-4/3)}{(+5)}$$

$$v=-6.42\,\text{cm}$$

Magnification,

$$m=\frac{n_1 v}{n_2 u}=\frac{(4/3)(-6.42)}{(3/2)(-5)}$$

$$=+1.141$$

The image is virtual, erect and magnified.

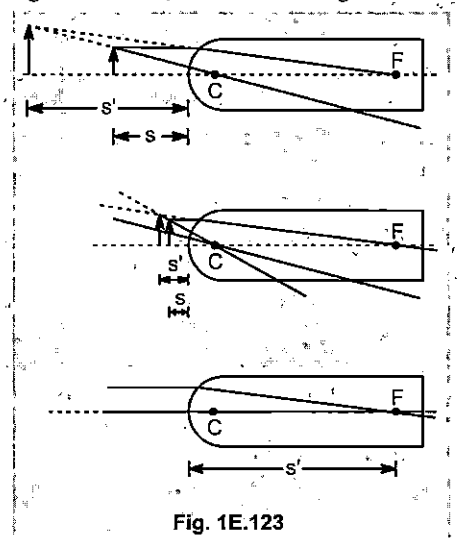


Fig. 1E.123

(c) When the object is very far from surface, $u = \infty$; so we have

$$\frac{(3/2)}{v}-\frac{(4/3)}{(\infty)}=\frac{(3/2-4/3)}{+5}$$

$$v=+45\ \text{cm}$$

The image is real and formed at second focus of refracting surface.

Example 124

A ring of radius 1 cm is placed 1 m infront of a spherical glass ball of radius 25 cm with refractive index 1.50. Determine the position of the final image of the ring and its magnification.

Solution: Light rays from object are refracted through the glass ball twice; once at surface S_1, from air to glass, and second, at surface S_2 from glass to air. We use paraxial approximation, so that single surface refraction equation can be used.

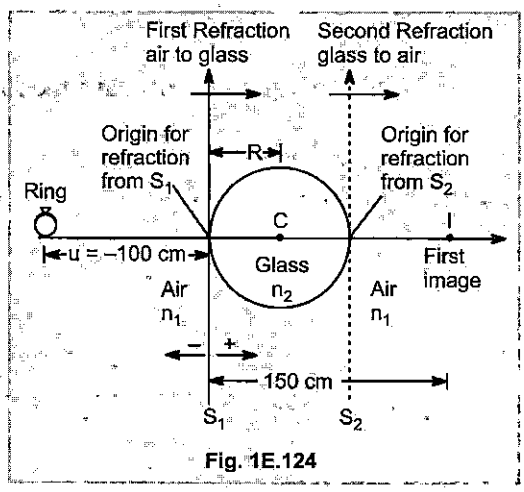


Fig. 1E.124

Refraction at First Surface

$n_1 = 1,\ n_2 = 1.5, u = -100\,\text{cm}, R = +25\,\text{cm}$

The radius of curvature is positive because centre of curvature is to the right.

Substituting these values in single surface refraction equation,

$$\frac{1.5}{v}-\frac{1}{(-100)}=\frac{(1.5-1)}{25}$$

On solving for v, we get

$$v=+150\,\text{cm}$$

The image is located 150 cm to the right of the first refracting surface. The magnification due to refraction at first surface,

$$m_1=\frac{n_1 v}{n_2 u}=\frac{1(150)}{1.5(-100)}=-1$$

Refraction at Second Surface

For refraction at second surface the origin of the cartesian coordinate system has to be shifted to the vertex of the second refracting surface.

The object distance for refraction at S_2 is

$$u' = +(150 - 50) = 100\,\text{cm}$$

This is the virtual object for S_2, the light rays converging to I_1 are refracted at S_2 before they can actually converge to form the image.

Example 125

A transparent glass sphere of radius R = 10 cm and refractive index n = 1.5 has its one half silvered so that it acts like a concave mirror. Find the position of the final image seen by one eye to the left of the object and the ball for an object at (a) 30 cm and (b) 20 cm to the left of the front surface of the ball.

Solution: The light ray is refracted at surface S_1, then reflected at S_2 and finally refracted out through S_1. From single surface refraction equation,

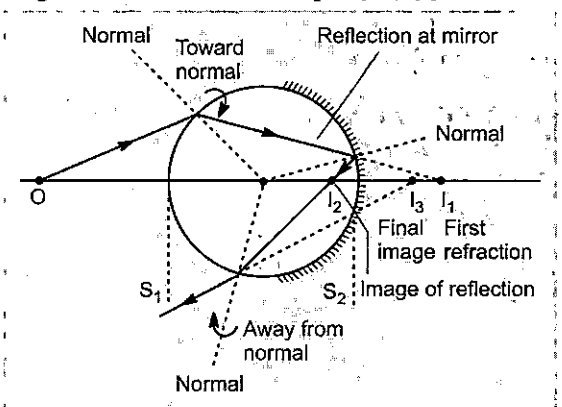


Fig. 1E.125

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

(a) For the object at 30 cm

$$\frac{1.5}{v_1} - \frac{1}{(-30)} = \frac{(1.5-1)}{+10}$$

$$v_1 = 90\,\text{cm}$$

The object distance for concave mirrored surface of sphere,

$$u_2 = 90 - 20 = 70\,\text{cm}$$

$$f = -10/2 = -5\,\text{cm}$$

From mirror equation, $\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f}$

$$\frac{1}{v_2} + \frac{1}{70} = -\frac{1}{5}$$

$$v_2 = -\frac{14}{3}\,\text{cm}$$

The second image acts as object for refraction at curved surface S_1, with $n_1 = 1.5, n_2 = 1$.

$$u_3 = -\left[20 - \frac{14}{3}\right]$$

$$= -\frac{46}{3}\,\text{cm}, R = -10\,\text{cm}$$

$$\frac{1}{v_3} - \frac{1.5}{(-46/3)} = \frac{1-1.5}{(-10)}$$

$$v_3 = -20.9\,\text{cm}$$

(b) For the object at 20 cm

$$\frac{1.5}{v_1} - \frac{1}{(-20)} = \frac{(1.5-1)}{10}$$

$$\frac{1.5}{v_1} = \frac{1}{20} - \frac{1}{20} = 0$$

$$v_1 = \infty$$

The rays are parallel to principal axis after refraction. The parallel rays are brought to the focus of the curved mirror. Hence the second image is at the focus, at $f = R/2 = 5$ cm from the vertex of the mirror. The final image is obtained after refraction from S_1

$$\frac{1}{v_3} - \frac{1.5}{+(20-5)} = \frac{1-1.5}{+10}$$

$$v_3 = 20\,\text{cm}$$

The final image is at the vertex of the mirrored surface.

On substituting the values into the single surface refraction equation,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1.00}{v'} - \frac{1.50}{100} = \frac{1.00-1.50}{-25}$$

Note the values of n_1 and n_2.

Solving for v, we get $v' = 29\,\text{cm}$

The image is located 29 cm to the right of the second refracting surface. The final image is real because light after refraction at S_2 actually reaches this point. The magnification, m_2 at S_2 is

$$m_2 = \frac{n_1 v'}{n_2 u'} = \frac{(1.50)(29)}{(1.00)(100)} = 0.44$$

Total magnification,

$$m_{\text{total}} = m_1 m_2$$

$$= (-1.00)(0.44)$$

$$= -0.44$$

The minus sign indicates that the final image is inverted.

$$\text{Image size} = m_{\text{total}}\ (\text{object size})$$

$$= -0.44(2.0)$$

$$= -0.88$$

The final image is located below the optic axis.

Example 126

For refraction at a spherical surface, the first focal length is defined as the value of u corresponding to $v = \infty$ as shown in Fig. 1E.126(a). The second focal length f_2 is defined as the value of v when $u = \infty$ as shown in Fig. 1E.126(b).

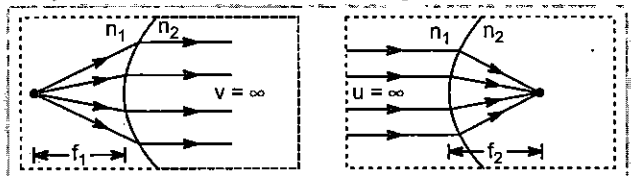


Fig. 1E.126

(a) Show that $\frac{n_1}{n_2} = \frac{f_1}{f_2}$

(b) Show that the general relation between object and image distance is

$$\frac{f_1}{u} + \frac{f_2}{v} = 1$$

Solution: We will apply single surface refraction equation for determining f_1 and f_2.

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \text{... (1)}$$

$$\frac{n_2}{f_2} - \frac{n_1}{\infty} = \frac{n_2 - n_1}{R}$$

$$f_2 = \frac{n_2 R}{n_2 - n_1} \quad \text{... (2)}$$

and $$\frac{n_2}{\infty} - \frac{n_1}{f_1} = \frac{n_2 - n_1}{R}$$

$$f_1 = -\frac{n_1 R}{n_2 - n_1} \quad \text{... (3)}$$

From equations (2) and (3), we have

$$\frac{f_1}{n_1} = -\frac{R}{n_2 - n_1} \text{ and } \frac{f_2}{n_2} = \frac{R}{n_2 - n_1} \quad \text{... (4)}$$

$$\frac{f_1}{n_1} + \frac{f_2}{n_2} = 0 \quad \text{... (5)}$$

or $$\frac{f_1}{f_2} = -\frac{n_1}{n_2} \quad \text{.. (6)}$$

- As n_1 and n_2 are always positive and unequal, f_1 and f_2 will have opposite sign and also unequal. The two focal points lie on the opposite side of refracting surface.

Further, from eqn. (1), we have

$$\frac{n_2 R}{v(n_2 - n_1)} - \frac{n_1 R}{u(n_2 - n_1)} = 1$$

Now substituting f_1 and f_2 from eqn. (4), we have

$$\frac{f_2}{v} + \frac{f_1}{u} = 1$$

Example 127

A thin lens with refractive index n and radius of curvature R_1 and R_2 is located at the interface between two media with indices of refraction n_1 and n_2 as shown in the figure below.

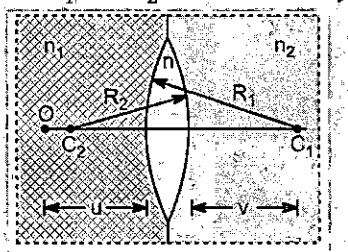


Fig. 1E.127

(a) If u and v are the object and image distances respectively and f_1 and f_2 the respective focal lengths, show that

$$\frac{f_1}{v} + \frac{f_2}{u} = 1$$

(b) Find f_1 and f_2 in terms of n_1, n_2, R_1 and R_2.

Solution: We will apply the general thin lens equation to obtain f_1 and f_2.

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n - n_1}{R_1} + \frac{n_2 - n}{R_2} \quad \text{... (1)}$$

The focal lengths are obtained by substituting $v = \infty$ (for f_1) and $u = \infty$ (for f_2).

$$-\frac{n_1}{f_1} = \frac{n - n_1}{R_1} + \frac{n_2 - n}{R_2} \quad \text{.. (2)}$$

$$f_1 = -\frac{n_1}{\frac{n - n_1}{R_1} + \frac{n_2 - n}{R_2}} \quad \text{...(3)}$$

and $$\frac{n_2}{f_2} = \frac{n - n_1}{R_1} + \frac{n_2 - n}{R_2} \quad \text{.. (4)}$$

$$f_2 = \frac{n_2}{\frac{n - n_1}{R_1} + \frac{n_2 - n}{R_2}} \quad \text{..(5)}$$

From eqn. (3),

$$\frac{f_1}{u} = -\frac{n_1/u}{\frac{n - n_1}{R_1} + \frac{n_2 - n}{R_2}} \quad \text{.. (6)}$$

From eqn. (5),

$$\frac{f_2}{v} = \frac{n_2/v}{\frac{n - n_1}{R_1} + \frac{n_2 - n}{R_2}} \quad \text{.. (7)}$$

Adding eqns. (6) and (7),

$$\frac{f_1}{u}+\frac{f_2}{v}=\frac{(n_2/v)-(n_1/u)}{\left(\frac{n-n_1}{R_1}\right)+\left(\frac{n_2-n}{R_2}\right)} \quad \text{.. (8)}$$

From eqn. (1) right side of eqn.(8) becomes unity, so we have

$$\frac{f_1}{u}+\frac{f_2}{v}=1$$

Example 128

Figure shows a glass slab of refractive index n, with two curved ends ABC and DEF as shown in Fig. 1E.128. Locate the final image of an object point B located at a distance 2R from the vertex of surface ABC. What is the condition to be satisfied if the image is to be real ?

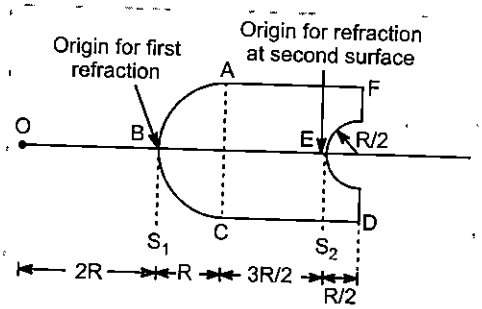


Fig. 1E.128

Solution: For refraction at curved surface S_1,

$$\frac{n}{v_1}-\frac{1}{(-2R)}=\frac{(n-1)}{R}$$

$$\frac{n}{v_1}=\frac{2n-3}{2R} \quad \text{...(1)}$$

The first image acts as object for refraction at second surface S_2. The origin of our cartesian coordinate system is now at vertex of surface S_2. Object distance for second refraction,

$$u_2=-(3R-v_1)$$

$$=-\left[3R-\frac{2nR}{(2n-3)}\right]$$

$$=-\left[\frac{4n-9}{2n-3}\right]R$$

For refraction at curved surface S_2,

$$\frac{1}{v_2}-\frac{n}{-\left[\frac{(4n-9)}{(2n-3)}\right]R}=\frac{(1-n)}{(R/2)} \quad \text{...(2)}$$

On solving the above expression for v_2, we get

$$v_2=\frac{(4n-9)}{(10n-9)(n-2)} \quad \text{...(3)}$$

The image will be real if v_2 is positive, *i.e.*,

$$=\frac{(4n-9)}{(10n-9)(n-2)}>0 \quad \text{... (4)}$$

The equation (4) is satisfied if :

Case (i) $(4n-9)>0 \Rightarrow n>9/4$

and $(10n-9)(n-2)>0 \Rightarrow 0.9<n<2$

There is no common solution.

Case (ii) $4n-9<0 \Rightarrow n<9/4$

$(10n-9)(n-2)<0 \Rightarrow n>2, n<0.9$

Hence the result is $2<n<2.25$

Example 129

A transparent sphere of radius 20 cm and refractive index 1.6 is fixed in a hole of the partition separating the two media: A (refractive index $n_1=1.2$) and B (refractive index $n_3=1.7$). A luminous point object is placed 120 cm from the surface of the sphere in medium A. It is viewed from D in medium B in a direction normal to the sphere. Find the position and nature of the image formed by the rays in the direction OD.

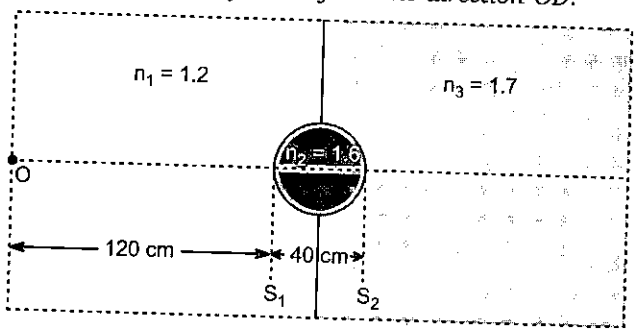


Fig. 1E.129 (a)

Solution: We will have to consider two refractions: first at surface S_1 and the second at surface S_2.

For refraction at surface S_1 :

$$\frac{1.6}{v_1}-\frac{1.2}{(-120)}=\frac{(1.6-1.2)}{20}$$

$$v_1=+160\,\text{cm}$$

The positive sign implies that first image is formed to the right of S_1.

For refraction at surface S_2: The first image is object for refraction at second surface.

$$u_2=+(160-40)\,\text{cm}$$

$$\frac{1.7}{v_2}-\frac{1.6}{+(160-40)}=\frac{1.6-1.7}{(-20)}$$

$$v_2=+204\,\text{cm}$$

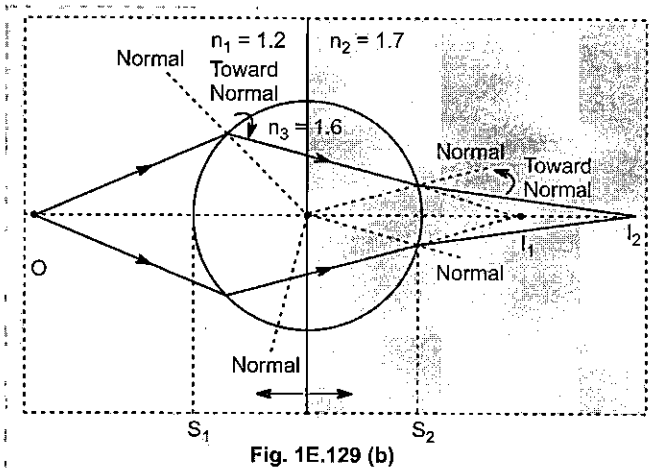


Fig. 1E.129 (b)

The final image is formed at 204 cm from the vertex and on the right side.

Example 130

A cubical block of glass, refractive index 1.5, has a spherical cavity of radius r = 9 cm inside it as shown in Fig. 1E.130. A luminous point object O is at a distance of 18 cm from the cube (see figure). What is the apparent position of O as seen from A.

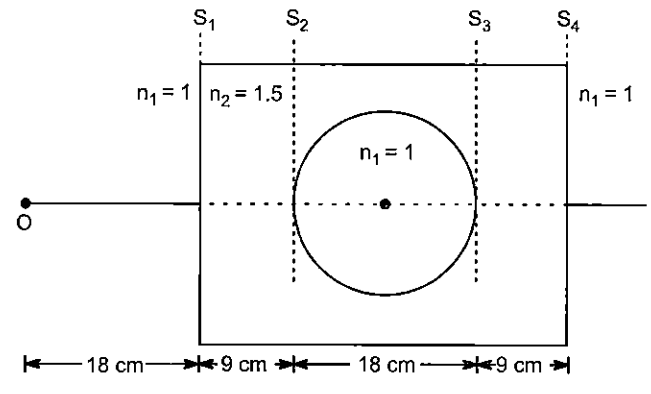


Fig. 1E.130

Solution: We have to consider four refractions at S_1, S_2, S_3 and S_4 respectively. At each refraction we will apply single surface refraction equation.

For refraction at first surface S_1:

$$\frac{3/2}{v_1} - \frac{1}{(-18)} = 0, \quad v_1 = -27\text{cm}$$

First image lies to the left of S_1.

For refraction at second surface S_2:

$$\frac{1}{v_2} - \frac{3/2}{-(27+9)} = \frac{(1-3/2)}{+9}, v_2 = -\frac{72}{7}\text{ cm}$$

Note that origin of cartesian coordinate system lies at vertex of surface S_2. The object distance is (27 + 9) cm. The second image lies to left of S_2.

For refraction at third surface S_3:

$$u_3 = -\left(\frac{72}{7} + 8\right) = -\frac{128}{7}$$

$$\frac{1.5}{v_3} - \frac{1}{(-128/7)} = \frac{(1.5-1)}{(-9)}$$

$$v_3 = -16.5\,\text{cm}$$

For refraction at fourth surface S_4 :

$$u_4 = -(16.5 + 9) = -25.5\,\text{cm}$$

$$\frac{1}{v_4} - \frac{3/2}{(-25.5)} = \frac{(1-3/2)}{\infty} = 0$$

$$v_4 = -17\text{cm}$$

The final image lies at 17 cm to the left of surface S_4.

Example 131

A glass sphere, refractive index 1.5 and radius 10 cm, has a spherical cavity of radius 5 cm concentric with it. A narrow beam of parallel light is directed into the sphere. Locate the final image.

Solution: We will have single surface refractions successively at the four surfaces S_1, S_2, S_3 and S_4. Do not forget to shift origin to vertex of respective surface.

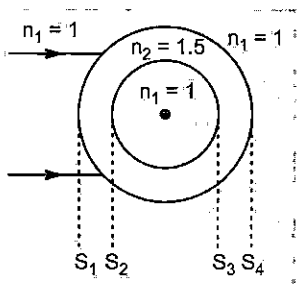


Fig. 1E.131

Refraction at first surface S_1 : Light travels from air to glass.

$$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{(1.5-1)}{(+10)}$$

$$v_1 = 30\,\text{cm}$$

First image is object for the refraction at second surface.

Refraction at second surface S_2: Light travels from glass to air.

$$\frac{1}{v_2} - \frac{1.5}{(+25)} = \frac{1-1.5}{(+5)}$$

$$v_2 = -25\text{cm}$$

Refraction at third surface S_3: Light travels from air to glass.

$$\frac{1.5}{v_3} - \frac{1}{(-35)} = \frac{(1.5-1)}{(-5)}$$

$$v_3 = -35/3\,\text{cm}$$

Refraction at fourth surface S_4: Light travels from glass to air.

$$\frac{1}{v_4} - \frac{1.5}{-(35/3+5)} = \frac{1-1.5}{-10}, v_4 = -25\,\text{cm}$$

The final image is virtual formed at 25 cm to the left of the vertex of surface S_4.

Example 132

A biconvex lens separates two media of refractive indices 1.3 and 1.7. The refractive index of the lens is 1.5 and the radii of curvatures of the two sides of the lens are $r_1 = 10$ cm and $r_2 = 60$ cm. The medium of refractive index 1.3 extends to 78 cm from the lens and that of refractive index extends to 34 cm from the lens. A luminous object O is at a distance of 144 cm from the lens. Find the position of final image from the lens.

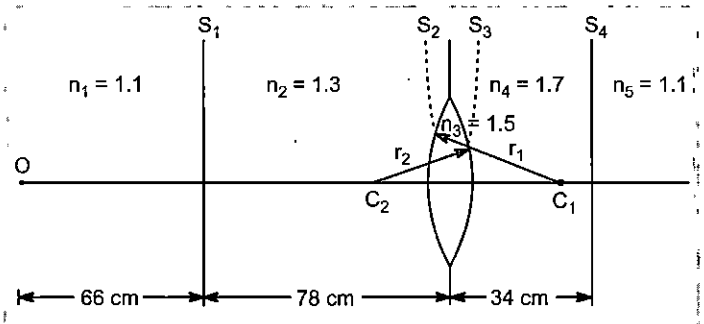


Fig. 1E.132 (a)

Solution: Once again we have to apply single surface refraction equation to the four surfaces S_1, S_2, S_3 and S_4. But we have learned the general thin lens equation

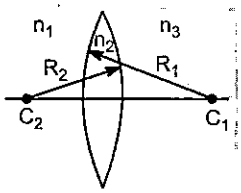


Fig. 1E.132 (b)

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

which will take account of the two refraction at S_2 and S_3.

For refraction at surface S_1 :

$$\frac{1.3}{v} - \frac{1.1}{(-66)} = \frac{(1.3-1.1)}{\infty} = 0$$

$$v = -78\,\text{cm}$$

Image lies to the left of the surface S_1. This acts as an object for the lens.

For refraction through lens: We apply the general thin lens equation. Object distance for lens.

$$u' = -(78+78)\,\text{cm}$$

$$\frac{1.7}{v'} - \frac{1.3}{(-156)} = \frac{1.5-1.3}{+10} + \frac{1.7-1.5}{(-60)}$$

$$v' = 340\,\text{cm}$$

For refraction at surface S_4: Object distance for refraction at S_4 is

$$340 - 34 = 306\,\text{cm}$$

$$\frac{1.1}{v''} - \frac{1.7}{(+306)} = 0$$

or $$v'' = \frac{1.1 \times 306}{1.7}$$

$$= 198\,\text{cm to the right of } S_4$$

Example 133

A glass sphere of radius 2R, refractive index n has a spherical cavity of radius R, concentric with it. A black spot on the inner surface of the hollow sphere is viewed from the left as well as right. Obtain the shift in position of the object.

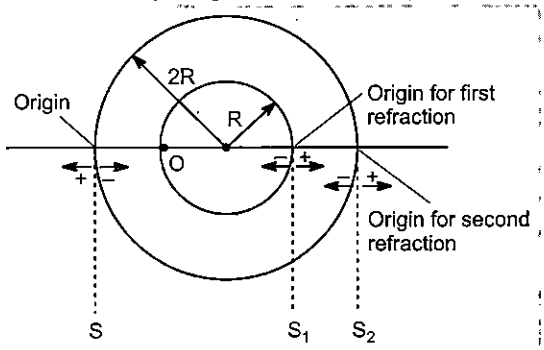


Fig. 1E.133

Solution: (i) Viewer on the left of hollow sphere : Single refraction takes place at surface S. From the single surface refraction equation we have

$$\frac{1}{v} - \frac{n}{(-R)} = \frac{(1-n)}{(-2R)}$$

which on solving for v yields

$$v = -\left(\frac{2R}{n+1}\right)$$

Image is on the right of refracting surface S.

Shift = Real depth – Apparent depth

$$= R - \left(\frac{2R}{n+1}\right) = \frac{(n-1)}{(n+1)}R$$

(ii) When the viewer is on the right, two refractions take place at surfaces S_1 and S_2.

For refraction at surface S_1:

$$\frac{n}{v_1} - \frac{1}{(-2R)} = \frac{(n-1)}{(-R)}$$

which on solving for v_1 yields

$$v_1 = -\frac{2nR}{2n-1}$$

The first lies to the left of S_1 and acts as object for refraction at the second surface. We have to shift the origin of cartesian coordinate system to the vertex of S_2. The object distance for the second surface is

$$v_2 = -\left[\frac{2nR}{2n-1} + R\right] = -\left(\frac{4n-1}{2n-1}\right)R$$

$$\frac{1}{v_2} = -\frac{n}{-\left[\frac{4n-1}{2n-1}\right]R} = \frac{1-n}{-2R}$$

On solving for v_2 we get $v_2 = -\dfrac{2(4n-1)}{(3n-1)}R$

The minus sign shows that image is virtual and lies to the left of S_2.

Shift = Real depth – Apparent depth

$$= 3R - \frac{2(4n-1)R}{(3n-1)} = \frac{(n-1)}{(3n-1)}R$$

THIN LENSES

In a thin lens there are two closely spaced refracting surfaces. The thickness of the lens is considered negligible compared with the object distances, the image distances and the radii of curvature of the two refracting surfaces.

Fig. 1.176 (a) shows a **converging** lens (**convex** lens). Rays of light entering a convex lens parallel to its axis are converged at its focal point.

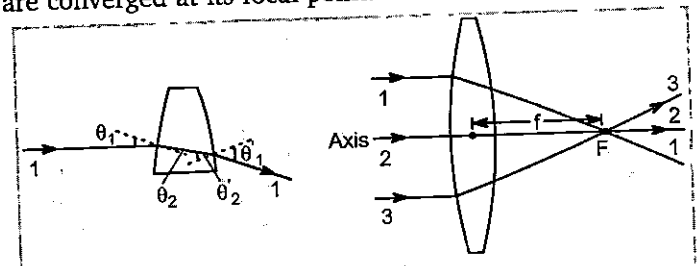


Fig. 1.176 (a)

Fig. 1.176 (b) shows a **diverging** lens (**concave** lens). Rays of light entering a diverging lens parallel to its axis are diverged and all these appear to originate at its focal point F.

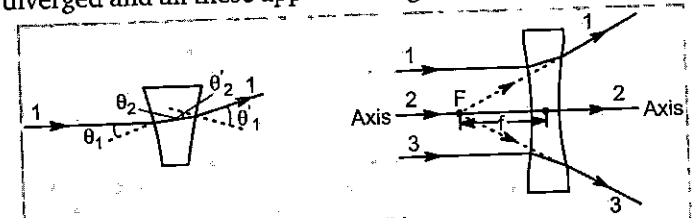


Fig. 1.176 (b)

Fig. 1.176 (c) shows a variety of lenses. Lenses thicker in the middle are converging while lenses thinner in the middle are diverging.

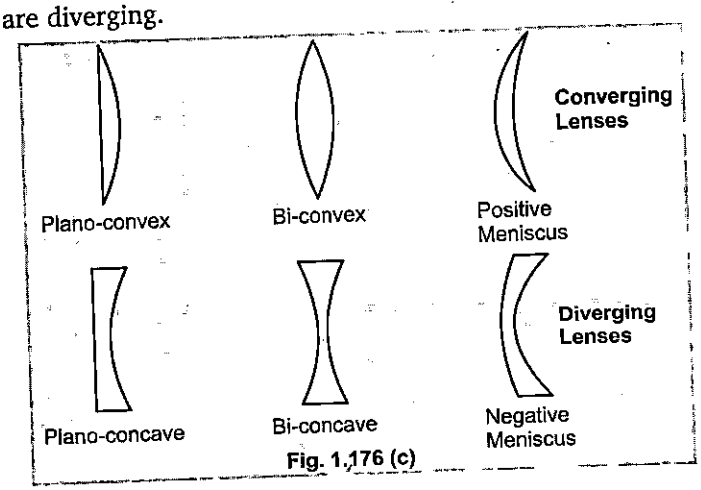
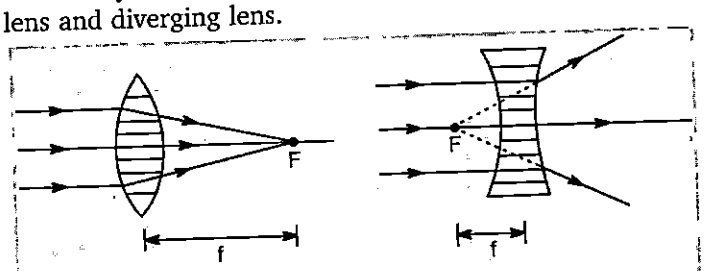


Fig. 1.176 (c)

A lens is a transparent object with two refracting surfaces whose central axes coincide; that common centre axis of the lens is called the optical axis or principal axis. The function of a lens is to deviate light; it can either converge or diverge a parallel beam of light. Depending on this property lenses may be characterized into two categories: converging lens and diverging lens.



(a) A converging lens (b) A diverging lens

Fig. 1.177

A thin lens may be considered as made-up of a large number of small-angled prism placed together; the angle of prism is zero at the optical centre and it increases as we move away from the optical axis.

A thin lens is a lens in which the thickness of the lens is small compared to the object, the image distance, or either of the two radii of curvature of the lens.

Derivation of the Thin Lens Equation

We will consider refraction at the two surfaces S_1 and S_2 (Fig. 1.178)

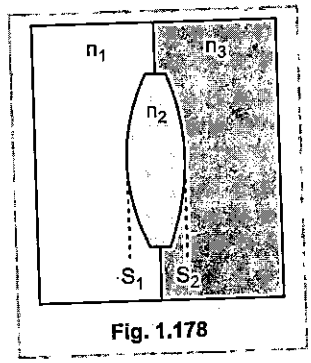


Fig. 1.178

Refraction at first surface S_1: Applying the single surface refraction equation, we get

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R_1} \quad \ldots (1)$$

The magnification produced,

$$m_1 = \frac{n_1 v}{n_2 u} \quad \ldots (2)$$

Refraction at second surface S_2 : The single surface refraction equation now is

$$\frac{n_3}{v'} - \frac{n_2}{u'} = \frac{n_3 - n_2}{R_2} \quad \ldots (3)$$

Note that the image formed after refraction at the first surface acts as an object for refraction at the second surface, *i.e.*, $v = u'$; eqn. (3) becomes

$$\frac{n_3}{v'} - \frac{n_2}{v} = \frac{n_3 - n_2}{R_2} \quad \ldots (4)$$

The magnification produced,

$$m_2 = \frac{n_2 v'}{n_3 v} \quad \ldots (5)$$

Note the two terms encircled in eqns. (1) and (4) ; they are same. If we add eqns. (1) and (4), we get

$$\frac{n_3}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \quad ...(6)$$

Eqn. (6) is the most general form of the thin lens equation. Total magnification is

$$m = m_1 m_2 = \frac{n_1 v}{n_2 u} \times \frac{n_2 v'}{n_3 v} = \frac{n_1 v'}{n_3 u} \quad ...(7)$$

❒ If media are the same on both sides of the lens, then $n_3 = n_1$; the eqn.(6) becomes

$$\frac{n_1}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R}$$

which on rearrangement gives

$$\frac{1}{v} - \frac{1}{u} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(8)$$

We have dropped prime v', the magnification now becomes

$$m = \frac{v}{u} \quad ...(9)$$

Focal point and focal length : When an object is placed infinitely far away from the lens, the incident rays are parallel. The image point is the focal point of the lens, the distance of the focal point from the lens is called the **focal length.**

From eqn. (8), on substituting $v = f, u = \infty$, we get

$$-\frac{1}{\infty} + \frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad ...(10)$$

Eqn. (10) is called **lens maker's equation.**

The thin lens equation now reduces to

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ... (11)$$

and magnification $m = \frac{v}{u}$ ... (12)

❒ Focal length of a biconvex lens :

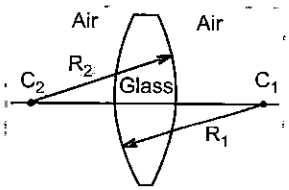


Fig. 1.179

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{(+R_1)} - \frac{1}{(-R_2)}\right)$$

$$= \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

❒ Focal length of a biconcave lens :

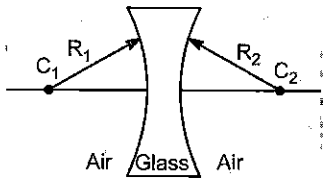


Fig. 1.180

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{(-R_1)} - \frac{1}{(+R_2)}\right]$$

$$= \left(\frac{n_2 - n_1}{n_1}\right)\left[-\frac{1}{R_1} - \frac{1}{R_2}\right]$$

❒ Focal length of a plano-convex lens :

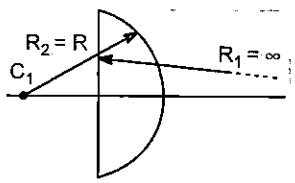


Fig. 1.181

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{\infty} - \frac{1}{(-R_2)}\right]$$

$$= \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_2}\right) = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R}\right)$$

❒ Focal length of a plano-concave lens :

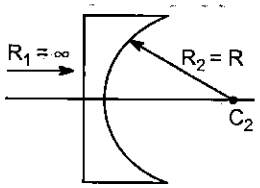


Fig. 1.182

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{\infty} - \frac{1}{(+R_2)}\right]$$

$$= \left(\frac{n_2 - n_1}{n_1}\right)\left(-\frac{1}{R_2}\right) = \left(\frac{n_2 - n_1}{n_1}\right)\left(-\frac{1}{R}\right)$$

Note that focal length of a lens is same, irrespective of the side on w hich the object is kept.

❒ Each lens has two focal points. The focal point located from lens maker's equation is **primary focal point.** Any ray incident on the lens parallel to the optic axis ($u = \infty$) after refraction from lens passes through or appears to pass through the primary focal point as shown in Fig. 1.183.

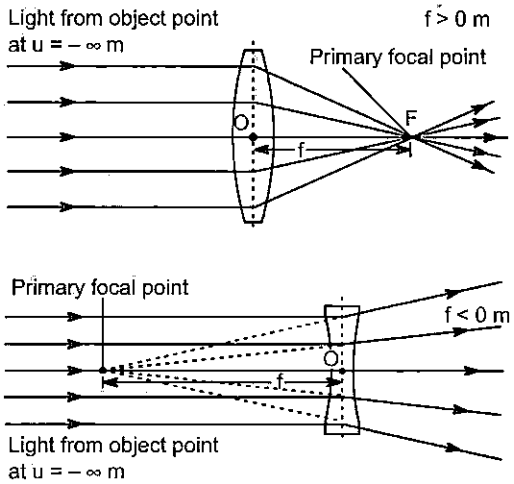


Fig. 1.183

Each lens also has a **secondary focal point.** Any ray passing through or appearing to pass through the secondary focal point, after refraction from lens, passes parallel to principal axis as shown in Fig. 1.184.

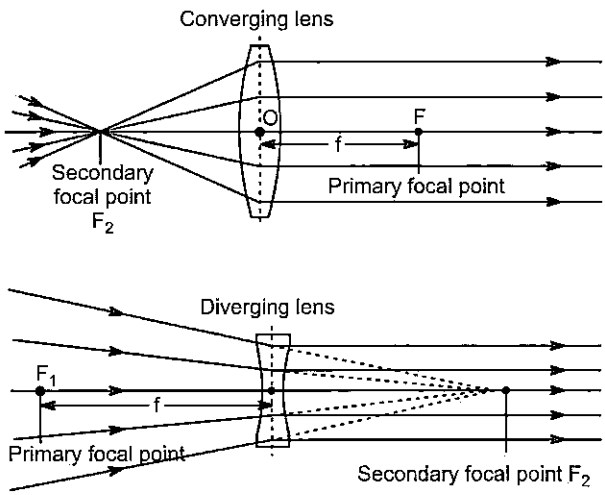


Fig. 1.184

OPTICAL

The second general rule for drawing diagrams involves the use of the so-called *optical centre* of a lens. Consider a ray *AB* incident upon a converging lens at *B* (Fig. 1.185) and being refracted to *C*, such that the inclination of *BC* to both surfaces is the same. Then the path of the emergent ray *CD* is, obviously, parallel to *AB*. Thus the effect of the lens on the ray *AB* is merely slightly to displace it laterally. It can be shown geometrically that all rays passing through the lens so that the incident and emergent directions are parallel must pass through a fixed point *O* in the lens. This point *O* is called the *optical centre* of the lens, and, when the surfaces are of the same curvature, is at the centre of the lens. For a thin lens the lateral displacement is negligible, and so, for our work with lenses, *rays passing through the optical centre are considered to be undeviated. Thus in graphical work it is legitimate to treat rays passing through the optical centre, taken as the intersection of the lens position and the principal axis, as continuing their path undisturbed.*

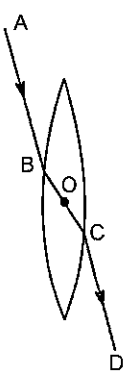


Fig. 1.185

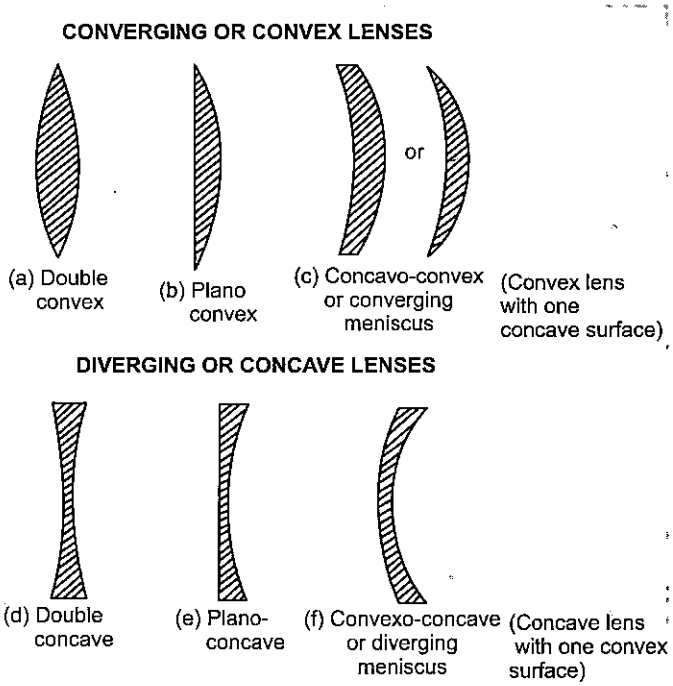


Fig. 1.186

Ray Diagram for Thin Lenses

Three rays are employed to locate the position of an image :

(1) A ray passing through the centre of lens, the origin of our cartesian coordinate system, passes essentially straight through the lens. A lens behaves as a glass slab in this region.

(2) A ray from an object parallel to the principal axis passes through or appears to pass through the primary focus.

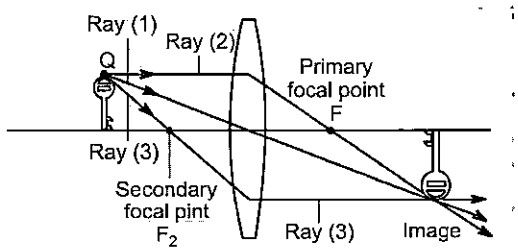


Fig 1.187 (a) Converging lens

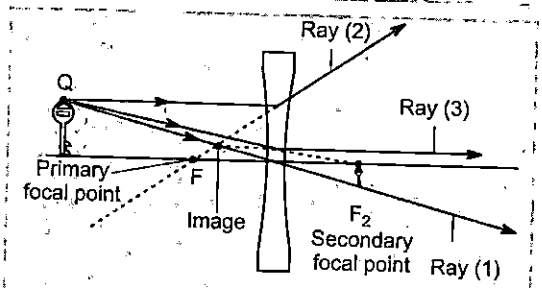
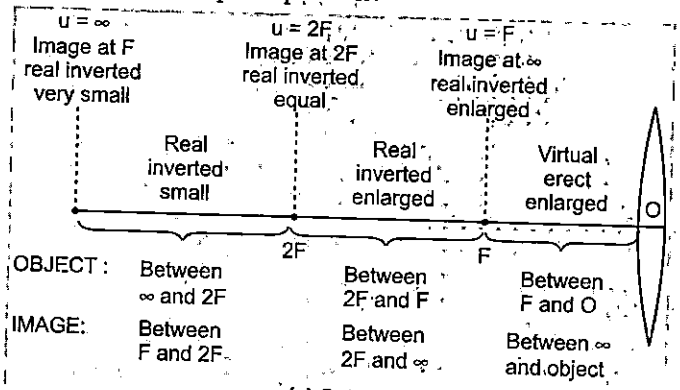
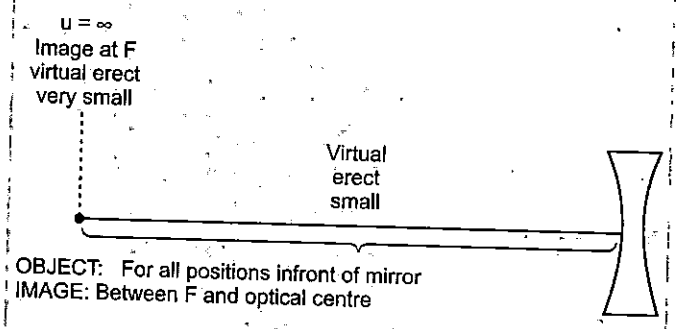


Fig 1.187 (b) Diverging lens

(3) A ray from the object passing through or appearing to pass through the secondary focal point emerges from the lens parallel to its principal axis.



(a) Convex lens



(b) Concave lens

Fig. 1.188

Example 134

A sphere of radius R made of material of refractive index μ_2. Where would an object be placed so that a real image is formed at equidistant from the sphere ?

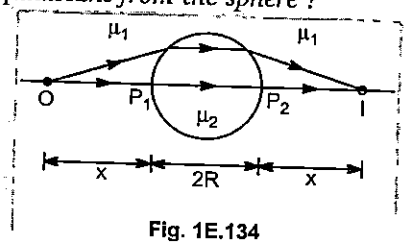


Fig. 1E.134

Solution: Let the object be placed at a distance x from the pole P_1 of the sphere. If a real image is to be formed at equidistant from the sphere, then the ray must pass symmetrically through the sphere, as shown in the figure. Applying the equation at the first surface, we get

$$\frac{\mu_2}{+\infty} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{-R} \quad \text{or} \quad x = \left(\frac{\mu_1}{\mu_2 - \mu_1}\right) R$$

Note that the real image is formed only when the refractive index of the sphere is more than that of the surrounding, *i.e.*, $\mu_2 > \mu_1$.

Example 135

A small object of height 0.5 cm is placed in front of a convex surface of glass ($\mu = 1.5$) of radius of curvature 10 cm. Find the height of the image formed in glass.

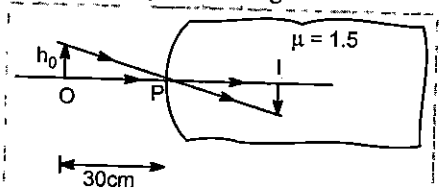


Fig. 1E.135

Solution: According to Cartesian sign convention

$$u = -30\text{cm}; \quad R = +10\text{cm}; \mu_1 = 1; \mu_2 = 1.5$$

Applying equation, we get

$$\frac{1.5-1}{v} - \frac{1}{-30} = \frac{1.5-1}{+10}$$

or $v = 90\,\text{cm}$ (real image)

Let h_i be the height of the image, then

$$\frac{h_i}{h_O} = \frac{\mu_1 v}{\mu_2 u} = \frac{(1)(90)}{(1.5)(-30)} = -2$$

$$h_i = -2h_O = -2(0.5) = -1\text{ cm}$$

The negative sign shows that the image is inverted.

Example 136

A glass sphere of index 1.5 and radius 40 cm has half its hemispherical surface silvered. The point where a parallel beam of light, coming along a diameter, find the focus (or appear to) after coming out of sphere.

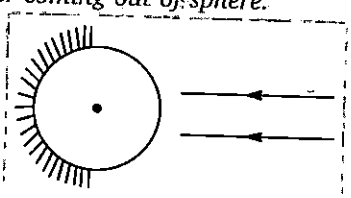


Fig. 1E.136 (a)

Solution: $R = 40$ cm

for refracting surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$n = 120 \text{ cm}$$

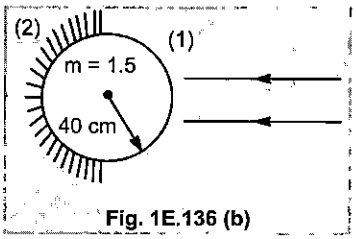


Fig. 1E.136 (b)

For mirror (2)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{40} = \frac{1}{(-20)}$$

$$v = -40/3$$

for refracting surface (1)

$$u = 80 - \frac{40}{3} = \frac{200}{3}$$

$$\frac{1}{v} - \frac{1.5}{-(200/3)} = \frac{1-1.5}{-40}$$

$$\frac{1}{v} + \frac{4.5}{200} = \frac{0.5}{40}$$

$$\frac{1}{v} = \frac{1}{80} - \frac{4.5}{200}$$

$$v = -100 \text{ cm}$$

So distance of image from centre will be

$100 - 40 = 60$ cm (left of the centre)

Example 137

An object 'O' is kept in air in front of a thin plano convex lens of radius of curvature 10 cm. It's refractive index is 3/2 and the medium towards right of plane surface is water of refractive index 4/3. What should be the distance 'x' of the object so that the rays become parallel finally.

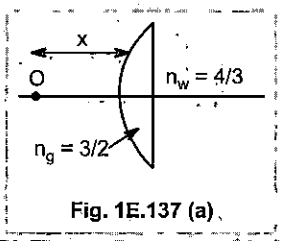


Fig. 1E.137 (a)

Solution: $\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$

$$\frac{\mu_3}{\infty} - \frac{1}{x} = \frac{1.5-1}{10} + \frac{4/3 - 3/2}{\infty}$$

$$\frac{-1}{x} = \frac{0.5}{10}$$

$$x = -20 \text{ cm}$$

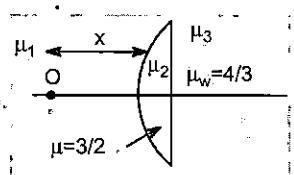


Fig. 1E.137 (b)

Example 138

The diagram shows an equiconvex lens. What should be the condition on the refractive indices so that the lens become diverging ?

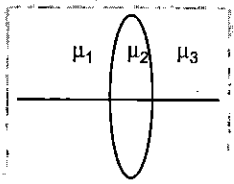


Fig. 1E.138 (a)

Solution: $\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$

$$\frac{\mu_3}{v} - \frac{\mu_1}{\infty} = \frac{\mu_2 - \mu_1}{R} + \frac{\mu_3 - \mu_2}{(-R)}$$

$$\frac{\mu_3}{v} = \frac{\mu_2 - \mu_1}{R} - \frac{(\mu_3 - \mu_2)}{R} < 0$$

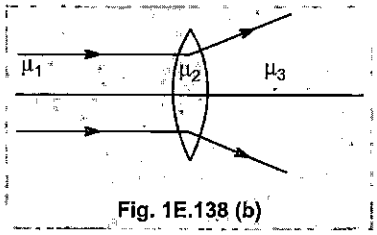


Fig. 1E.138 (b)

For diverging nature $v < 0$

$$\frac{\mu_2 - \mu_1}{R} < \frac{\mu_3 - \mu_2}{R}$$

$$2\mu_2 < \mu_1 + \mu_3$$

Focal Lengths for a Lens with Different Medium an either Side

For paraxial rays only.

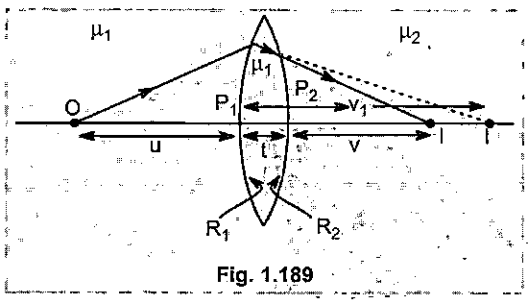


Fig. 1.189

For first refractory surface,

$$\frac{\mu_l}{v_1}-\frac{\mu_1}{u}=\frac{\mu_l-\mu_1}{R_1} \quad ...(1)$$

For second refractory surface,

$$\frac{\mu_2}{v}-\frac{\mu_l}{(v_1-t)}=\frac{\mu_2-\mu_l}{R_2}$$

For thin lens thickness 't' is negligible,

$$\frac{\mu_2}{v}-\frac{\mu_l}{v_1}=\frac{\mu_2-\mu_l}{R_2} \quad ...(2)$$

Adding equations (1) and (2) we get,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_l-\mu_1)}{R_1}-\frac{(\mu_l-\mu_2)}{R_2} \quad ...(A)$$

First principal focus ($u=f_1; v=\infty$)

$$\frac{\mu_2}{\infty}-\frac{\mu_1}{f_1}=\frac{(\mu_l-\mu_1)}{R_1}-\frac{(\mu_l-\mu_2)}{R_2}$$

$$\frac{1}{f_1}=-\frac{1}{\mu_1}\left\{\frac{(\mu_l-\mu_1)}{R_1}-\frac{(\mu_l-\mu_2)}{R_2}\right\} \quad ...(3)$$

Second principal focus ($u=\infty; v=f_2$)

$$\frac{\mu_2}{f_2}-\frac{\mu_1}{\infty}=\frac{(\mu_l-\mu_1)}{R_1}-\frac{(\mu_l-\mu_2)}{R_2}$$

$$\frac{1}{f_2}=\frac{1}{\mu_2}\left\{\frac{(\mu_l-\mu_1)}{R_1}-\frac{(\mu_l-\mu_2)}{R_2}\right\} \quad ...(4)$$

Dividing equation (4) by (3), we get,

Result-1: $$\frac{f_1}{f_2}=-\frac{\mu_1}{\mu_2} \quad ...(B)$$

Relation between first principal focus and second principal focus.

Since ratio is –ve, so first and second principal focus always lie on the opposite sides of lens.

From equation (3) and (4) we get,

$$f_1=-\frac{\mu_1}{k};\quad f_2=\frac{\mu_2}{k}$$

$$k=\left[\frac{(\mu_l-\mu_1)}{R_1}-\frac{(\mu_l-\mu_2)}{R_2}\right] \Rightarrow \text{say}$$

$$\frac{f_1}{u}=-\frac{\mu_1}{ku};\quad \frac{f_2}{v}=\frac{\mu_2}{kv}$$

$$\frac{f_1}{u}+\frac{f_2}{v}=\frac{1}{k}\left(\frac{\mu_2}{v}-\frac{\mu_1}{u}\right)$$

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=k \quad \text{from (A)}$$

$$\frac{f_1}{u}+\frac{f_2}{v}=\frac{1}{k}\times k$$

Result-2: $$\frac{f_1}{u}+\frac{f_2}{v}=1 \quad ...(C)$$

Transverse Magnification (m_t):

$$m_t=m_1\times m_2$$

$m_1 \Rightarrow$ transverse magnification for first surface

$m_2 \Rightarrow$ transverse magnification for second surface

$$m_1=\frac{\mu_1}{\mu_l}\left(\frac{v_1}{u}\right);\quad m_2=\frac{\mu_l}{\mu_2}\left(\frac{v}{v_1}\right)$$

Result-3: Transverse magnification

$$(m_t)=\frac{\mu_1}{\mu_2}\left(\frac{v}{u}\right) \quad ...(D)$$

Longitudinal magnification (for very small object, placed along the principle axis)

$$m_l=\frac{dv}{du}$$

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_l-\mu_1}{R_1}-\frac{\mu_l-\mu_2}{R_2}$$

Differentiating both sides w.r.t. u;

$$\left(-\frac{\mu_2}{v^2}\right)\frac{dv}{du}+\frac{\mu_1}{u^2}=0$$

$$\frac{dv}{du}=\frac{\mu_1}{\mu_2}\left(\frac{v^2}{u^2}\right)$$

$$m_l=\frac{\mu_1}{\mu_2}\left(\frac{v^2}{u^2}\right) \quad ...(E)$$

$$m_l=\left(\frac{\mu_1}{\mu_2}\frac{v}{u}\right)^2\left(\frac{\mu_2}{\mu_1}\right)$$

$$m_l=\frac{\mu_2}{\mu_1}(m_t)^2 \quad ...(F)$$

IMPORTANT POINTS

1. If nature of medium on two sides of lens is same ($\mu_1=\mu_2=\mu_s)\mu_s \Rightarrow$ refractive index of surrounding. Equation (A) transforms to,

$$\frac{\mu_s}{v}-\frac{\mu_s}{u}=(\mu_l-\mu_s)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(G)$$

Equation (3) transforms to,

$$\frac{1}{f_1}=-\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Equation (4) transforms to,

$$\frac{1}{f_2}=-\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Result-1 $f_1=-f_2; |f_1|=|f_2|=f$

First and second principal focal lengths are numerically equal,

Result-2 $\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f_2}=\frac{1}{f}$

Sign of 'f' is according to sign of second principal focal length. Equation (D) transforms t,

Result-3 $m_t=\frac{v}{u}=\frac{f-v}{f}=\frac{f}{f+u}$

Equation (E) transforms to,

Result-4 $m_l=\frac{v^2}{u^2}=m_t^2$

Equation (B) transforms to,

Result-5 $\frac{f_1}{f_2}=-1$

Equation (C) transforms to,

$$\frac{-f_2}{u}+\frac{f_2}{v}=1 \qquad \frac{1}{v}-\frac{1}{u}=\frac{1}{f_2}$$

2. Lensmaker's Formula

It relates the focal length of the lens to the refractive index μ of the lens material and the radii of curvature of the two surfaces.

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where R_1 is the radius of curvature of the lens surface first reached by the light, and R_2 is the radius of curvature of the second surface.

(i) If the lens is immersed in a medium whose refractive index is not equal to unity, then μ in the above formula is replaced by $\mu=\frac{\mu_{\text{lens}}}{\mu_{\text{medium}}}$.

(ii) For converging lens secondary focal length is +ve and for diverging lens secondary focal length is –ve.

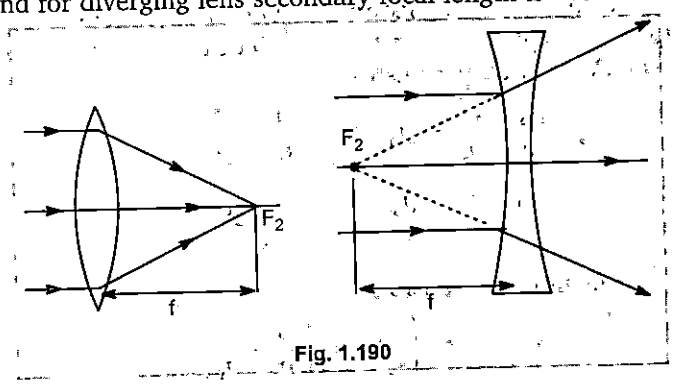


Fig. 1.190

3. Application and limitation:

The Lensmaker's formula is application for thin lenses only. The value of R_1 and R_2 are to be put in accordance with the Cartesian sign convention.

Double or Bi-Convex Lens

$$\frac{1}{R_1}-\frac{1}{R_2}=\frac{1}{R_1}-\frac{1}{-R_2}$$

$$=\frac{1}{R_1}+\frac{1}{R_2}\Rightarrow +\text{ve}$$

$$\frac{1}{f_2}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

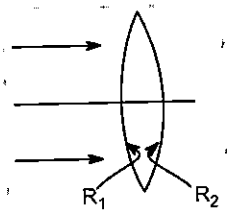


Fig. 1.191

If $\mu_l>\mu_s$, then $f_2\Rightarrow$ +ve (Converging nature)

If $\mu_l<\mu_s$, then $f_2\Rightarrow$ –ve (Diverging nature)

If $R_1=R_2=R$ (Equi-convex)

$$\frac{1}{f_2}=\left(\frac{\mu_l}{\mu_s}-1\right)\frac{2}{R}$$

Plano-Convex Lens

$$\frac{1}{R_1}-\frac{1}{R_2}=\frac{1}{R_1}\Rightarrow +\text{ve}$$

$$\frac{1}{f_2}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

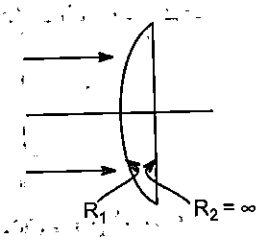


Fig. 1.192

If $\mu_l>\mu_s$, then $f_2\Rightarrow$ +ve (Converging nature)

If $\mu_l<\mu_s$, then $f_2\Rightarrow$ –ve (Diverging nature)

Double or Bi-Concave Lens

$$\frac{1}{R_1}-\frac{1}{R_2}=-\frac{1}{R_1}\frac{1}{R_2}=-\left[\frac{1}{R_1}+\frac{1}{R_2}\right]\Rightarrow -\text{ve}$$

$$\frac{1}{f_2}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

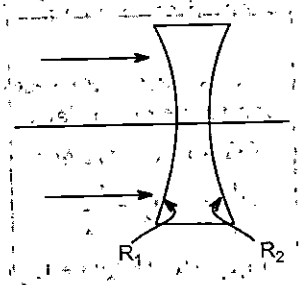


Fig. 1.193

If $\mu_l>\mu_s$, then $f_2\Rightarrow$ – ve (Diverging nature)

If $\mu_l<\mu_s$, then $f_2\Rightarrow$ + ve (Converging nature)

$(R_1=R_2=R)$ (Equi-concave lens)

$$\frac{1}{f_2}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{-2}{R}\right)$$

Plano-Concave Lens

$$\frac{1}{R_1}-\frac{1}{R_2}=-\frac{1}{R_1}\Rightarrow -\text{ve}$$

$$\frac{1}{f_2}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

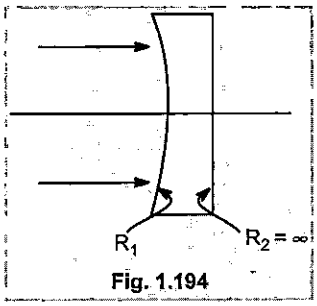


Fig. 1.194

If $\mu_l > \mu_s$, then $f_2 \Rightarrow$ –ve (Diverging nature)

If $\mu_l < \mu_s$, then $f_2 \Rightarrow$ +ve (Converging nature)

Convexo-Concave Lens

$$\frac{1}{R_1}-\frac{1}{R_2}=-\frac{1}{R_1}+\frac{1}{R_2}$$

$\because$ $R_1 > R_2$ (numerically)

$\therefore$ $\frac{1}{R_1}<\frac{1}{R_2}; \frac{1}{R_1}-\frac{1}{R_2}$ becomes + ve

$$\frac{1}{f_2}=\left(\frac{\mu_l}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

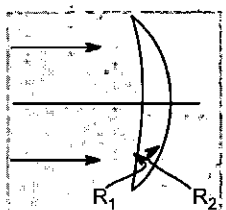


Fig. 1.195

If $\mu_l > \mu_s$, then $f_2 \Rightarrow$ +ve (Converging nature)

If $\mu_l > \mu_s$, then $f_2 \Rightarrow$ –ve (Diverging nature)

Concave-Convex Lens

$\because$ $R_1 < R_2$ (numerically)

$$\frac{1}{R_1}>\frac{1}{R_2}$$

$$\frac{1}{R_1}-\frac{1}{R_2} \Rightarrow +ve$$

$$\frac{1}{f_2}=\left(\frac{\mu_1}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

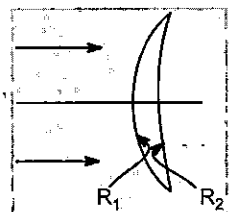


Fig. 1.196

If $\mu_l > \mu_s$, then $f_2 \Rightarrow$ + ve (Converging nature)

If $\mu_l > \mu_s$, then $f_2 \Rightarrow$ – ve (Diverging nature)

OPTIONAL

S. No.	Type of Lens	Shape of Lens	The Factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$	Reciprocal of the focal length in air or vacuum	Nature of Lens
(1)	Concavo-convex	) or (	$\left(\frac{1}{x}-\frac{1}{y}\right)$	$(\mu-1)\left(\frac{1}{x}-\frac{1}{y}\right)$	Converging
(2)	Convexo-concave	) or (	$-\left(\frac{1}{x}-\frac{1}{y}\right)$	$-(\mu-1)\left(\frac{1}{x}-\frac{1}{y}\right)$	Diverging
(3)	Biconvex	◊	$\left(\frac{1}{x}+\frac{1}{y}\right)$	$(\mu-1)\left(\frac{1}{x}+\frac{1}{y}\right)$	Converging

(4)	Biconcave		$-\left(\frac{1}{x}+\frac{1}{y}\right)$	$-(\mu-1)\left(\frac{1}{x}+\frac{1}{y}\right)$	Diverging
(5)	Planoconvex		$\frac{1}{R}$	$(\mu-1)/R$	Converging
(6)	Planoconcave		$-\frac{1}{R}$	$-(\mu-1)/R$	Diverging
(7)	Symmetrical biconvex		$\frac{2}{R}$	$(\mu-1)\left(\frac{2}{R}\right)$	Converging
(8)	Symmetrical biconcave		$-\frac{2}{R}$	$-(\mu-1)\left(\frac{2}{R}\right)$	Diverging

Graph of u vs. v for a lens : According to lens equation, it is a hyperbola, as shown in Fig. 1.197.

(a) Convex lens :

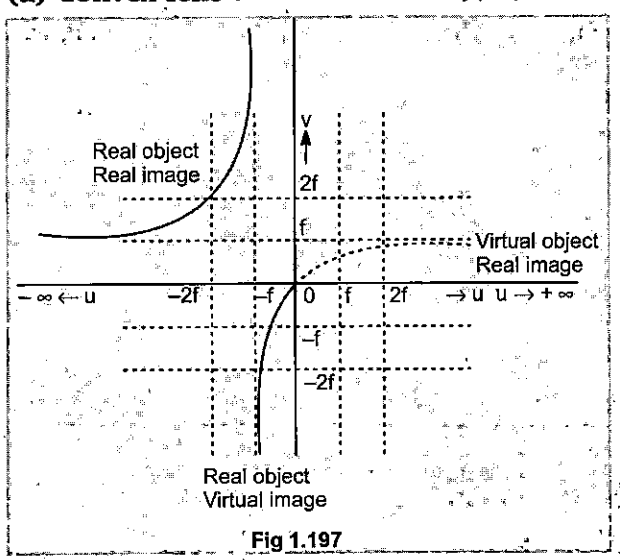


Fig 1.197

$u =$	$-\infty$	$-2f$	$-f$	$-\frac{f}{2}$	$-\frac{f}{4}$	0	$+f$	$+2f$	$+\infty$
$v =$	$+f$	$+2f$	$+\infty$	$-f$	$-\frac{f}{3}$	0	$+\frac{f}{2}$	$+\frac{2f}{3}$	$+f$

(b) Concave lens :

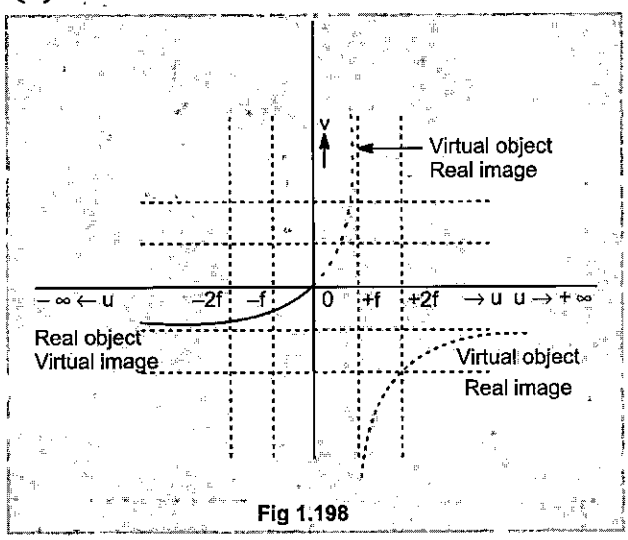


Fig 1.198

$u =$	$-\infty$	$-2f$	$-f$	$-\frac{f}{2}$	0	$+\frac{f}{2}$	$+f$	$+f$	$+2f$	$+\infty$
$v =$	$-f$	$\frac{-2f}{3}$	$-\frac{f}{2}$	$-\frac{f}{3}$	0	$+f$	$+\infty$	$-\infty$	$-2f$	$-f$

ANALYSIS OF GRAPHS

Convex Lens

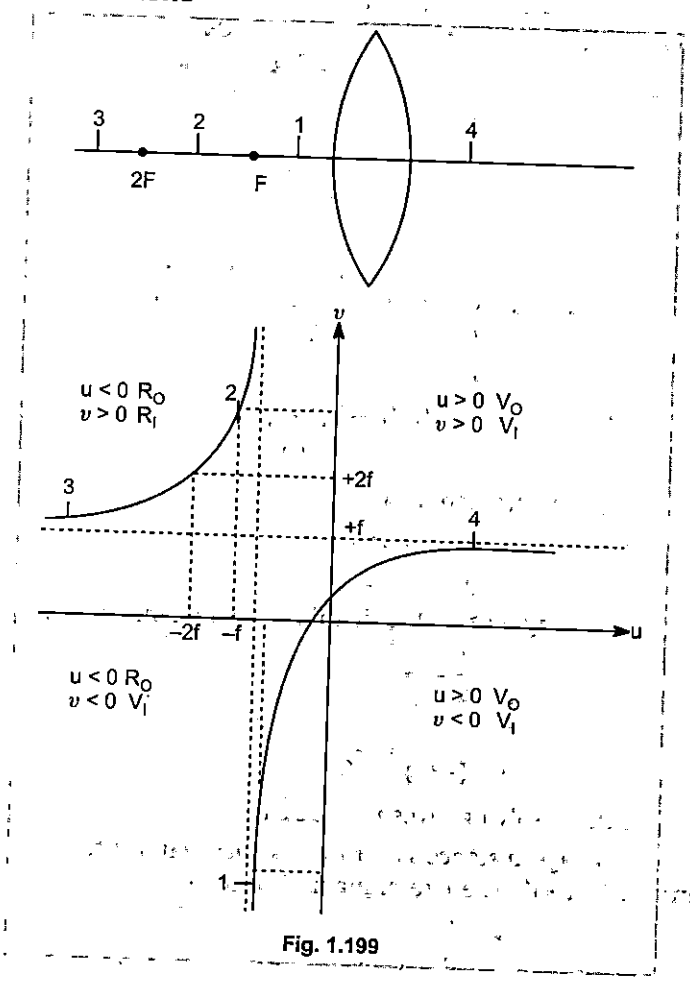


Fig. 1.199

Region 1: Object lies between optical centre and focus real object, virtual image $|v|>|u|$.

$m = \frac{v}{u}$ thus $|m|>1$ image is enlarged is $u < 0 : v < 0$ thus $m > 0$ image is erect. As object moves from focus towards optical centre magnification goes on decreasing.

Region 2: Object lies between F and $2F$ object is real image is also real $|v|>|u|$.

$|m|>1$ image is enlarged $v > 0; u < 0$ $m < 0$ image is inverted as object moves from F to $2F$ magnification decreases and equals 1 at $2F$.

Region 3: Object lies between $2F$ and infinity object is real and image is also real. $|v|<|u|$.

$|m|<1$ image is smaller in size $v > 0; u < 0, m < 0$ image is inverted as object moves farther away magnification goes on decreasing.

Region 4: Object is virtual and it lies between optical centre and infinity. Object is virtual and image is real $|v|<|u|$.

$|m|<1$ image is smaller in size $v > 0, u > 0, m > 0$ image is erect as object moves away magnification goes on decreasing.

Concave Lense

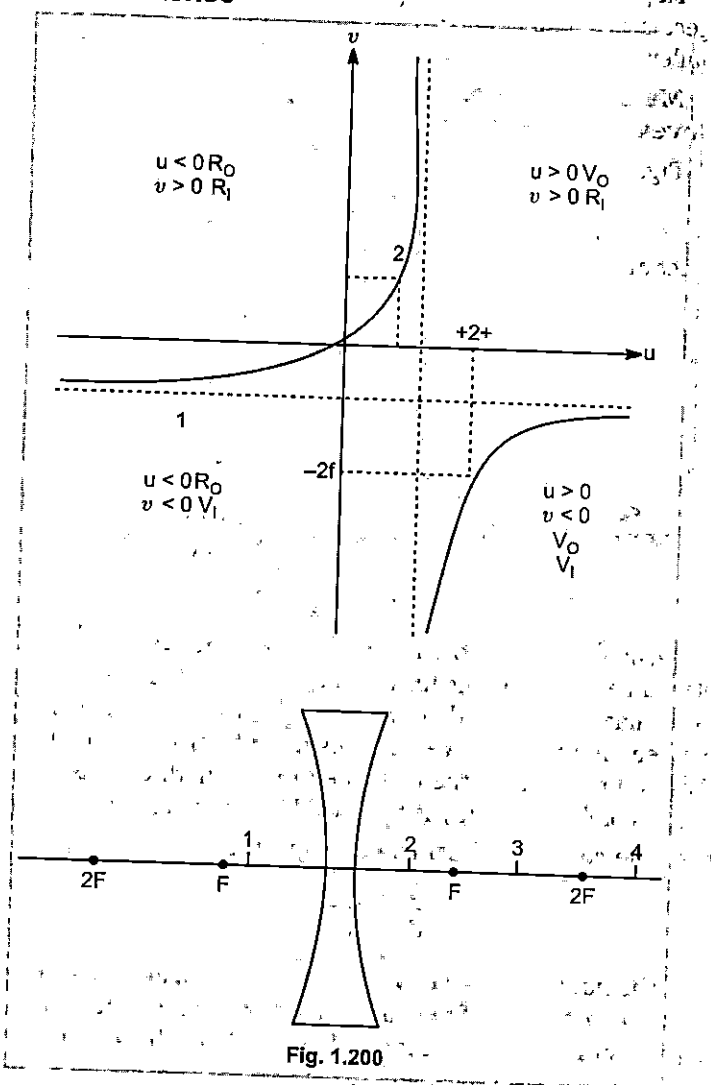


Fig. 1.200

Region 1: For all the positions of a real object infront of a lens. Object is real image is virtual $|v|<|u|$.

$|m|<1$ image is smaller in size $v < 0, u < 0$ $m > 0$ image is erect. As object is moved away from lens image size *i.e.*, magnification goes on decreasing.

Region 2: Virtual object that lies between optical centre and focus.

Object is virtual and image is real $|v|>|u|; |m| > 1$ image is enlarged $v >, u > 0, m > 0$ image is erect.

As object is moved towards F image size goes on increasing *i.e.*, m increases.

Region 3: Virtual object lies between $+F$ and $+2F$ object is virtual and image is virtual $|v|>|u|; |m|>1$ image is enlarged $u > 0; v < 0; m < 0$ image is inverted.

As object moves from $+F$ to $+2F$ magnification goes an decreasing.

Region 4: Virtual object lies between $2F$ and infinity object is virtual $|v|<|u|$, $|m|<1$ image is smaller in size $u > 0$; $v < 0$; $m < 0$ image is inverted.

Measurement of refractive index of liquid by a convex lens

Fig. 1.201 shows an equiconvex lens placed on a plane mirror. An object pin is moved up and down. When the pin lies at the focus of the lens, there is no parallax between the object and the image.

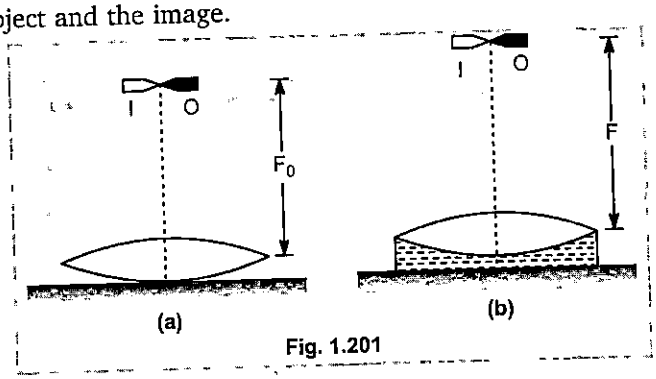


Fig. 1.201

When a liquid whose refractive index is to be obtained is placed between a plane mirror and a convex lens, the object pin O has to be shifted *downward* so that no parallax exists between it and its image. The position of object O from the lens is now *equal* to the combined focal length of *'lens and liquid'* combination. If f is the focal length of the liquid lens, then the combined focal length is given by

$$\frac{1}{F}=\frac{1}{f_0}+\frac{1}{f}$$

The liquid lens is *plano-concave* type as its lower surface is the *plane* surface of the mirror and the upper surface is the *curved* surface of the convex lens. If R is the radius of curvature of the convex lens, then

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)$$

or $$\frac{1}{f}=-\frac{(\mu-1)}{R}$$

Thus, $$\mu=1+\frac{R}{f}$$

Example 139

A pencil of height 1 cm is placed 30 cm from an equiconvex lens, refractive index n=3/2, radius of curvature for both the surfaces, $R_1 = R_2 = R = 10$ cm. Find the location of the image and describe its characteristics.

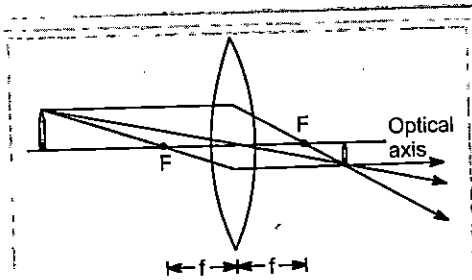


Fig 1E.139

Solution: The rays of light diverge from the object; it is a **real object.** Focal length of a convex lens is positive.

$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{2}{R}\right)$$

$$=\frac{(3/2)-1}{1}\left(\frac{2}{10}\right)=\frac{1}{10}$$

The given parameters are

$$u=-30\,\text{cm}$$

$$f=+10\,\text{cm}$$

Substituting these values in thin lens equation, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{(-30)}=\frac{1}{10}$$

which on solving gives $v = +15$ cm.

The image distance is positive, which implies that the **image is real.** The lateral magnification is

$$m=\frac{v}{u}=\frac{15}{(-30)}=-\frac{1}{2}$$

The image is half the size of the object ; size of the image is 0.50 cm. The negative sign with magnification shows **inverted image.**

Example 140

If the pencil in the previous problem is kept at 6 cm from the lens, locate and characterize the image.

Solution: From lens equation,

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}+\frac{1}{(-6)}=-\frac{1}{15}$$

$$v=-15\,\text{cm}$$

Lateral magnification, $m=\frac{v}{u}=\frac{-15}{-6}=2.50$

The minus sign with image distances shows that the image is located on the side of the object. The image is upright, because magnification is positive.

Example 141

If in the previous example we use a diverging lens with a focal length 10.0 cm to form an image of the pencil kept 15 cm infront of the lens, locate and characterize the image.

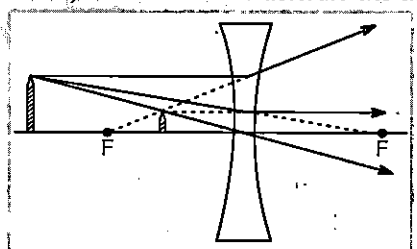


Fig. 1E.141

Solution: In accordance with cartesian sign convention the given parameters are,

$$f = -10\,\text{cm}, u = -15\,\text{cm}$$

From lens equation, we have

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{(-10)} + \frac{1}{(-15)} = -\frac{1}{6}$$

$$v = -6\,\text{cm}$$

Lateral magnification,

$$m = \frac{v}{u} = \frac{-6}{-15} = 0.40$$

The minus sign with image shows that the image is located on the side of the object. The magnification is positive and $m < 1$, which shows that the image is upright and diminished.

Example 142

A pencil with a height of 5 mm is placed 45 cm to the left of a converging lens of focal length 20 cm. A diverging lens of focal length 15 cm is at a distance 10 cm from the first lens. What is (a) the image distance v_1 and (b) the height I_1 of the image produced by the first lens ? (c) What is the object distance for the second lens ? Find (d) the image distance v_2 and (e) the height I_2 of the image formed by the second lens.

Solution: The image formed by a converging lens can be real or virtual. It depends on where the object is located relative to the focal point. If the object is to the left of the focal point as in this case, the image is real and formed on the right side of the lens. If on the other hand the object is located between the focal point and the lens, the image is virtual, located on the left of the lens.

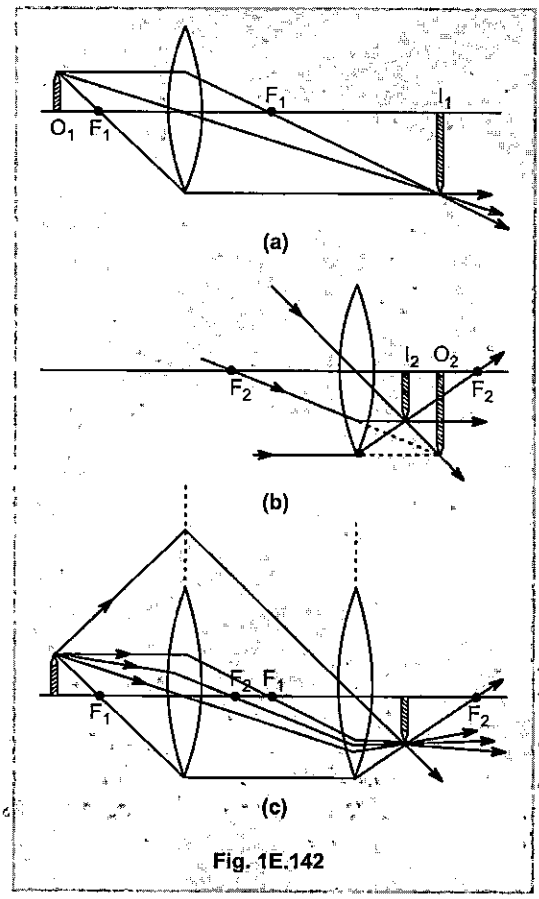


Fig. 1E.142

The image produced by the first lens acts as an object for the second lens; this image is called the first image. the first image can lie between the lenses or to the right of the second lens. In the first case it will be a real object and in the second case it will be a virtual object for the second lens.

(a) For the image formed by the first lens,

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{20} + \frac{1}{(-45)} = \frac{1}{36}$$

$$v_1 = +36\,\text{cm}$$

We thus find that the first lens is attempting to form a real image, 36 cm behind it.

However, before this image can be formed the rays are intercepted by the second lens located 10 cm behind it, see Fig. 1E.142 (a). The point of convergence of these rays is a **virtual object** for the second lens.

(b) The height of the image,

$$I_1 = O_1\left(\frac{v_1}{u_1}\right) = (5\times10^{-1})\left(\frac{36}{-45}\right) = -4\,\text{mm}$$

The minus sign indicates that the image is inverted.

(c) The object distance for the second lens

$= (36-10)\,\text{cm} = 26\,\text{cm}$

to the right of second lens ; hence $v_2 = +26\,\text{cm}$

(d) From lens equation for the second lens,

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{(-15)} + \frac{1}{(+26)} = -\frac{1}{35.5}$$

$$v_2 = -35.5\,\text{cm}$$

The negative sign shows that it is on the left of the diverging lens (virtual image)

(e) The image height for the first lens is object height for the second lens. Height of the image formed by the second lens

$$I_2 = O_2\left(\frac{v_2}{u_2}\right) = (-4\times10^{-1})\left(\frac{-35.5}{+26}\right) = 5.46\,\text{mm}$$

The positive sign for I_2 means that the final image has the same orientation as the first image.

Example 143

A diverging lens of focal length 10 cm is placed 10 cm in front of a plane mirror as shown in the Fig. 1E.143 (a). Light from a very far away source falls on the lens. What is the distance of final image ?

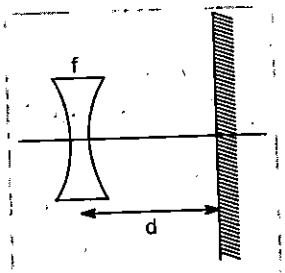


Fig. 1E.143 (a)

Solution:

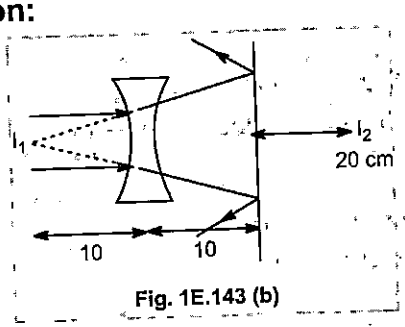


Fig. 1E.143 (b)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-30} = +\frac{1}{-10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-1}{10} - \frac{1}{30} = -\frac{4}{30} \quad \Rightarrow \quad v = -7.5$$

The final image distance = 2.5 cm in front of the mirror.

Example 144

The values of d_1 and d_2 for final rays to be parallel to the principle axis are: (focal lengths of the lenses are written on the lenses)

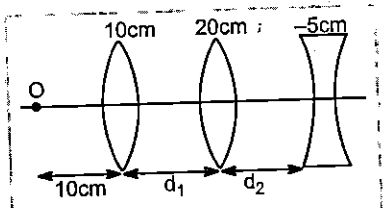


Fig. 1E.144 (a)

(a) $d_1 = 10\,cm,\ d_2 = 15\,cm$

(b) $d_1 = 20\,cm,\ d_2 = 15\,cm$

(c) $d_1 = 30\,cm,\ d_2 = 15\,cm$

(d) None of these

Solution: (a), (b), (c)

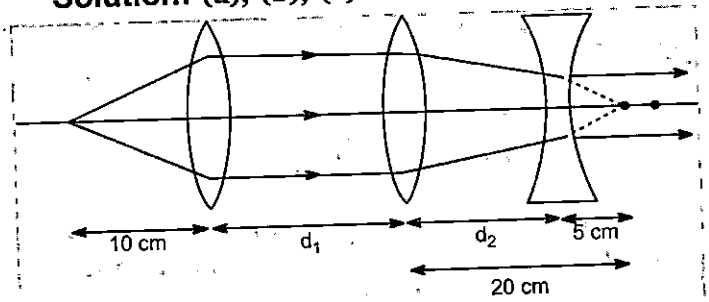


Fig. 1E.144 (b)

$d_2 = 20 - 5 = 15$ cm

d_1 can take any value.

Example 145

An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm on the other side of the lens, a convex mirror is placed at its focus such that the image formed by combination coincides with the object itself. Find the focal length of concave mirror.

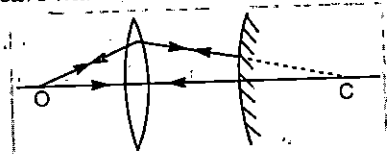


Fig. 1E.145 (a)

Solution: For retracing of ray; ray must fall normally on mirror *i.e.*, towards the centre of curvature.

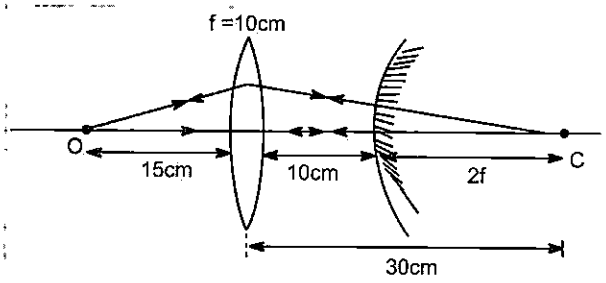


Fig. 1E.145 (b)

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad u=-15$$

$$\frac{1}{v}+\frac{1}{15}=\frac{1}{10} \qquad f=10\text{ cm}$$

$$v=30\text{cm}$$

For mirror $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $u=2f$

$$\frac{1}{v}+\frac{1}{2f}=\frac{1}{f}$$

$$v=2f$$

Hence from the ray diagram

$$2f=30-10$$

$$f=10\text{ cm}$$

Example 146

A point source of light is placed 60 cm away from screen. Intensity detected at point P is I. Now a diverging lens of focal length 20 cm is placed 20 cm away from S between S and P. The lens transmits 75% of light incident on it. Find the new value of intensity at P.

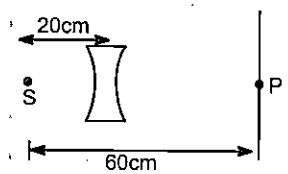


Fig. 1E.146 (a)

Solution: $u=-20, f=-20$

gives, $v=-10$

Let P = Power of source

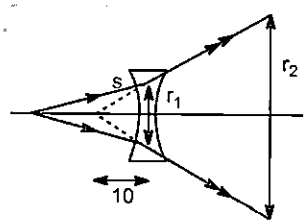


Fig. 1E.146 (b)

$$I=\frac{P}{4\pi(60)^2} \qquad ...(1)$$

Energy received by lens

$$E_2=\frac{P}{4\pi(20)^2}A_1$$

$$\therefore \qquad I_2=\frac{0.75E_2}{A_2} \qquad ...(2)$$

From similar triangles

$$\frac{A_2}{A_1}=25$$

$$\therefore \qquad I_2=0.27\,I$$

Example 147

A point source of light S is placed on the axis of a lens of focal length 20 cm as shown. in the Fig. 1E.147. A screen is placed normal to the axis of lens at a distance x from it. Treat all rays as paraxial.

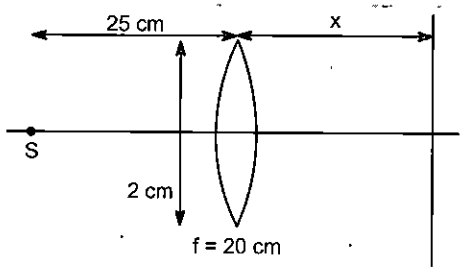


Fig. 1E.147 (a)

(a) As x is increased from zero intensity continuously decreases

(b) As x is increased from zero intensity first increases then decreases

(c) Intensity at centre of screen for x = 90 cm and x = 110 cm, is same

(d) Radius of bright circle obtained on screen is equal to 1 cm for x = 200 cm

Solution: (b, c, d) $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{(-25)}=\frac{1}{20}$

$$\Rightarrow \qquad \frac{1}{v}=\frac{1}{20}-\frac{1}{25}=\frac{1}{100} \Rightarrow v=100\text{ cm}$$

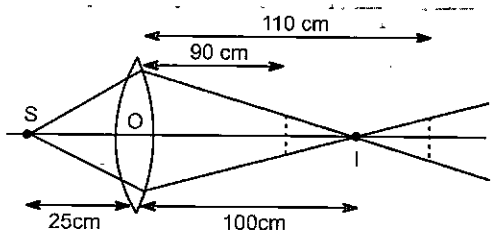


Fig. 1E.147 (b)

From O to I intensity increases and then decreases at $x = 90$ cm and 110 cm intensity is same.

(d) Radius at $x = 200$ cm is equal to radius of lens.

DISPLACEMENT METHOD

If a thin converging lens of focal length 'f' is placed between an object and a screen fixed at a distance D apart and if $D > 4f$, then there are two positions of the lens at which a sharp image of the object is formed on the screen.

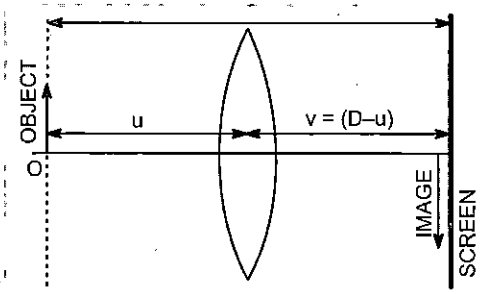


Fig. 1.202

If the object is at a distance 'u' from the lens, the distance of image from the lens $v = (D-u)$. So from lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we have, $$\frac{1}{(D-u)}-\frac{1}{-u}=\frac{1}{f}$$

i.e., $$u^2 - Du + Df = 0$$

So that $$u=\frac{1}{2}[D \pm \sqrt{D(D-4f)}] \quad ...(1)$$

Now there are three possibilities:

(a) If $D < 4f$: u will be imaginary, so physically no position of lens is possible.

(b) If $D = 4f$: In this situation $u = D/2 = 2f$. So only one position is possible and in this situation, $v = D - u = 4f - 2f = 2f (= u)$

(c) If $D > 4f$: In this situation both the roots of equation (1) will be real.

i.e., $$u_1=\frac{1}{2}D-\sqrt{D(D-4f)}$$

and $$u_2=\frac{1}{2}[D+\sqrt{D(D-4f)}] \quad ...(2)$$

So if $d > 4f$, there are two position of lens at distance u_1 and u_2 from the object for which real image is formed on the screen.

This method is called 'Displacement method' and is used in laboratory to determine the focal length of convergent lens. In case of displacement method.

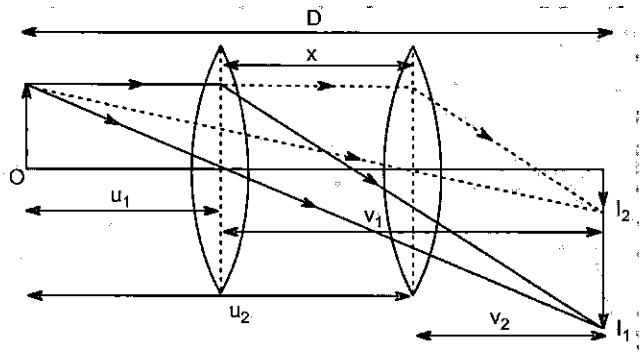


Fig. 1.203

(i) If the distance between two positions of the lens is x,

$$x = u_2 - u_1 = \sqrt{D(D-4f)} \text{ from equation (2),}$$

i.e., $$x^2 = D^2 - 4Df$$

So, $$f = (D^2 - x^2)/4D \quad ...(3)$$

(ii) The image distance corresponding to two positions of lens will be,

$$v_1 = D - u_1 = D - \frac{1}{2}[D - \sqrt{D(D-4f)}]$$

$$= \frac{1}{2}[D + \sqrt{D(D-4f)}] = u_2$$

and $$v_2 = D - u_2 = D - \frac{1}{2}[D + \sqrt{D(D-4f)}]$$

$$= \frac{1}{2}[D - \sqrt{D(D+4f)}] = u_1$$

i.e., for two positions of the lens object and image distance are interchangeable.

(iii) As $x = u_2 - u_1$

and $D = u_1 + v_1 = u_1 + u_2$

as $v_1 = u_2$,

i.e., $u_1 = (v_2) = (D - x)/2$

and $u_2 = (v_1) = (D + x)/2$

So the magnification for two position of the lens will be respectively,

$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \left(\frac{D+x}{D-x}\right)$$

and $$m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \left(\frac{D-x}{D+x}\right) \quad ...(4)$$

And hence,

(a) $m_1 \times m_2 = (I_1 I_2 / O^2) = 1$

i.e., $$O = \sqrt{I_1 I_2} \quad ...(5)$$

(b) $$\frac{m_1}{m_2} = \frac{I_1}{I_2} = \left(\frac{D+x}{D-x}\right) \times \left(\frac{D+x}{D-x}\right) = \left(\frac{D+x}{D-x}\right)^2$$

$$m_1 - m_2 = \left(\frac{D+x}{D-x}\right) - \left(\frac{D-x}{D+x}\right) = \frac{4Dx}{D^2 - x^2}$$

With the help of equation (3) it transforms to,

$$m_1 - m_2 = \frac{x}{f}$$

i.e., $$f = \frac{x}{m_1 - m_2}$$

(a) When Lenses are in Contact

Consider two thin lenses with respective focal lengths f_1 and f_2. If u be the object distance, and v' the image distance due to the first lens, then from lens equation,

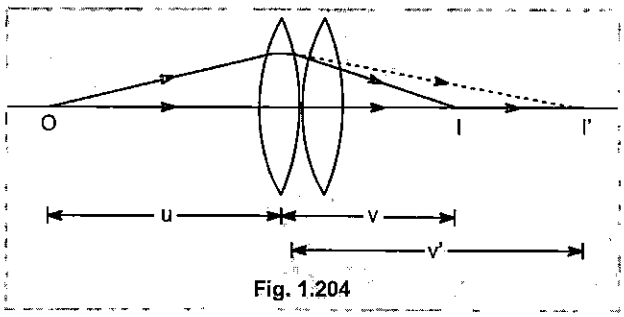


Fig. 1.204

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \quad ...(1)$$

Evidently, the image I' formed by the refraction at the first lens behaves as an object for the second lens. Let the distance of the final image be v'. Again from lens equation,

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \quad ...(2)$$

Adding equations (1) and (2), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad ...(3)$$

If f be the focal length of the equivalent lens *i.e.*, corresponding to the same object distance u, if f be the focal length of a single lens, for which the image distance is also v, then from lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ...(4)$$

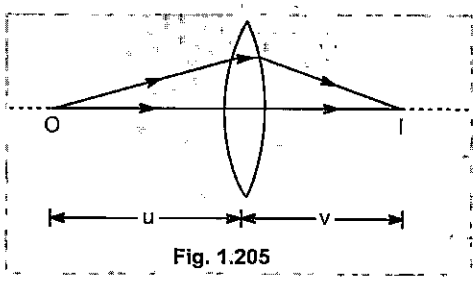


Fig. 1.205

Comparing the R.H.S. of Eqns. (3) and (4), we get

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Similarly, if P be the effective optical power of the lens system, and P_1, P_2 the individual powers of the two lenses, then

$$P = P_1 + P_2$$

$$\frac{1}{f} = \sum_{i=1}^{n} \frac{1}{f_i}$$

$$P = \sum_{i=1}^{n} P_i$$

Note that the effective focal length or power is independent of the order in which different lenses are arranged.

(b) When the lenses are separated from each other coaxially: Determine the deviation produced by a lens. is given by $h\left(\frac{1}{f}\right)$

where, h is the height above the principal axis where the light ray PQ strikes the lens.

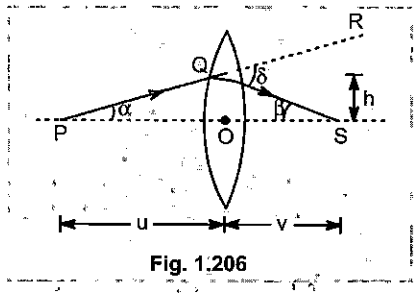


Fig. 1.206

Now, consider two thin lenses L_1 and L_2 with respectively focal lengths f_1 and f_2 and placed co-axially with separation of d. (Fig. 1.207)

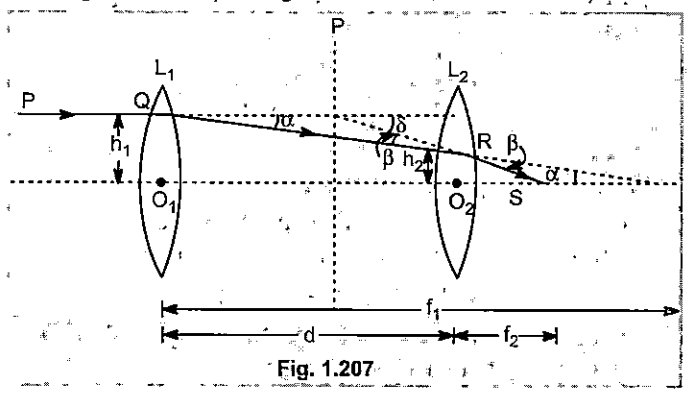


Fig. 1.207

A light ray PQ parallel to the common axis strikes the lens L_1 at a height h_1 above the axis and gets deviated by an angle α, such that

$$\therefore \quad \alpha = \frac{h_1}{f_1}$$

Similarly, for the second lens, the point of striking of light ray QR is at a height h_2 above the axis and its deviation is β.

$$\therefore \quad \beta = \frac{h_2}{f_2}$$

Evidently, the total angle of deviation

$$\delta = \alpha + \beta$$

or, $$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \quad ...(1)$$

where f is the effective focal length, of a single lens kept in plane P which serves the purpose of the lens system.

From similar triangles QO_1T and RO_2T, we have

$$\frac{QO_1}{RO_2} = \frac{O_1T}{O_2T}$$

or, $$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or $$h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Substituting for h_2 in Eq. (1), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2}$$

or $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad ...(2)$$

Here it is worthwhile mentioning that the values of f_1 and f_2 should be substituted in terms of proper sign and the sign of the resulting f automatically reveals the nature of the lens system.

Equivalent Focal Length of A Lens Combination:

If n number of lenses of focal lengths $f_1, f_2, f_3, \ldots\ldots f_n$ are joined together then the equivalent focal length of the combination is:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots + \frac{1}{f_n}$$

Sign Convention

Focal length of a converging lens is taken as positive and that of a diverging lens is taken as negative. In terms of power, the above expression may be written as

$$P = P_1 + P_2 + P_3 + \ldots\ldots + P_n$$

Power of a lens is defined as the reciprocal of focal length, where f is measured in metre.

$$P = \frac{1}{f}$$

The unit of power is dioptre $\quad 1D = 1m^{-1}$

When several lenses or mirrors are used co-axially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens (or mirror) acts as an object for the third, and so on. The total magnification in such situations will be given by,

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \ldots\ldots\ldots$$

i.e., $$m = m_1 \times m_2 \times \ldots\ldots\ldots$$

If lens is cut parallel to principal axis, then radius of curvature of none of the refractory surfaces changes, so focal length remains unchanged. If lens is cut perpendicular to principal axis then radius of curvature of one of the refractory surfaces changes and so focal length changes.

Silvering of One of the Surfaces of Lens.

Silvered lens is equivalent to mirror. Focal length of equivalent mirror is given by,

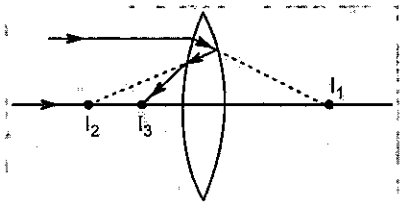


Fig. 1.208

$$\frac{1}{F} = -\left(\frac{n}{f_1} - \frac{m}{f_m}\right)$$ (proper sign has to be considered).

$F \Rightarrow$ Focal length of equivalent mirror

$n \Rightarrow$ Number of refractions taking place

$m \Rightarrow$ Number of reflections taking place

$f_1 \Rightarrow$ Focal length of lens in unsilvered condition

$f_m \Rightarrow (R/2)$

$R \Rightarrow$ Radius of curvature of silvered surface.

If F is – ve, then equivalent mirror is converging and if F is +ve, then equivalent mirror is diverging.

Case (I): Plane surface silvered:

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R}\right)$$

$$f_m = \frac{R}{2} = \infty$$

$$\frac{1}{F} = -\left[\frac{2}{f_1} - \frac{1}{f_m}\right] = \left[\frac{2(\mu - 1)}{R} - 0\right]$$

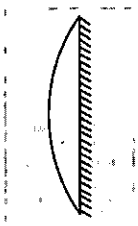


Fig. 1.209

$$F = -\frac{R}{2(\mu - 1)}\left[\mu = \left(\frac{\mu_1}{\mu_s}\right)\right]$$

Case (II): Curve surface silvered:

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R}\right)$$

$$f_m = -\frac{R}{2}$$

$$\frac{1}{F} = -\left[\frac{2}{f_1} - \frac{1}{f_m}\right] = -\left[\frac{2(\mu - 1)}{R} + \frac{2}{R}\right]$$

$$F = -\frac{R}{2\mu}$$

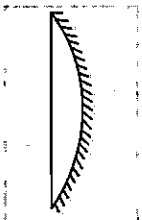


Fig. 1.210

Case (III): Surface R_4 mirror and system is dipped in alcohol:

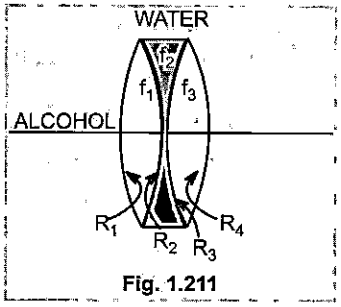


Fig. 1.211

$$\frac{1}{f_1}=\left(\frac{\mu_g}{\mu_{Al}}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \quad \text{(Glass lens in alcohol)}$$

$$\frac{1}{f_2}=\left(\frac{\mu_w}{\mu_{Al}}-1\right)\left(-\frac{1}{R_1}-\frac{1}{R_3}\right) \quad \text{(Water lens in alcohol)}$$

$$\frac{1}{f_3}=\left(\frac{\mu_g}{\mu_{Al}}-1\right)\left(\frac{1}{R_3}+\frac{1}{R_4}\right) \quad \text{(Glass lens in alcohol)}$$

$$\frac{1}{f_1}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3};\ f_m=-\frac{R_4}{2}$$

$$\frac{1}{F}=-\left(\frac{2}{f_1}-\frac{1}{f_m}\right);$$

$$\frac{1}{F}=-\left(\frac{2}{f_1}+\frac{2}{f_2}+\frac{2}{f_3}+\frac{2}{R_4}\right)$$

Equivalent focal length F in surrounding alcohol.

Example 148

The adjacent Fig. 1E.148 shows a thin plano-convex lens of refractive index μ_1 and a thin plano-concave lens of refractive index μ_2, both having same radius of curvature R of their curved surfaces. The thin lens of refractive index μ_3 has radius of curvature R of both its surfaces. This lens is so placed in between the plano-convex and plano-concave lenses that the plane surfaces are parallel to each other. What is the focal length of the combination.

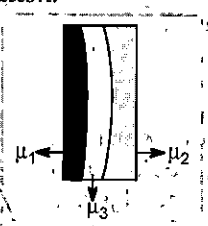


Fig. 1E.148

Solution: $\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{\infty}+\frac{1}{R}\right)=\frac{\mu_1-1}{R}$

$$\frac{1}{f_2}=(\mu_3-1)\left(-\frac{1}{R}+\frac{1}{R}\right)=0$$

$$\frac{1}{f_3}=(\mu_2-1)\left(-\frac{1}{R}-\frac{1}{\infty}\right)=-\frac{(\mu_2-1)}{R}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}=\frac{(\mu_1-1)}{R}-\frac{(\mu_2-1)}{R}=\frac{1}{R}(\mu_1-\mu_2)$$

$$f=\frac{R}{(\mu_1-\mu_2)}$$

Concept: *If lens of focal length 'f' is cut into two equal parts by a horizontal plane AB (Fig. 1.212), then as none of μ, R_1 and R_2 will change, the focal length of each part will be equal to that of original lens.*

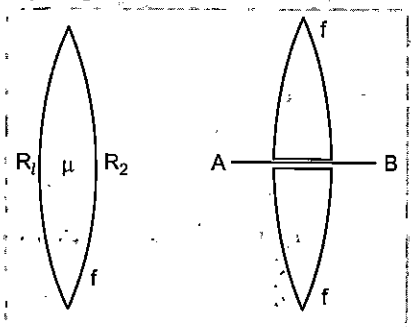


Fig. 1.212 (a)

However in this situation as light transmitting area of each part becomes half of initial, so intensity will be reduced to half.

However if the same lens is cut into two equal parts by a vertical plane CD [Fig. 1.212 (b)] then radius of curvature of one of the refractory surfaces changes and so focal length changes.

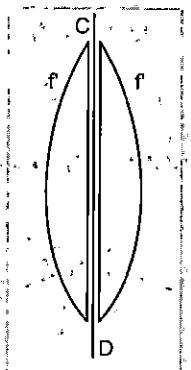


Fig. 1.212 (b)

The focal length of each part will becomes,

$$\frac{1}{f'}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{(\mu-1)}{R}=\frac{1}{2f}$$

i.e., $f'=2f$ (Here original lens is considered as equi-convex lens)

i.e., focal length of each part will be double of initial value. In this situation as the light transmitting area of each part of lens remains equal to initial, intensity and aperture will not change.

If a lens of focal length ' f' is divided into two equal parts as shown in Fig. 1.212 (c) and each part has a focal length f' then as,

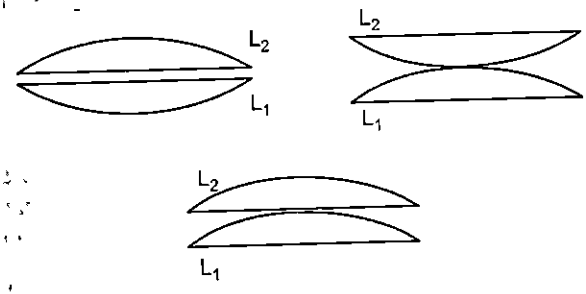


Fig. 1.212 (c)

$$\frac{1}{f}=\frac{1}{f'}+\frac{1}{f'} \quad i.e., \quad f'=2f$$

i.e., each part will have focal length 2f.

Now if these part are put in contact as in Fig.1.212 (c) the resultant focal length of the combination will be,

$$\frac{1}{F}=\frac{1}{2f}+\frac{1}{2f} \quad i.e., \quad F=f(=\text{initial value})$$

If a lens of focal length ' f' is cut in two equal parts as shown in Fig. 1.212 (d) [i], each part will have focal length ' f'. Now if these parts are put in contact as shown in Fig. 1.212 (d)[ii], the resultant focal length will be,

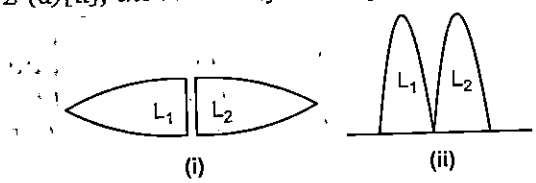


Fig. 1.212 (d)

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f} \; i.e., \quad f=(f/2)$$

Angle of deviation of ray when it passes through a lens.

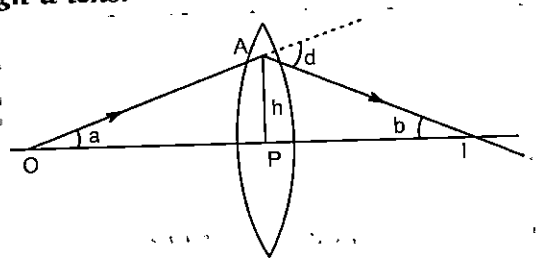


Fig. 1.213

$$\delta=h\left(\frac{1}{v}-\frac{1}{u}\right)$$

or,
$$\delta=\frac{h}{f}$$

Two Thin Lenses Separated by a Distance

A co-axial system of two thin convex lenses of focal length ' f' each separated by a distance ' d'. In case of two thin lenses separated by a distance ' d', we have,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

For
$$f_1=f, f_2=f$$
$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}-\frac{d}{f^2}$$

i.e.,
$$F=\frac{f^2}{2f-d}\text{. So}$$

(a) If $d<f, F$ will be positive and $<f$, so the system will behave as converging lens of focal length $<f$.

(b) If $d=f, F=f$, *i.e.*, the system will behave as converging lens of focal length ' f' .

(c) If $f<d<2f, F$ will be positive and $>f$, so the system will behave as converging lens of focal length $>f$.

(d) If $d=2f, F$ will be infinite *i.e.*, the system will behave as a plane glass plate of infinite focal length.

(e) If $d>2f, F$ will because negative *i.e.*, the system will behave as diverging lens.

Concept of Image Forming at Object itself:

In some unique situations the image is formed at the same point as the object. For a lens, if we apply the lens formula we get,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

or
$$\frac{1}{u}-\frac{1}{u}=\frac{1}{f}$$

which is not possible.

On the other hand, for the case of refraction across a single curved surface, the formula is

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$$

which reduces to

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$$

The solution for which is $u=R$. Therefore the object and image coincide when the object is placed at the center of curvature (Fig. 1.214).

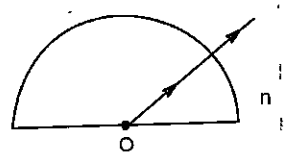


Fig. 1.214

Similarly, in the case of mirrors, the formula is

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

which simplifies to

$$\frac{1}{u}+\frac{1}{u}=\frac{2}{R}$$

and the solution once again is $u = R$. Therefore for mirrors and refraction across a single curved surface, we can say that object and image will coincide only when the object is kept at the centre of curvature.

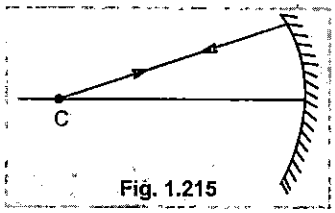


Fig. 1.215

In problems in optics, we will usually have a train of optical elements with the stipulation that the image is formed on the object itself. In such cases, there will have to be a mirror at the end of the optical train and the rays have to be incident normally on the mirror in order to retrace their paths. Three kinds of mirrors are possible.

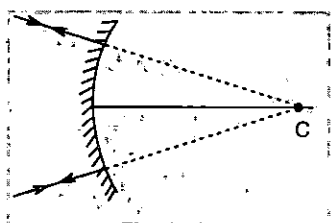


Fig. 1.216

Example 149

A glass slab of thickness 3 cm and refractive index 1.5 is placed in front of a concave mirror of focal length 20 cm. Where should a point object be placed if it is to image on to itself ? The glass slab and the concave mirror are shown in Fig. 1E.149.

Solution: Let the distance of the object from the mirror be x. We know that the slab simply shifts the object. The shift being equal to

$$s = t\left[1-\frac{1}{\mu}\right] = 1 \text{ cm}$$

The direction of shift is towards the concave mirror.

∴ the apparent distance of the object from the mirror is $x - 1$.

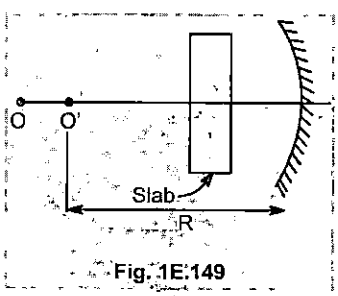


Fig. 1E.149

If the rays are to retrace their paths, the object should appear to be at the center of curvature of the mirror.

∴ $x - 1 = 2f = 40$ cm

or $x = 41$ cm from the mirror.

Example 150

A convex lens of focal length 20 cm is placed 10 cm in front of a convex mirror of radius of curvature 15 cm. Where should a point object be placed in front of the lens so that it images on to itself ?

Solution: The convex lens and the convex mirror are shown in Fig. 1E.150. The combination behaves like a concave mirror

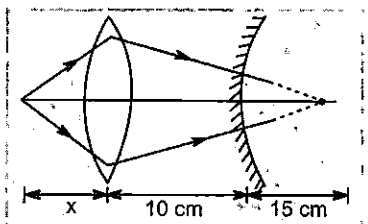


Fig. 1E.150

Let the distance of the object from the lens be x.

For the ray to retrace is path it should be incident normally on the convex mirror, or in other words the rays should pass through the center of curvature of the mirror.

From the diagram we see that for the lens

$u = -x, f = +20$ cm, $v = +10 + 15 = +25$ cm

From the lens equation we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{1}{25}-\frac{1}{-x}=\frac{1}{20}$$

or $x = 100$ cm

Example 151

A convex lens of focal length 10 cm is placed 30 cm in front of a second convex lens also of the same focal length. A plane mirror is placed after the two lenses. Where should a point object be placed in front of the first lens so that it images on to itself ?

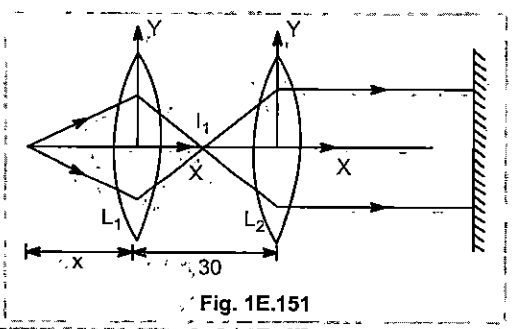


Fig. 1E.151

Solution: The convex lenses and the plane mirror are shown in figure.

The combination behave like a concave mirror.

Let the distance of the object from the first lens be x.

For the ray to retrace its path it should be incident normally on the plane mirror.

From the diagram we see that for lens L_2,

From the lens equation we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad u=-10\text{ cm}$$

From the diagram we see that for lens L_1,

$$v=30-10=20\text{ cm}, \quad f=+10\text{ cm}, u=-x$$

From the lens equation we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{1}{20}-\frac{1}{-x}=\frac{1}{10}$$

or $x=20\text{ cm}$

METHODS FOR DETERMINING FOCAL LENGTH OF A CONVEX LENS

(a) Graphical method : The convex lens forms real and inverted images of an object placed between infinity and focus.

From lens equation, using cartesian sign convention, we have

$$u=-x, \quad v=+y, \quad f=+f$$

$$\frac{1}{+y}-\frac{1}{-x}=\frac{1}{+f}$$

If a graph is plotted between $\frac{1}{+v}$ and $\frac{1}{-u}$, it represents a **straight line** with slope equal to -1, as shown in Fig. 1.217. The *intercept* with the horizontal axis is $\frac{1}{-f}$ and that with the vertical axis is $\frac{1}{+f}$. If a line OP is drawn at angle 45°, it intersects the line AB at the point P. The *coordinates* of the point P are $\left(\frac{1}{-2f},\frac{1}{2f}\right)$.

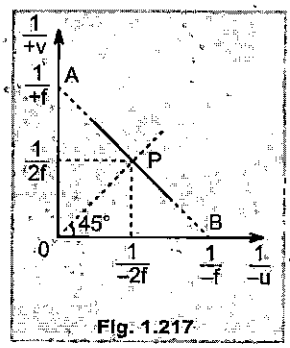


Fig. 1.217

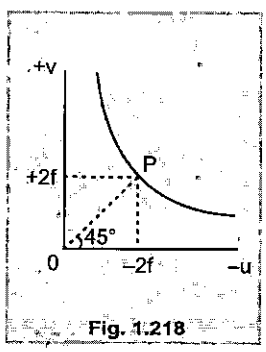


Fig. 1.218

Alternatively, a graph between v and $-u$ can be plotted as shown in Fig. 1.218. A line drawn at angle 45° from the origin intersects the curve at the point P whose coordinates are $(-2f, +2f)$. By measuring the coordinates of this point, the focal length of the mirror can also be measured.

(b)Plane-Mirror method : A convex lens is placed on a plane mirror. An object pin O is moved along the principal axis of the lens. At a certain position, the image coincides with the object. The distance of object pin O from the lens is the focal length of the lens. When an object is placed at the focus of a lens, the refracted rays become parallel to the principal axis. There parallel rays are incident on the mirror normally, therefore they retrace their path and converge at the focus.

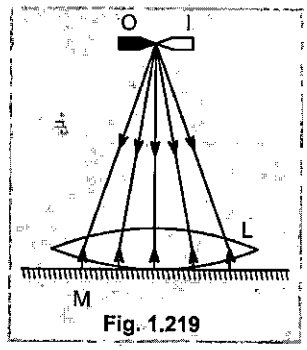


Fig. 1.219

(c) Measurement of focal length of a concave lens : Since a concave lens does not form a real image of a real object, therefore, a convex lens is used to form a real image as in the case with the *convex* mirror.

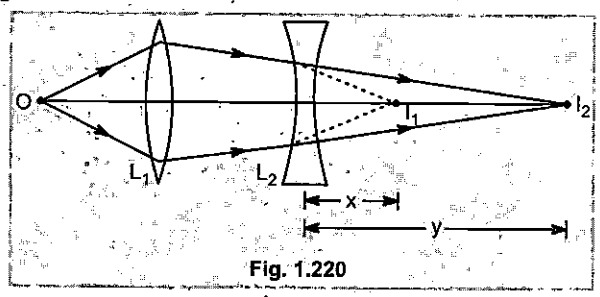


Fig. 1.220

In Fig. 1.220 a convex lens L_1 forms a real image I_1 of an object O. When a concave lens L_2 is placed in between I_1 and L_1, then the convergence of the beam coming from the convex lens is shifted to position I_2 instead of I_1.

The point I_1 acts as *virtual* object to the lens L_2 and it forms a real image I_2 of I_1, whose distance from I_2 is y, then according to *Cartesian Sign Convention*,

$$u=+x, \quad v=+y$$

By using the lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we get $$\frac{1}{+y}-\frac{1}{+x}=\frac{1}{f}$$

or $$f=-\left(\frac{xy}{y-x}\right)$$

Thus, by measuring x and y, the focal length of the *concave* lens can be easily calculated.

Note: The focal length of the convex lens used in this method must be *greater than* the focal length of the concave lens, otherwise, the real image by the concave lens will not be formed.

Example 152

A biconcave lens $f_1 = -25$ cm is placed 20 cm infront of a concave mirror $f_2 = 5$ cm as shown in Fig. 1E.152. A 2 cm high object is placed 15 cm to the left of the lens. Using the lens and mirror equations and a ray diagram, find the location of the three images : (a) the image formed by the lens as the rays travel to the right, (b) the image formed after the rays reflect from the mirror, and (c) the final image after the leftward travelling rays once again pass through the lens.

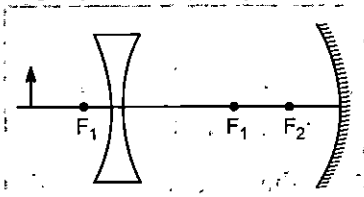


Fig. 1E.152

Solution: (a) From lens equation,

$$\frac{1}{v}-\frac{1}{(-15)}=\frac{1}{(-25)}, v=-9.375$$

Magnification, $m_1=\frac{v}{u}=\left(\frac{-9.375}{-15}\right)=+0.625$

The first image is a virtual, erect, diminished and formed 9.375 cm to the left of the lens.

(b) The first image acts as an object for the mirror; object distance for the mirror is (20 + 9.375) cm.

From mirror equation,

$$\frac{1}{v'}+\frac{1}{(-29.375)}=\frac{1}{(-5)}, v'=-6.026\,\text{cm}$$

Magnification, $m_2=-\left(\frac{v'}{u'}\right)=-\left(\frac{-6.026}{-29.375}\right)$

$$=-0.205$$

The second image is real (infront of the mirror), inverted diminished, formed at 6.026 cm infront of the mirror.

(c) The second image is an object for second refraction at the lens when the rays return.

Object distance = +(20 – 6.026) cm

$$\frac{1}{v''}-\frac{1}{(+13.974)}=+\frac{1}{25}$$

$$v''=+8.96$$

Note the sign of focal length of lens and object distance.

Magnification, $m_3=\frac{v''}{u''}=\left(\frac{8.96}{13.974}\right)=0.641$

Final image is 8.96 cm to the right of the lens.

Overall magnification,

$$m = m_1 \times m_2 \times m_3$$
$$=(+0.625)(-0.205)(0.641)$$
$$=-0.0822$$

Final image is virtual and inverted.

Example 153

A converging lens, $f_1 = 12$ cm, is separated by 50.0 cm from a diverging lens, $f_2 = -10.0$ cm. An object 1.0 cm high is placed 25.0 cm to the left of the first lens. Find the position of the final image and its height. Complete the ray diagram and characterize the image.

Solution: The object O lies outside the first focal point F_1 of the first lens, so this lens produces a real image I. The light rays that strike the second lens diverge from this real image just as if I were an actual object. Hence the image made by the first lens acts as an object for the second lens.

We first find the position of the first image. From lens equation,

$$\frac{1}{v}-\frac{1}{(-25)}=\frac{1}{12}, v=+\frac{300}{13}\,\text{cm}$$

The first image I is $\frac{300}{13}$ cm to the right of the first lens.

The magnification is

$$m_1=\frac{v}{u}=\left(\frac{300/13}{-25}\right)=-0.923$$

Hence the height of the first image $= (-0.923)(1)$
$$=-0.923\,\text{cm}$$

The first image is $\left(50-\frac{300}{13}\right)$ cm to the left of the second lens; hence the object distance for the second lens is $u'=\frac{350}{13}$ cm.

From lens equation, for the second lens,

$$\frac{1}{v'}-\frac{1}{(-350/13)}=\frac{1}{-10}, v'=-7.29\,\text{cm}$$

The final image is 7.29 cm to the left of the second lens and 42.71 cm to the right of the first lens. The magnification produced by the second lens is

$$m_2=\left(\frac{-7.29}{-350/13}\right)=0.270$$

Height of the second image = (0.270)(0.923) = 0.250

Overall magnification $m = m_1 \times m_2$

$$=(0.270)(-0.923)=-0.250$$

The final image is virtual and inverted.

Example 154

A concave mirror has radius of curvature $R = 30$ cm. A pencil lies perpendicular to the axis of the mirror at each of the following object distances: (a) 40 cm (b) 30 cm (c) 15 cm (d) 5 cm. Find the image of the pencil, magnification of each image and show it graphically.

Solution: The focal length of mirror is $f = R/2 = 15\,\text{cm}$.

From mirror equation,

(a) $\frac{1}{v}+\frac{1}{(-40)}=\frac{1}{(-15)}, v=-24\,\text{cm}$

The minus sign shows that the image is infront of the mirror, real.

$$m=-\frac{v}{u}=-\left(\frac{-24}{-40}\right)=-0.6$$

As $m < 1$, the image is diminished in size; minus sign shows that the image is inverted.

(b) $\frac{1}{v}+\frac{1}{(-30)}=\frac{1}{(-15)}, u=-30\,\text{cm}$

The image is real, formed infront of the mirror.

$$m=-\left(\frac{v}{u}\right)=-\left(\frac{-30}{-30}\right)=-1$$

As $m = -1$, the object and the image are equal, minus sign shows that the image is inverted.

(c) $\frac{1}{v}+\frac{1}{(-15)}=\frac{1}{(-15)}, v=\infty$

In this case outgoing rays are parallel, corresponding to an infinite image distance.

$$m=-\frac{v}{u}=-\frac{\infty}{(-15)}=-\infty \text{ or } (+\infty)$$

(d) $\frac{1}{v}+\frac{1}{(-5)}=\frac{1}{(-15)}, v=+7.5$

The plus sign shows that the image is behind the mirror, virtual. It is a general observation that an object placed inside the focal point of a concave mirror produces a virtual image.

$$m=-\left(\frac{v}{u}\right)=-\left(\frac{+7.5}{-5}\right)=+1.5$$

As $m > 1$, the image is magnified; plus sign shows that it is erect.

Example 155

A thin prism, refracting angle $A = 1° = \frac{\pi}{180}$ rad, is kept infront of a concave mirror. The object is kept at the origin of the coordinate system as shown in Fig. 1E.155 (a).

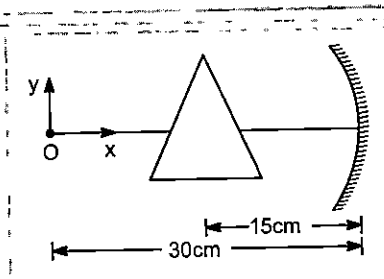


Fig. 1E.155 (a)

If the image is formed at coordinates $\left(10\,\text{cm}, -\frac{\pi}{24}\,\text{cm}\right)$, assuming paraxial ray approximation, determine :

(a) the radius of curvature of the mirror,

(b) the refractive index of the prism.

Solution: A thin angle prism produces a deviation $\delta = (n-1)A$, toward the base of the prism. The object appears to be at O' due to deviation as shown in Fig. 1E.150 (b).

$$OO' = Y_0 = (15)\delta$$

$$= (15)(\mu-1)\left(\frac{\pi}{180}\right)$$

$$= (\mu-1)\frac{\pi}{12}\,\text{cm}$$

The image is formed at (30 − 10) cm from the mirror, at I'

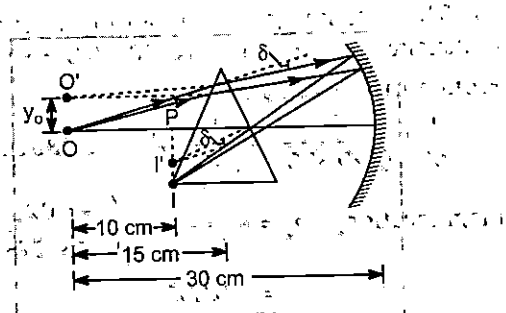


Fig. 1E.155 (b)

Magnification, $m=\frac{v}{u}=\frac{2}{3}=\frac{PI'}{OO'}$

$$PI'=\frac{2}{3}OO'=\frac{2}{3}(\mu-1)\frac{\pi}{12}=(\mu-1)\frac{\pi}{18}$$

The ray of light, after reflection from the mirror, passes through the prism and is again deviated by δ, so that the final image is formed at I where

$$II' = (15-10)\delta$$

$$= (\mu-1)\frac{\pi}{36}\,\text{cm}$$

y-coordinate of the final image I,

$$-PI = -(PI' + II')$$

$$= -\left[(\mu-1)\frac{\pi}{18}+(\mu-1)\frac{\pi}{36}\right]$$

$$= -(\mu - 1)\frac{\pi}{12}$$

Hence, $-(\mu - 1)\frac{\pi}{12} = -\frac{\pi}{24}$ (given)

or $\mu = 1.5$

as $\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$

$$\frac{1}{(-20)} + \frac{1}{(-30)} = \frac{2}{R}$$

$$R = -24\,\text{cm}$$

Example 156

A thin equiconvex lens of glass of refractive index $n_g = 3/2$ and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water, $n_w = 4/3$. On the opposite of the lens a mirror is placed inside the tank on the tank wall perpendicular to the lens axis as shown in the Fig. 1E.151. The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank infront of the lens at a distance 0.9 m from the lens along its axis. Find the position (relative to the lens) of the image of the object formed by the system.

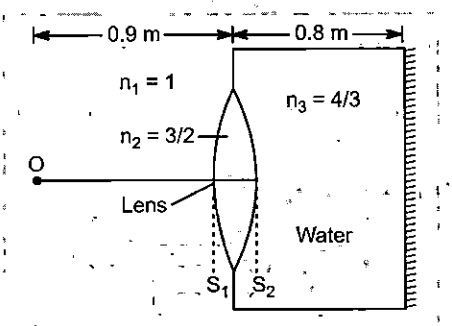


Fig. 1E.156

Solution: Let us trace the path of light from the object. First it is refracted at surface S_1 (air to glass), next refraction takes place at surface S_2 (glass to water), then reflection at the mirror. *Where does the light go after being reflected from the mirror ?* It is reflected back and refracted through surfaces S_2 and S_1 once again in the order of their appearance. First we will apply general thin lens equation for a lens. The given parameters are

$$u = -0.9\,\text{m}, n_1 = 1,\ n_2 = 3/2 \quad n_3 = 4/3$$

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} + \frac{n_3 - n_2}{R}$$

But R is not given ; we will determine it from the lens maker's formula :

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{0.3} = \left(\frac{1.5-1}{1}\right)\left(\frac{2}{R}\right)$$

$$R = 0.3$$

So we have $\frac{4/3}{u} - \frac{1}{0.9} = \frac{(3/2)-1}{0.3} + \frac{(4/3)-(3/2)}{0.3}$

or $v = 1.2\,\text{m}$

The image formed is real, to the right of the lens and behind the mirror. It is a virtual object at a distance 0.4 m for the mirror. Now the mirror forms an image infront of it. This image is an object for the lens system, with

$$n_1 = 4/3,\ n_2 = 3/2, n_3 = 1,\ R = +0.3\,\text{m},\ u = -0.4\ \text{m}$$

$$\frac{n_3}{v'} - \frac{n_1}{u'} = \frac{n_2 - n_1}{R} + \frac{n_3 - n_2}{R}$$

$$\frac{1}{v'} - \frac{4/3}{(-0.4)} = \frac{(3/2)-(4/3)}{0.3} + \frac{1-(3/2)}{(0.3)}$$

$$v' = 90\,\text{cm}$$

Example 157

A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in Fig. 1E.157.

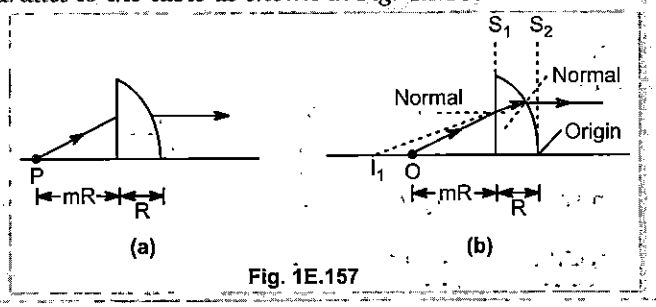


Fig. 1E.157

Solution: We will consider two refractions at plane surface S_1 and curved surface S_2. Refraction at S_1,

$$\frac{n}{v_1} - \frac{1}{(-mR)} = 0$$

$$v = -nmR$$

The image is virtual and on left of surface S_1. It acts as an object for refraction at the curved surface

Refraction at S_2:

$$\frac{1}{v_2} - \frac{n}{-(nmR + R)} = \frac{1-n}{(-R)}$$

or $$\frac{n}{(nmR + R)} = \frac{1-n}{(-R)}$$

as emergent rays are parallel to principal axis $v_2 = \infty$. On solving for m, we get

$$m = \frac{1}{n^2 - n} = \frac{1}{(3/2)^2 - (3/2)} = \frac{4}{3}$$

Example 158

A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in Fig.1E.158. The space between the lens and the mirror is filled with a liquid of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid.

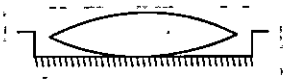


Fig. 1E.158

Solution: The light retraces its path if it is incident normally on a mirror. The rays after refraction through the lens and the liquid are parallel. We will apply the general thin lens equation with parameters,

$$n_1 = 1,\quad n_2 = 3/2,\ n_3 = 4/3,\ u = -15\,\text{cm and } v = \infty$$

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R} + \left(\frac{n_3 - n_2}{R}\right)$$

$$\frac{n_3}{\infty} - \frac{1}{(-15)} = \frac{(3/2)-1}{R} - \frac{[(4/3)-(3/2)]}{R}$$

On solving for R we get $R = 10\,\text{cm}$. Similarly, when second liquid is filled, we have

$$\frac{n_3'}{\infty} - \frac{1}{(-25)} = \frac{(3/2)-1}{10} - \frac{n_3' - (3/2)}{-10}$$

On solving for n_3' we get, $n_3' = 1.6$

Example 159

A diverging lens, focal length $f_1 = 20$ cm, is separated by 5 cm from a converging mirror, focal length $f_2 = 10$ cm. where should an object be placed so that a real image is formed at the object itself ?

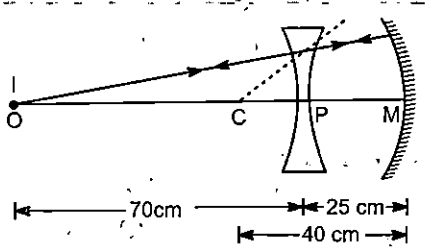


Fig. 1E.159

Solution: Let the object be placed at a distance x to the left of the lens.

From the lens equation,

$$\frac{1}{v} = \frac{1}{(-20)} + \frac{1}{(-x)}$$

$$v = -\left(\frac{20x}{x+20}\right)$$

A virtual image is formed due to first refraction at the lens. This image is an object for reflection from the concave mirror. Object distance is

$$-\left(5 + \frac{20x}{20+x}\right) = -\left(\frac{25x+100}{x+20}\right)$$

From mirror equation,

$$\frac{1}{v'} + \left[-\frac{(x+20)}{25x+100}\right] = \frac{1}{-10}$$

$$\frac{1}{v'} = -\frac{1}{10} + \frac{x+20}{25x+100} = \frac{10x+200-25x-100}{250(x+4)}$$

$$v' = -\frac{50(x+4)}{(3x-20)}$$

This image is formed to the left of the mirror.

Object distance for second refraction through concave lens,

$$u'' = -\left[5 - \frac{50(x+4)}{(3x-20)}\right]$$

We assumed that second image lies between lens and mirror.

The final image is produced at the object itself; hence

$$v'' = +x$$

From lens equation,

$$\frac{1}{x} - \frac{1}{\left[5 - \frac{(50x+4)}{3x-20}\right]} = \frac{1}{-20}$$

On solving for x, we get

$$25x^2 - 1400x - 6000 = 0$$

$$x^2 - 56x - 240 = 0$$

$$(x-60)(x+4) = 0$$

Hence $x = 60\,\text{cm}$.

The object must be placed at 60 cm to the left of the diverging lens.

Example 160

A parallel beam of light is incident normally on the flat surface of a hemisphere of radius $R = 6$ cm and refractive index 1.56. Assuming paraxial ray approximation,

(a) determine the point at which the beam is focussed as measured along the axis from the curved surface;

(b) determine the new focal length measured from the flat surface if the rays are incident at the curved surface.

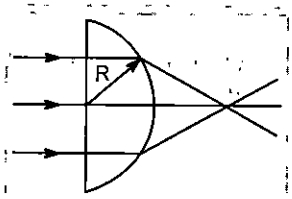


Fig. 1E.160 (a)

Solution: Rays pass without deviation at flat surfaces. From single surface refraction equation for surface S_2, we have

$$\frac{1}{v} - \frac{n}{\infty} = \frac{(1-n)}{(-R)}$$

$$v = \frac{R}{n-1} = \frac{6}{1.56-1}$$

$$= \frac{6}{0.56} = 10.7\,\text{cm}$$

This focal point as rays travel from left.

From single surface refraction equation at S_1, we have

$$\frac{n}{v} - \frac{1}{\infty} = \frac{(n-1)}{R}$$

$$v = \frac{nR}{n-1}$$

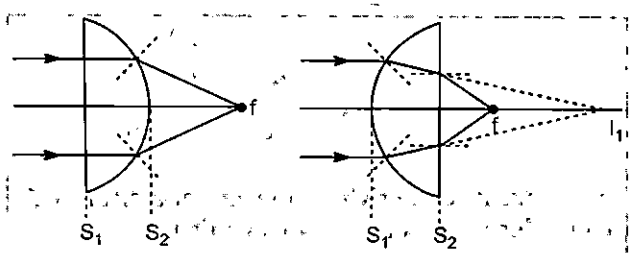


Fig. 1E.160 (b)

This is the first image, it acts as an object for refraction at the plane surface.

Object distance for refraction at S_2,

$$= \frac{R}{n-1} - R = \frac{R}{(n-1)}$$

So we have $$\frac{1}{v'} - \frac{n}{+(R/n-1)} = 0$$

$$v' = \frac{R}{n(n-1)}$$

$$= \frac{6}{1.56(1.56-1)} = 6.9\,\text{cm}$$

Hence focal point is at 6.9 cm from the plane surface.

Example 161

A equiconvex lens, f_1 = 10 cm, is placed 40 cm infront of a concave mirror, f_2 = 7.50 cm, as shown in Fig. 1E.161. An object 2 cm high is placed 20 cm to the left of the lens. Find the location of three images : (a) the image formed by lens as rays travel to the right, (b) the image formed after rays reflect from mirror and (c) the final image after leftward travelling rays once again pass through lens. Complete the ray diagram and characterize the image.

Solution: (a) From lens equation,

$$\frac{1}{v} - \frac{1}{(-20)} = \frac{1}{10},\ v = +20\,\text{cm}$$

$$\text{Magnification, } m_1 = \frac{v}{u} = \left(\frac{+20}{-20}\right) = -1$$

Image is real and inverted, same size as object.

(b) The first image acts as object for concave mirror. Object distance for mirror is (40 – 20) cm.

From mirror equation,

$$\frac{1}{v'} + \frac{1}{(-20)} = \frac{1}{(-7.5)},\ v' = -12\,\text{cm}$$

$$\text{Magnification, } m_2 = -\left(\frac{u'}{v'}\right) = -\left(\frac{-12}{-20}\right) = -0.6$$

The second image is 12 cm to the left of the mirror, real, erect (that is reinverted).

(c) The second image acts as objects for the lens. The object distance for second refraction at the lens,

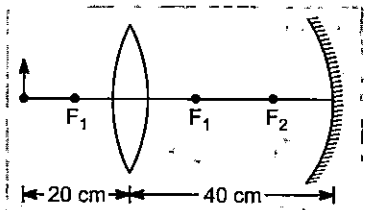


Fig. 1E.161 (a)

$$u'' = +28\,\text{cm}$$

From lens equation,

$$\frac{1}{v''} - \frac{1}{(+28)} = \frac{1}{-10},\ v'' = -15.6\,\text{cm}$$

Note the sign convention for f and u.

$$\text{Magnification, } m_3 = \frac{v''}{u''} = \left(\frac{-15.6}{+28}\right) = -0.556$$

The final image is real, inverted and lies 15.6 cm to the left of the lens.

Overall magnification,

$$m = m_1 \times m_2 \times m_3$$

$$= (-1) \times (-0.6) \times (-0.556)$$

$$= -0.333$$

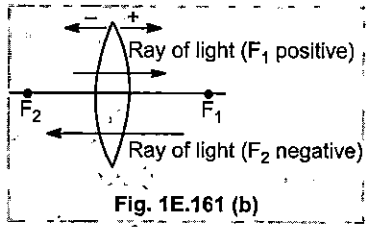


Fig. 1E.161 (b)

Example 162

A diverging lens (f_1 = 20 cm) is separated by 25 cm from a concave mirror (f_2 = 20 cm). An object is placed 70 cm to the left of the lens. Find the focal length of the lens if the image coincides with the object.

Solution: The light refracts through the lens, reflects at the mirror and finally passes once again through the lens. The final image will coincide with the object if it retraces its path after reflection from the mirror, *i.e.*the ray strikes the mirror normally. The normal rays at the mirror after being extended must pass through the centre of curvature of the mirror. Thus, the object for second refraction at the lens is at *C* and its image is at *I*, see Fig. 1E.162.

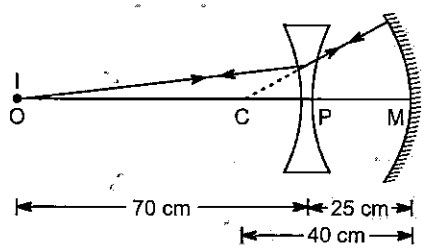


Fig. 1E.162

From figure, $PC = (40 - 25) = 15$ cm

From lens equation, $\frac{1}{(-15)} - \frac{1}{(-70)} = \frac{1}{f}$

$$f = -19.1 \text{ cm}$$

Example 163

A biconvex lens, f_1 = 20 cm, is placed 5 cm infront of a convex mirror, f_2 = 15 cm. An object of length 2 cm is placed at a distance 10 cm from the lens. Find the location of three images : (a) the image formed by the lens as the rays travel to the right, (b) the image formed after the rays reflect from the mirror and (c) the final image after the leftward travelling rays once again pass through the lens.

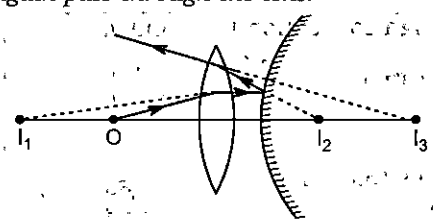


Fig. 1E.163

Solution: (a) From lens equation,

$$\frac{1}{v} - \frac{1}{(-10)} = -\frac{1}{+20}, \; v = -20 \text{ cm}$$

Magnification, $m_1 = \frac{(-20)}{(-10)} = +2$

Image is virtual, erect and magnified.

(b) The first image acts as an object for the convex mirror. Object distance for the mirror, $u' = (20 + 5) = 25$ cm.

From mirror equation,

$$\frac{1}{v'} + \frac{1}{(-25)} = \frac{1}{+15}, \quad v' = +\frac{75}{8} \text{ cm}$$

Magnification, $m_2 = \frac{(+75/8)}{(-25)} = \frac{3}{8}$

Image is virtual (to the right of the mirror), erect and diminished.

(c) The object distance for second refraction at the lens

$$= \frac{75}{8} + 5 = \frac{115}{8}$$

From lens equation,

$$\frac{1}{v''} - \frac{1}{(+115/8)} = \frac{1}{-20},$$

$$v'' = \frac{460}{9} = +51.1 \text{ cm}$$

Magnification, $m_3 = \left(\frac{+460/9}{115/8}\right) = \frac{32}{9}$

Overall magnification,

$$m = m_1 \times m_2 \times m_3$$

$$= (2) \times \left(\frac{3}{8}\right) \times \left(\frac{32}{9}\right) = \frac{8}{3}$$

Hence size of image $= \left(\frac{8}{3} \times 2\right)$ cm $= 5.33$ cm

Final image is to the right of the lens at a distance 51.1 cm from the lens, real, erect and magnified.

Example 164

A luminous object and a screen are at a fixed distance D apart. (a) Show that a converging lens of focal length f, placed between object and screen will form a real image on the screen for two lens positions that are separated by a distance

$$d = \sqrt{D(D - 4f)}$$

(b) Show that $\left(\frac{D-d}{D+d}\right)^2$ gives the ratio of the two image sizes for these two positions of the lens.

(c) If the heights of two images are h_1 and h_2 respectively, the height of the object is $h = \sqrt{h_1 h_2}$

(d) If the distance between two positions of the lens is d,

$$f = \frac{D^2 - d^2}{4D}$$

and

$$f = \frac{d}{m_1 - m_2}$$

where m_1 and m_2 are magnifications in the two positions of the lens.

Solution: Let the object distance be x; then the image distance is $D - x$.

From lens equation, $\frac{1}{x} + \frac{1}{D-x} = \frac{1}{f}$

On algebraic rearrangement we get

$$x^2 - Dx + Df = 0$$

On solving for x, we get

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2}$$

$$x_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)}$$

(b) If the object is at $u = x_1$,

$$m_1 = \frac{I_1}{O} = \frac{D - x_1}{x_1}$$

Now, $x_1 = \frac{1}{2}(D-d)$, where $d = \sqrt{D(D-f)}$.

So, $$m_1 = \frac{D-(D-d)/2}{(D+d)/2} = \left(\frac{D+d}{D-d}\right)$$

Similarly, when the object is at x_2, the magnification is

$$x_2 = \frac{1}{2}(D+d)$$

$$m_2 = \frac{I_2}{O} = \frac{D-x_2}{x_2}$$

$$= \frac{D-(D+d)/2}{(D+d)/2} = \frac{D-d}{D+d}$$

The ratio of magnifications is

$$\frac{m_2}{m_1} = \frac{(D-d)/(D+d)}{(D+d)/(D-d)} = \left(\frac{D-d}{D+d}\right)^2$$

(c) As $$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1}$$

and $$m_2 = \frac{I_2}{O} = \frac{v_2}{u_2}$$

$$m_1 \times m_2 = \frac{I_1 I_2}{O^2} = \frac{v_1}{u_1} \times \frac{v_2}{u_2} = 1$$

Hence $$O = \sqrt{I_1 I_2}$$

(d) As $$d = \sqrt{D(D-4f)}$$

$$d^2 = D^2 - 4Df$$

$$f = \frac{D^2 - d^2}{4D}$$

Also $$m_1 - m_2 = \frac{D+d}{D-d} - \frac{D-d}{D+d} = \frac{4Dd}{D^2-d^2}$$

Hence $$m_1 - m_2 = \frac{d}{f}$$

or $$f = \frac{d}{m_1 - m_2}$$

Example 165

A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis. The separation between the object and image planes is 1.8 m. The magnification of the image formed by one of the half lenses is 2. Find the focal length of the lens and separation between the two halves. Draw the ray diagram for image formation.

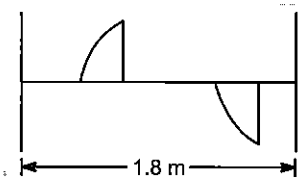


Fig. 1E.165 (a)

Solution: Splitting of a lens in two parts does not affect the position of the image. Each half forms an image at the same position but of reduced intensity. The previous problem shows that for a fixed object and screen there are two positions of the lens for which the image is formed at the same position. Let the object and image distances for the two lenses be u, v and u', v' respectively. In accordance with principle of reversibility of light,

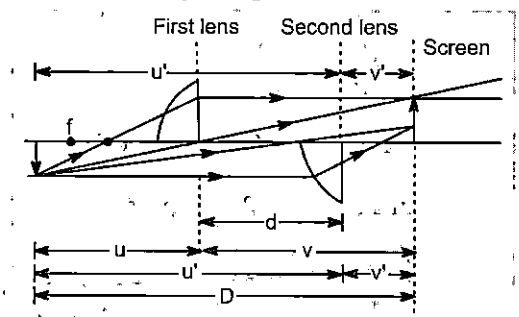


Fig. 1E.165 (b)

$$u' = v \quad \text{and} \quad v' = u$$

Hence $$u + d + v' = u + d + u = D$$

or $$u = \frac{D-d}{2}$$

Also $$v = D - u = \frac{D+d}{2}$$

As $$m = \frac{v}{u} = \frac{D+d}{D-d} = 2 \text{ (given)},$$

$$d = \frac{D}{3} = \frac{1.8}{3} = 0.6\,\text{m}$$

Hence $$u = \frac{1}{2}(D-d) = 0.6\,\text{m}$$

and $v = \frac{1}{2}(D + d) = 1.2\,\text{m}$

From lens equation, $\frac{1}{1.2} - \frac{1}{-(0.6)} = \frac{1}{f}$

$f = 0.4\,\text{m}$

Example 166

A point object O is placed at a distance of 0.3 m from a convex lens (focal length 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in Fig. 1E.166. Find the position of the image. If more than one image is formed, find their number and the distance between them.

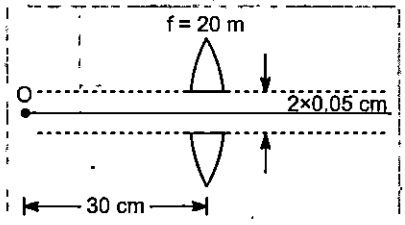


Fig. 1E.166 (a)

Solution: As explained in the previous problem, each half lens will form an image in the same plane. The optic axes of the lenses are displaced,

From lens equation,

$$\frac{1}{v} - \frac{1}{(-30)} = \frac{1}{20}, \quad v = 60\,\text{m}$$

From similar triangles OI_1I_2 and OP_1P_2, we have

$$\frac{I_1I_2}{P_1P_2} = \frac{u+v}{u}$$

or $$I_1I_2 = \frac{90}{30}(2\times 0.05) = 0.3\,\text{cm}$$

Thus the two images are 0.3 cm apart.

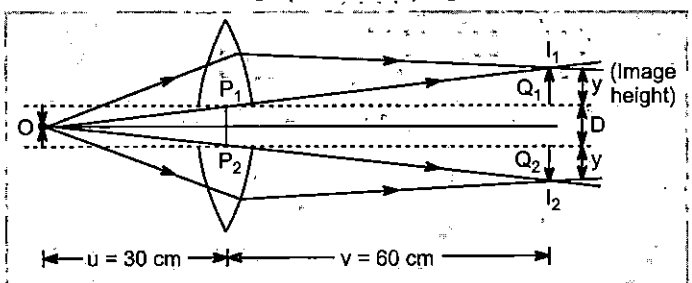


Fig. 1E.166 (b)

Alternately, imagine two arrows (see figure) that act as objects for the lens.

$$\text{Magnification, } m = \frac{v}{u} = \frac{(+60)}{(-30)} = -2$$

Image of height of arrows is

$$y = 2\times(0.05) = 0.10\,\text{cm}$$

Thus two inverted images are formed whose tips are at I_1 and I_2 respectively.

Thus $I_1I_2 = 2y + \Delta = (2\times 0.1) + 0.1 = 0.3\,\text{cm}$

Example 167

Consider an equiconvex lens with one of its surfaces mirrored. (a) Determine the focal length in terms of refractive index, n, and radius of curvature, R. (b) Consider a plano-convex lens of refractive index n and radius R. (c) Determine its focal length (i) if its plane surface is mirrored, (ii) if its curved surface is mirrored.

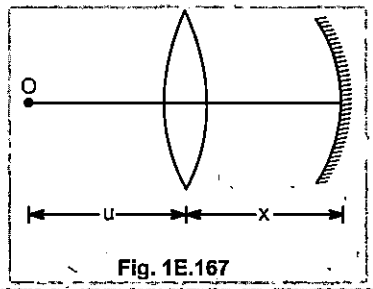


Fig. 1E.167

Solution: (a) Consider a concave mirror (same radius of curvature as lens) separated by x from the lens. Let an object be placed at a distance u from the lens.

From lens equation, $\frac{1}{v_1} - \frac{1}{(-u)} = \frac{1}{f_l}$ ...(1)

The first image will act as object for mirror. Object distance for mirror is $v_1 - x$. From mirror equation,

$$\frac{1}{(v_1 - x)} + \frac{1}{v_2} = \frac{1}{f_m} \quad \text{...(2)}$$

The second image will be an object for second refraction through the lens. The object distance $= v_2 - x$.

From lens equation, $\frac{1}{v} - \frac{1}{-(v_2 - x)} = \frac{1}{f_l}$ ...(3)

where v is final image distance.

Adding eqns. (1), (2) and (3), we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l} - x\left[\frac{1}{v_1(v_1 - x)} + \frac{1}{v_2(v_2 - x)}\right]$$

If we set $x = 0$, we get $\frac{1}{v} + \frac{1}{u} = \frac{1}{F}$

where $$\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l}$$

- This result can be generalised ; the optical power of a composite system of lenses or mirrors is the sum of the optical powers of individual components.
- The optical power of the lens has been considered twice because light travels twice through the lens before the final image is formed.

$$\frac{1}{f_l} = \frac{2(n-1)}{R} \Rightarrow \frac{1}{f_m} = \frac{2}{R}$$

Thus $$\frac{1}{F}=2.\frac{2(n-1)}{R}+\frac{2}{R}=\frac{2(2n-1)}{R}$$

$$F=\frac{R}{2(2n-1)}$$

(b) When the plane surface of a plano-convex lens is mirrored,

$$\frac{1}{f_l}=\frac{(n-1)}{R},\frac{1}{f_m}=\frac{1}{\infty}=0$$

as radius of curvature of a plane mirror is infinite

Hence $$\frac{1}{F}=\frac{2(n-1)}{R}+0$$

$$F=\frac{R}{2(n-1)}$$

(c) When the curved surface is mirrored,

$$\frac{1}{f_l}=\frac{(n-1)}{R},\ \frac{1}{f_m}=\frac{2}{R}$$

$$\frac{1}{F}=\frac{2(n-1)}{R}+\frac{2}{R}=\frac{2\mu}{R}$$

$$F=\frac{R}{2\mu}$$

❒ In case (b) the composite optical system will behave as a concave mirror of focal length

$$F=-\frac{R}{(2\mu-1)}$$

and in case (c) $$F=-\frac{R}{2\mu}$$

Example 168

A plano-convex lens, refractive index $n = 1.5$, radius of curvature $R = 12$ cm, has the plane surface silvered. (a) At what distance from the lens will parallel rays incident on the convex face converge ? (b) Determine the position of image for an object at a distance 20 cm from the lens.

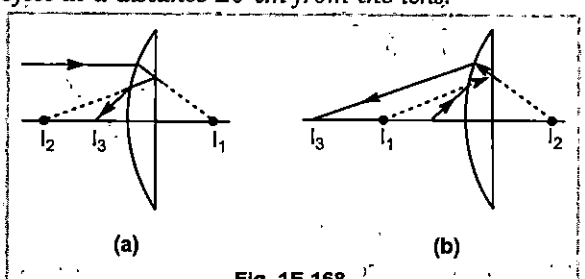


Fig. 1E.168

Solution: From thin lens equation,

$$\frac{1}{f}=(n-1)\left(\frac{1}{12}-\frac{1}{\infty}\right)$$

$$\frac{1}{f_l}=(1.5-1)\left(\frac{1}{12}-\frac{1}{\infty}\right),\ f_l=24\text{cm}$$

$$\frac{1}{f_m}=0\ \text{[silvered surface is plane]}$$

Power of composite optical system,

$$\frac{1}{F}=\frac{1}{f_l}+\frac{1}{f_m}+\frac{1}{f_l}$$

$$=2\times\frac{1}{0.24}+0$$

$$=\frac{1}{0.12}\text{D}$$

The focal length of the equivalent concave mirror is

$$F=0.12\text{m}=12\text{cm}$$

The parallel rays will converge at focus of composite system as shown in Fig. 1E.108(a).

From mirror equation,

$$\frac{1}{v}+\frac{1}{\infty}=\frac{1}{(-12)},\ v=-12\text{cm}$$

Note that first image I_1 after refraction is virtual, second image I_2 after reflection is virtual and final image I_3 is infront of composite system, hence real.

Example 169

The diameter of aperture of a plano-convex lens is 6 cm and its maximum thickness is 3 mm. If the velocity of light in the material of lens is 2×10^8 m/s, calculate the focal length of the lens.

Solution: Given thickness of the lens at the centre $t = 0.3$ cm. If R be the radius of the spherical surface of the lens, then

$$(2R-t)t=y^2$$

or $$2Rt-t^2=y^2$$

As $$t\ll R,\quad 2Rt\simeq y^2$$

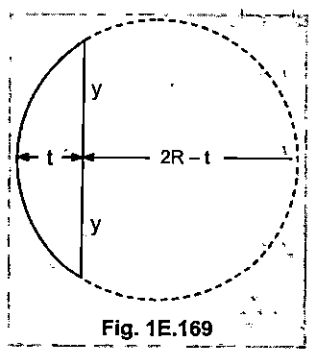


Fig. 1E.169

and $$R=\frac{y^2}{2t}=\frac{(3)^2}{2\times0.3}=15\text{cm}$$

If μ is the refractive index of material of the lens, then

$$\mu=\frac{3\times10^8}{2\times10^8}=1.5,$$

Now by lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{15} - \frac{1}{\infty}\right)$$

$\therefore$ $f = 30\,\text{cm}$.

Example 170

An observer holds in front of himself a thin symmetrical converging lens; a is the numerical value of the radius of curvature of each face. He sees two images of his eyes, one erect and the other inverted. Explain the formation of these images and assuming the refractive index of glass to be 1.50, prove that he will see two erect images if the distance of the lens is less than 0.25 a.

Solution: First image seen due to reflection at first surface of the lens since thin surface behave as convex mirror so the image seen is erect. Second image seen due to reflection at second surface of the lens. Since the lense is behave disk a lens sivered at back surface work as a concave mirror and if the objection placed within the focus and pole then the image will be erect or if distance of eye is more than 2F then enverted image is seen.

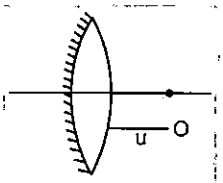


Fig. 1E.170

For reflection from see surface

$$P = IP_I + P_M = 2(\mu - 1)\frac{2}{a} + \frac{2}{a} = \frac{2}{a}(2\mu - 1)$$

$$F = -\frac{1}{P} = \frac{-a}{4\mu - 2}$$

To see erect image distance between eye and image should be less the effective focal length F.

$\Rightarrow$ $|u| < |F|$

$$u < \frac{a}{4\mu - 2}$$

$$u < \frac{a}{4 \times \frac{3}{2} - 2} = \frac{a}{6 - 2} = \frac{a}{4} = 0.25a$$

$$u < 0.25a$$

Example 171

A thin convergent lens is placed between an object and a screen whose positions are fixed. There are two positions of the lens at which the sharp image of the object is formed on the screen. Find the transverse dimensions of the object if at one position the transverse dimension of the image is $h_1 = 8\,mm$ and at the other $h_2 = 2\,mm$.

Solution: For a convex lens forming real image, we have

$$\frac{1}{+v} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Here $u + v = l$ (fixed)

$\therefore$ $$\frac{1}{l-u} + \frac{1}{u} = \frac{1}{f} \Rightarrow u^2 - ul + lf = 0$$

$$u = \frac{1}{2} \pm \frac{1}{2}\sqrt{l^2 - 4lf}$$

Similarly, we have

$$\frac{1}{v} + \frac{1}{l-v} = \frac{1}{f} \Rightarrow v^2 - vl + lf = 0$$

$$v = \frac{l}{2} \pm \frac{1}{2}\sqrt{l^2 - 4lf}$$

Since u and v, cannot be equal, we take

$$u_1 = \frac{l}{2} + \frac{1}{2}\sqrt{l^2 - 4lf};$$

$$v_1 = \frac{l}{2} - \frac{1}{2}\sqrt{l^2 - 4lf}$$

$$u_2 = \frac{l}{2} - \frac{1}{2}\sqrt{l^2 - 4lf};$$

$$v_2 = \frac{l}{2} + \frac{1}{2}\sqrt{l^2 - 4lf}$$

Now, $$m_1 = \frac{h_1}{h} = \frac{v_1}{u_1} = \frac{\frac{l}{2} - \frac{1}{2}\sqrt{l^2 - 4lf}}{\frac{l}{2} + \frac{1}{2}\sqrt{l^2 - 4lf}}$$

$$m_2 = \frac{h_2}{h} = \frac{v_2}{u_2} = \frac{\frac{l}{2} + \frac{1}{2}\sqrt{l^2 - 4lf}}{\frac{l}{2} + \frac{1}{2}\sqrt{l^2 - 4lf}}$$

We see that, $m_1 \cdot m_2 = 1$

$\Rightarrow$ $$\frac{h_1}{h} \cdot \frac{h_2}{h} = 1$$

$\Rightarrow$ $$h = \sqrt{h_1 \cdot h_2} = \sqrt{2 \times 8} = \sqrt{16}$$

$\Rightarrow$ $h = 4\,\text{mm}$

Example 172

A converging lens and a converging mirror are placed with their principal axis coinciding. Their separation equals 40 cm. A point source S is placed on the principal axis at a distance of 12 cm from the lens as shown in the figure. It is found that the final beam comes out parallel to the principal axis. Focal length of the lens equals 15 cm. Find the focal length of the mirror.

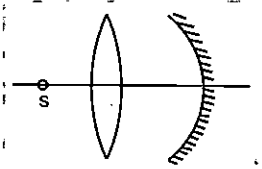


Fig. 1E.172 (a)

Solution: Refraction from lens:

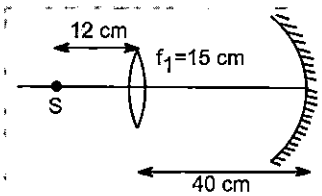


Fig. 1E.172 (b)

$$\frac{1}{f_L}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{15}=\frac{1}{v}+\frac{1}{12} \quad [f_L=15\,\text{cm},\ u=-12\,\text{cm}]$$

$$\frac{1}{15}-\frac{1}{12}=\frac{1}{v}\Rightarrow v_1=-60\,\text{cm}$$

Reflection from mirror:

u_1 for mirror $u_2=-100\,\text{cm}$

if finally rays coming parallel to principal axis then the object should be at focus between mirror and lens. Then image created by mirror will be

$v_2=-25\,\text{cm}$ from the pole of mirror

for mirror $\frac{1}{f_m}=\frac{1}{u}+\frac{1}{v}$

$$\frac{1}{f_m}=-\frac{1}{100}-\frac{1}{25}=-\left(\frac{1+u}{100}\right)$$

$$f_m=-20\,\text{cm}$$

Example 173

A thin bi-convex lens made up of glass of refractive index $\frac{3}{2}$ is placed in front of a plane mirror. The space between the lens and the mirror is filled with water of refractive index $\frac{4}{3}$.

The radii of curvatures of the lens are $R_1=15$ cm and $R_2=25$ cm. A point object is placed at distance x from the surface whose curvature is R_1. The distance x is greater than the separation between the lens and the mirror. Find the value of x so that image must coincide with the object.

Solution: For first refraction

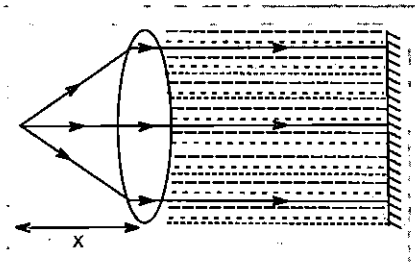


Fig. 1E.173

$u=x$

$v=?$

$R=15\,\text{cm}$

$\mu_1=1$ (air)

$\mu_2=\frac{3}{2}$

$$\therefore \quad \frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}\Rightarrow\frac{3}{2V}+\frac{1}{x}=\frac{1}{2\times15}$$

$$\Rightarrow \quad \frac{31}{2V}=-\frac{1}{x}+\frac{1}{30}$$

For accord refraction

$u=V$

$V'=?$

$R=-25\,\text{cm}$

$\mu_1=\frac{3}{2}$

$\mu_1=\frac{4}{3}$

$$\Rightarrow \quad \frac{4}{3V'}-\frac{3}{2V}=\frac{\frac{4}{3}-\frac{3}{2}}{-25} \quad \Rightarrow \frac{3}{2V}=\frac{1}{150}$$

$$\Rightarrow \quad -\frac{1}{x}+\frac{1}{30}=\frac{1}{150} \quad \Rightarrow \quad x=25\,\text{cm}$$

To coincide the final image to the object the reflected ray must retrace the path

$$\Rightarrow \quad V'=\infty$$

Example 174

A convex lens is held directly above a coin lying on a table and forms an image of it. After the lens has been moved vertically a distance equal to its focal length, it forms another image of the coin equal in size to the previous image. If the diameter of the coin is 16 mm, what is the diameter in millimeters of the image ?

Solution: Magnitude of magnification remains unchanged

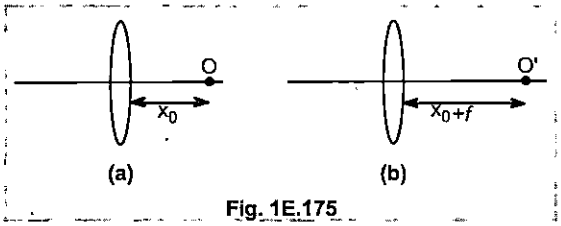


Fig. 1E.175

$$m=\left|\frac{f}{f+(-x_0)}\right|=\left|\frac{f}{f+(-x_0-f)}\right|=\frac{1}{f-x_0}=\pm\frac{1}{x_0}$$

$$\pm x_0=f-x_0$$

either $f=0$ or $x_0=f/2$

$$m=\frac{f}{f-f/2}=2=\frac{d_{image}}{d_{coin}}$$

$\Rightarrow$ $d_{image}=2\times d_{coin}=2\times 16=32\,mm$

Example 175

An equiconvex air lens ($R_1=R_2=10\,cm$) is made in an extended glass medium having refractive index ($\mu=3/2$). Find the refractive index of the material to be filled in, so that the power of the lens changes without change in its magnitude.

Solution: When no matter was filled, it behaves like an air lens

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow\quad \frac{1}{f}=\left(\frac{\mu_a}{\mu_g}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{2}{3}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right)$$

$f=-15\,cm$, divergent lens of focal length 15 cm

in opposite case it will behave as convergent lens of 15 cm focal length.

Let μ_m, be the refractive index of the material to be filled.

$$\Rightarrow\quad \frac{1}{f'}=\left(\frac{\mu_m}{\mu_g}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{2\mu}{3}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right)$$

$$\Rightarrow\quad \frac{1}{15}=\left(\frac{2\mu}{3}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right)\quad\Rightarrow\mu_m=2$$

Example 176

When an object is placed at a distance of 25 cm from a concave mirror, the magnification is m_1. The object is moved 15 cm farhter away with respect to the earlier position, and the magnification becomes m_2. If $m_1/m_2=4$ find the focal length of the mirror. (Assume image is real m_1, m_2 are numerical values)

Solution: $\frac{1}{v}-\frac{1}{25}=\frac{1}{f}$ $\qquad v=m_1u$

$$\frac{1}{-25m_1}-\frac{1}{25}=\frac{1}{-f}\quad\text{or}\quad\frac{1}{25m_1}+\frac{1}{25}=\frac{1}{f}\qquad\ldots(1)$$

Similarly, in second case

$$\frac{1}{40m_2}+\frac{1}{40}=\frac{1}{f}\qquad\ldots(2)$$

$$m_1=4m_2\qquad\ldots(3)$$

From eq. (1), (2) and (3) we get

$$m_2=1,\ m_1=4,\ f=20$$

Example 177

A lens is held directly above a coin lying on a table and forms an image of it. After the lens has been moved vertically a distance equal to its focal length, it forms another image of the coin equal in size to previous image. If the diameter of the coin is 4.0 cm. What is the diameter of the image?

Solution: Object distance = $2u$. Focal length = f and image distance = v

By lens formula $\longrightarrow$ $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ $\Rightarrow$ $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

$$\Rightarrow\quad \frac{1}{v}=\frac{u-f}{uf}\quad\Rightarrow\quad v=\frac{uf}{u-f}$$

So, $$m_1=\frac{v}{u}=\frac{f}{u-f}\qquad\ldots(1)$$

If lens has been moved by f vertically down, then

Object distance $(u')=-(u-f)$. Focal length $=f$ and Image distance $=v'$

$$\frac{1}{f}=\frac{1}{v'}+\frac{1}{(u-f)}$$

$$\Rightarrow\quad \frac{1}{v'}=\frac{u-f-f}{f(u-f)}$$

$$\Rightarrow\quad v'=\frac{f(u-f)}{(u-2f)}$$

So, $$m_2=\frac{v'}{(u-f)}=\frac{f}{(u-2f)}\qquad\ldots(2)$$

$$m_1=-m_2\qquad\ldots(3)$$

$$\frac{f}{(u-f)}=-\frac{f}{(u-2f)}$$

$$u-2f=-u+f$$

$$2u=3f$$

$$u=\frac{3}{2}f\qquad\ldots(4)$$

Put u in (1) from (4)

$$v = 3f \quad ...(5)$$

$$m = \frac{v}{u} = 2$$

$$I = 2 \times 4 = 8\,\text{cm}$$

Example 178

A lens having focal length 30 cm is cut along a plane parallel to the principal axis of the lens at a distance 5 mm above optical centre 'A' and upper part of lens is shifted by 5 mm from the x-axis as shown is the figure. A point object is placed on the principal axis of the mirror at origin O (0, 0). Find the number of images formed just after the reflection from the mirror and write their coordinates.

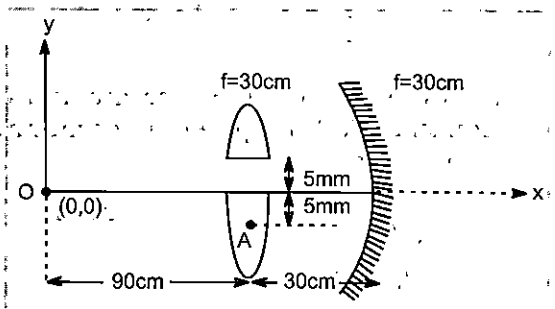


Fig. 1E.178

Solution: Image formed by upper lens

$$\frac{1}{v_1} - \frac{1}{(-90)} = \frac{1}{30} \Rightarrow v_1 = 45\,\text{cm}$$

This will act as an object for mirror

$$\therefore \quad \frac{1}{v^2} + \frac{1}{15} = \frac{-1}{30} \Rightarrow v_1 = -10\,\text{cm}$$

coordinates of image, (110, 0)

Image formed directly by mirror

$$\frac{1}{v} - \frac{1}{120} = \frac{-1}{30}$$

$$\frac{1}{v} = \frac{-1}{30} + \frac{1}{120} = \frac{-3}{120} = -\frac{1}{40}$$

$$v = -40\,\text{cm}$$

$\therefore$ coordinates (80, 0)

Image formed by lower lens

$$\frac{1}{v_0} - \left(\frac{1}{-90}\right) = \frac{1}{30} \Rightarrow v_c = 45\,\text{cm}$$

This acts as an objects from mirror

$$\therefore \quad \frac{1}{v_m} + \frac{1}{15} = \frac{-1}{30} \Rightarrow v_m = -10\,\text{cm}$$

Magnification due to lens $= -\frac{45}{90} = -\frac{1}{2}$

coordinates of image formed by lower lens = (135, –0.75)

Magnification due to mirror $= \frac{+10}{15} = \frac{+2}{3}$

$\therefore$ Coordinates of final image = (110, – 0.5)

Example 179

An equi-convex lens of focal length F in air is cut into two halves along the plane perpendicular to the optic axis. One of the halves is placed between a fixed object and a screen separated by distance of 90 cm. Two images are formed on the screen for two different positions of the lens with magnification 2 and 1/2 respectively. Find the value of F. What would be the value of the focal length if the lens is immersed in water ? Refractive index of the material of the lens is 1.5 and that of water is $\mu_w = 4/3$.

Solution: Let focal length of each part be

$$\Rightarrow \quad f = 2F$$

$$\frac{v_1}{u_1} = 2 \Rightarrow v_1 = 2u_1$$

$$\therefore \quad u_1 + 2u_1 = D$$

$$\Rightarrow \quad u_1 = D/3$$

$$\frac{v_2}{u_2} = \frac{1}{2}$$

also, : $v_2 + u_2 = D$

as $u_2 = u_1 + 30$

$$\Rightarrow \quad \frac{u_1 + 30}{2} + u_1 + 30 = D$$

$$\Rightarrow \quad 3u_1 + 90 = 2D$$

$\Rightarrow$ $D = 90\,\text{cm}$ (in the above discussion, we used the absolute values of the image + object distances)

Applying

$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u}$$

$$u_1 = \frac{D}{3} = \frac{90}{3} = 30\,\text{cm}$$

$$v_1 = D - u_1 = 90 - 30 = 60\,\text{cm}$$

$$\frac{1}{f} = \frac{1}{60} - \frac{1}{(-30)} = \frac{1+2}{60} = \frac{1}{20}$$

$$\Rightarrow \quad f = 20\,\text{cm}$$

$$F = f/2 = 10\,\text{cm}$$

In water, the focal length is

$$\frac{1}{f} = \left(\frac{\mu}{\mu_w} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{1}{40}$$

$$(1.5-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\frac{1}{40} \quad ...(1)$$

$$\Rightarrow \left(\frac{1.5\times 3}{4}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\frac{1}{f_{water}} \quad ...(2)$$

From eqn. (1) and (2) $\frac{(1.5-1)}{\left(\frac{4.5}{4}-1\right)}=\frac{f_{water}}{40}$

$$\Rightarrow \quad f_{water}=4\times 40=160\,cm$$

Example 180

A concave mirror of focal length = 20 cm is taken for some experiment. An object is placed on its axis at a distance of 30 cm from the mirror. Where should a plane mirror be placed so that the image of the above point is formed at a height of 20 cm from the principle axis and at the centre of curvature of the mirror. Consider one refraction from concave mirror and one reflection from plane mirror.

Solution: $\frac{1}{V_1}+\frac{1}{30}=\frac{1}{20}\Rightarrow V_1=60\,cm$

The first image of the object is formed at a distance of 60 cm from the pole P, on the same side of the object.

If a plane mirror is kept at C, perpendicular to the principle axis, after reflection from the plane mirror, the image is formed at a distance of 20 cm infront of the mirror *i.e.* at the focus of the spherical mirror

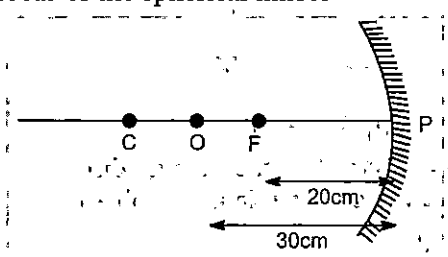


Fig. 1E.180 (a)

When the plane mirror will be rotated by 45° along the axis of the lens the reflected rays and hence the final image will be rotated by angle 90°, and hence, the image will be rotated by 90°

∴ The final image will be formed as shown in the 1E.180 (b)

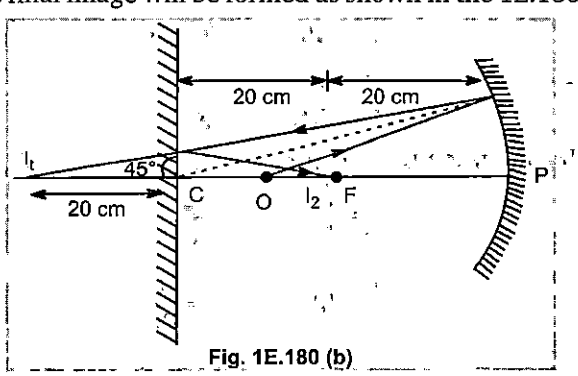


Fig. 1E.180 (b)

Example 181

A hollow equiconvex lens made of a very thin glass sheet has one of its curved surfaces silvered. It converges a parallel beam of light at a distance of 0.2 m infront of it. Where will it converge the same rays if filled with water, refractive index $n_w = 4/3$?

Solution: As glass is thin, we can ignore its refractive effects. The lens behaves as an air lens with a glass boundary.

$$\frac{1}{f_l}=(1-1)\left(\frac{1}{R}-\frac{1}{(-R)}\right)=0$$

If R is the radius of curvature of the curved surface,

$$\frac{1}{f_m}=\frac{2}{R}, \qquad f_m=\frac{R}{2}$$

With air inside, the system will behave as a concave mirror of focal length $R/2$. For parallel incoming rays, from mirror equation we have

$$\frac{1}{(-0.2)}+\frac{1}{-\infty}=\frac{1}{f_m},\ f_m=-0.2\,m$$

Hence, $R=-2f_m=0.4\,m$

When water is filled in the lens,

$$\frac{1}{f_l}=\left(\frac{4}{3}-1\right)\left[\frac{1}{0.4}-\frac{1}{(-0.4)}\right]=\frac{1}{0.6}\,D$$

Power of composite system,

$$\frac{1}{F}=\frac{1}{f_l}+\frac{1}{f_m}+\frac{1}{f_l}$$

$$=2\times\left[\frac{1}{0.6}\right]+\frac{2}{0.4}=\frac{5}{0.6}$$

Hence, $F=-0.12\,m=-12\,cm$

For parallel rays we have

$$\frac{1}{v}-\frac{1}{-\infty}=\frac{1}{-12}$$

$$v=-12\,cm$$

The final image is formed at 12 cm infront of the lens.

Illustration 13: Figure shows a mirrored lens place at 10 cm below water. Find position of object such that its image concider with the object.

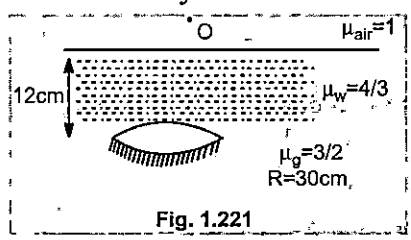


Fig. 1.221

In this situation if after refraction through water then refraction at curved surface of the lens the light is incident

normally on the mirrored curved surface light will retrace its path and according to principle of reversibility of light the final image will coincide with the object.

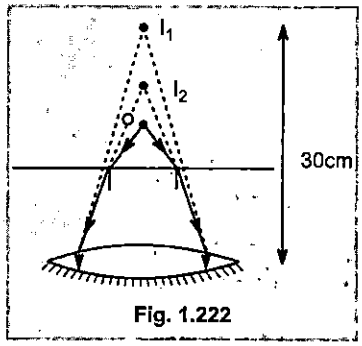


Fig. 1.222

We solve the problem after the light reflects from mirror.

Let image forms at v after event 1

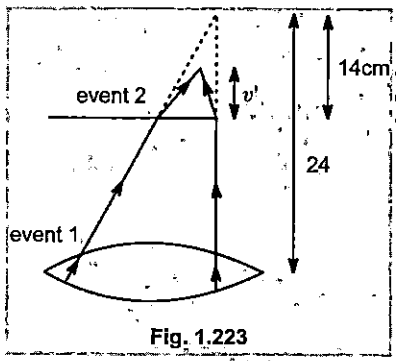


Fig. 1.223

$$\frac{4}{3v}-\frac{3}{2\times 30}=\frac{\left(\frac{4}{3}-3\frac{1}{2}\right)}{-30}$$

$$v = 24\,\text{cm}$$

Let image be formed at v'

$$\frac{1}{v'}=\frac{4}{3\times 12}$$

or $$v'=\frac{36}{4}=9\,\text{cm}$$

Illustration 14:

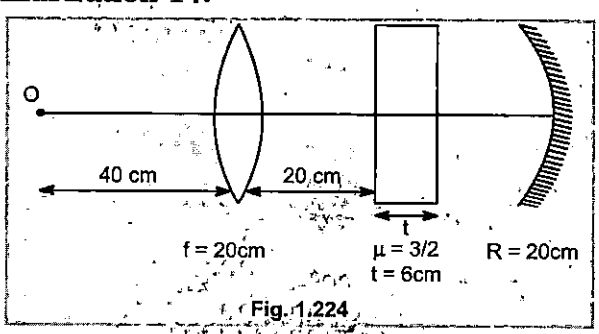


Fig. 1.224

Figure shows on object O at a distance 40 cm from lens, a glass slab at 20 cm from lens. A concave mirror is to be placed such that find image after reflection back from mirror is formed at object itself. What is/are separation of object from mirror.

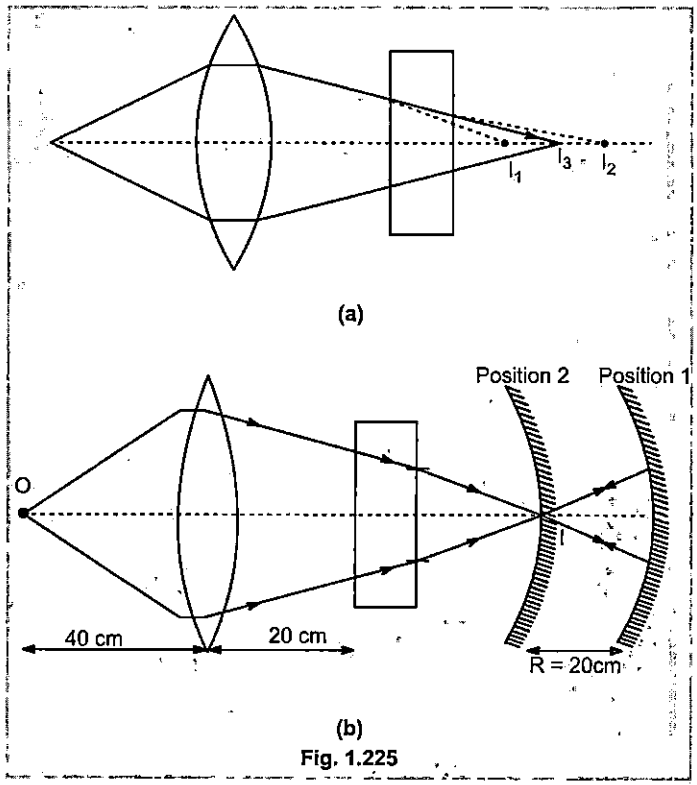


Fig. 1.225

The are two cases possible

(i) If final image is formed at centre of curvature of mirror.

(ii) If final image is formed at pole of mirror.

Due to presence of slab the image of lens shifts towards right by $t\left(1-\frac{1}{\mu}\right)=2\,\text{cm}$. Thus required distance is 102 cm or 82 cm.

Example 182

The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface [see Fig. 1E.182 (a)].

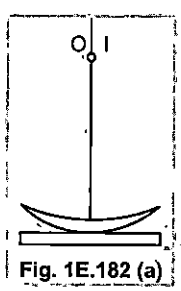


Fig. 1E.182 (a)

(a) Where should a pin be placed on the optic axis such that its image is formed at the same place ?

(b) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved, so that the image of the pin again coincides with the pin ?

Solution: (a) The optical arrangement is equivalent to a concave mirror of focal length F given by $\frac{1}{F}=\frac{1}{f_g}+\frac{1}{f_m}+\frac{1}{f_g}$, where f_g is the focal length of the lens without silvering and f_m is the focal length of the mirror.

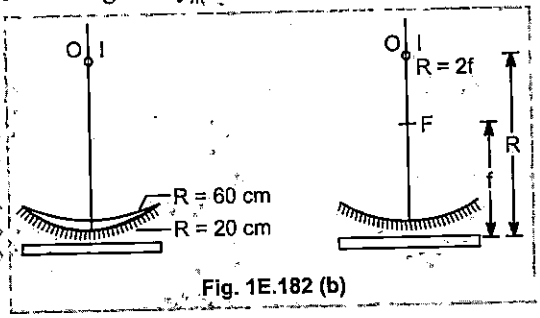


Fig. 1E.182 (b)

$$\frac{1}{f_g}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1.5-1)\left(\frac{1}{+20}-\frac{1}{+60}\right)=\frac{1}{60}$$

$$f_g=60\,\text{cm}$$

and $f_m=R_1/2=20/2=10\,\text{cm}$

$\therefore$ $\frac{1}{F}=\frac{1}{60}+\frac{1}{10}+\frac{1}{60}=\frac{8}{60}$

or $F=\frac{60}{8}=7.5\,\text{cm}$

For the image to be formed at the place of the object,

$$X=R=2F=7.5\times 2=15\,\text{cm}$$

Method 2 : We use the relation $\frac{n_2}{x_2}-\frac{n_1}{x_1}=\frac{n_2-n_1}{R}$

For the object and the image to coincide, the rays fall normally on the reflecting surface, i.e., on the silvered face of the lens.

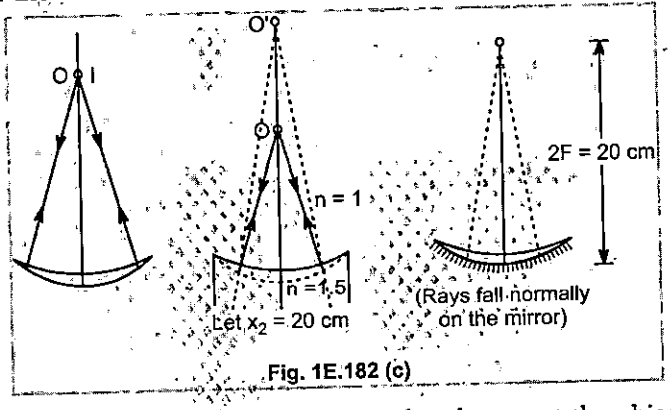


Fig. 1E.182 (c)

Then the rays retrace backward and meet at the object point again (optical reversibility).

For the refraction at the upper surface of the lens,

$n_1=1.0,\quad n_2=1.5,\quad x_1=?,\quad x_2=20,\quad R=+60$

($x_2=+20$ ensures that the rays fall on the silvered face normally).

$\therefore$ $\frac{1.5}{20}-\frac{1.0}{x_1}=\frac{1.5-1.0}{+60}$ or $\frac{1.0}{x_1}=\frac{1.5}{20}-\frac{0.5}{60}=\frac{3.0}{60}$

or $x_1=15\,\text{cm}$

Method 3: We use lens maker's formula and the equation $\frac{1}{f}=\frac{1}{x_2}-\frac{1}{x_1}$.

The given optical arrangement can be visualised as a convex lens of focal length 60 cm and a concave mirror of focal length 10 cm kept in contact as shown in the figure.

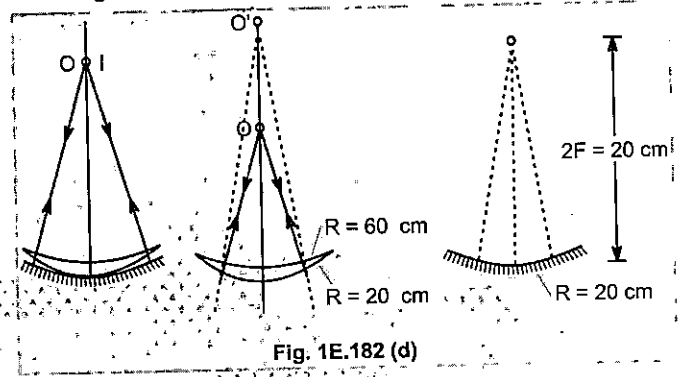


Fig. 1E.182 (d)

If the rays fall normally on the mirror after refraction through the lens, they will retrace backwards and meet at the point of the pin again.

For the lens,

$x_1=?$

$x_2=+20$ (for normal incidence on the mirror)

$f=-60$ (using cartesian-coordinate sign convention)

$\therefore$ $\frac{1}{-60}=\frac{1}{+20}-\frac{1}{x_1}$

or $x_1=15\,\text{cm}$

(b) When the concave part is filled with water of refractive index 4/3, the optical arrangement is equivalent to a concave mirror of focal length F such that

$$\frac{1}{F}=\frac{1}{f_w}+\frac{1}{f_g}+\frac{1}{f_m}+\frac{1}{f_g}+\frac{1}{f_w},$$

where $\frac{1}{f_w}=\left(\frac{4}{3}-1\right)\left(\frac{1}{60}-\frac{1}{\infty}\right)=\frac{1}{180}$

or $f_w=180\,\text{cm}$

and $f_g=60\,\text{cm}$ (calculated earlier)

$\therefore$ $\frac{1}{F}=\frac{1}{180}+\frac{1}{60}+\frac{1}{10}+\frac{1}{60}+\frac{1}{180}=\frac{26}{180}$

or $F=\frac{180}{26}\,\text{cm}$

Now, $x_1 = R = 2F = \frac{180}{26} \times 2 = \frac{180}{13} = 13.85\,\text{cm}$

$\therefore \quad \Delta x = 15.0 - 13.85 = 1.15\,\text{cm}$

Method 2: We use the equation $\frac{n_2}{x_2} - \frac{n_1}{x_1} = \frac{n_2 - n_1}{R}$

For refraction at the interface '1' (air-water)

$$\frac{4/3}{x_2} - \frac{1}{x_1} = \frac{4/3 - 1}{\infty} \quad \ldots (1)$$

The image of interface '1' is the object for the interface '2'

$$\therefore \quad \frac{1.5}{+20} - \frac{1}{x_1} = \frac{1.5 - 4/3}{+60}$$

or $\quad x_1 = \frac{360}{26} = 13.85$

$\therefore \quad \Delta x = 15.0 - 13.85 = 1.15\,\text{cm}$

Method 3: Using lens maker's formula and the relation

$$\frac{1}{F} = \frac{1}{x_2} - \frac{1}{x_1},$$

$f_w = 180\,\text{cm}$ (using lens maker's formula)

$f_g = 60\,\text{cm}$ (using lens maker's formula)

$$\therefore \quad \frac{1}{-180} = \frac{1}{x_2} - \frac{1}{x_1}, \quad \text{(for the water lens)} \quad \ldots(1)$$

and $\quad \frac{1}{-60} = \frac{1}{+20} - \frac{1}{x_1} \quad$ (for the glass lens) $\quad \ldots(2)$

(The image by the water lens is object for the glass lens and if the image by the glass lens is at +20, then the rays will fall normally on the mirror)

Adding equations (1) and (2)

$$\frac{1}{-180} + \frac{1}{-60} = \frac{1}{20} - \frac{1}{x_1}$$

or $\quad x_1 = \frac{180}{13} = 13.85\,\text{cm},$

$\therefore \quad \Delta x = 15.0 - 13.85 = 1.15\,\text{cm}$

Example 183

A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axes PQ and RS parallel but separated in the vertical direction by 0.6 cm as shown. The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optic axis PQ of the lens at a distance 20 cm from the lens and the reflection from the mirror, find the distance of A'B' from the pole of the mirror and obtain its magnification. Also locate the positions of A' and B' w.r.t. the optic axis RS.

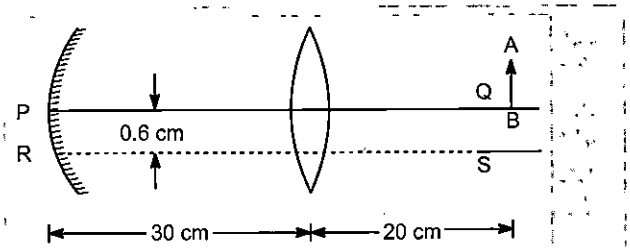


Fig. 1E.183

Solution: The image formed by the lens acts as an object for the mirror.

From lens equation,

$$\frac{1}{v} - \frac{1}{(-20)} = \frac{1}{15}, \quad v = 60\,\text{cm}$$

Image is formed 60 cm to the left of the lens.

$$\text{Magnification, } m = \frac{v}{u} = \frac{(60)}{(-20)} = -3$$

Height of image $= (-3)(1.2) = -3.6\,\text{cm}$

The image formed is real and inverted.

According to sign convention, all the distances longitudinally and laterally are measured from the optical centre or pole of the refracting or reflecting surface.

The first image acts as a virtual object for the mirror.

From mirror equation, $\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f}$

$$\frac{1}{v'} + \frac{1}{(+30)} = \frac{1}{(-30)}$$

$$v' = -15\,\text{cm}$$

$$\text{Magnification } m = -\frac{v'}{u'} = -\frac{(-15)}{(30)} = 0.5$$

Note that A_1B_1 is object for the mirror.

One part ($BB_1 = 0.6\,\text{cm}$) lies above the principal axis and the other part ($A_1B = 3\,\text{cm}$) lies below the principal axis.

Height of part B_2B' (*i.e.*, above principal axis.)

$$= (0.6) \times (0.5) = 0.3\,\text{cm}$$

Height of part B_2A' (*i.e.*, below principal axis)

$$= 3 \times (0.5) = 1.5\,\text{cm}$$

If we consider the pole of the mirror as origin of our coordinate system, the coordinates of B' and A' are respectively (15 cm, 0.3 cm) and (15 cm, −1.5 cm).

Total height of image $A'B' = (1.5 + 0.3) = 1.8\,\text{cm}$

Example 184

Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d ($d < f_1$ and $d < f_2$) and their centres are displaced by a vertical separation Δ as shown in Fig. 1E.184. Taking the origin of coordinates O at the centre of first lens, what would be the x and y-coordinates of the focal point of this lens system for a parallel beam of rays coming from left ?

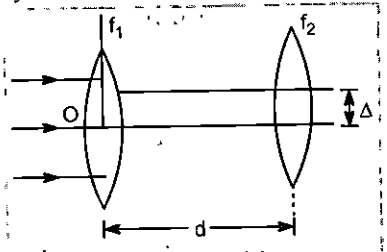


Fig. 1E.184 (a)

Solution: The parallel rays will be focussed at the focal point of the first lens. The first image lies at I, at a distance f_1 from the origin. This image I_1 will act as an object for refraction through the second lens. The object distance for the second lens, $u = (f_1 - d)$.

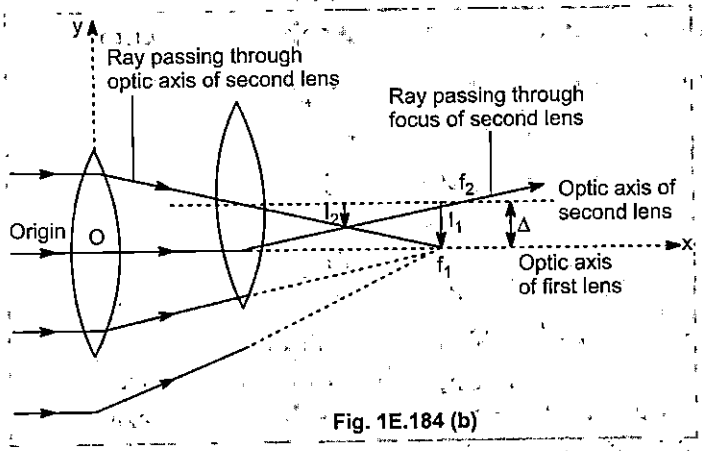


Fig. 1E.184 (b)

From lens equation, $\frac{1}{v} - \frac{1}{+(f_1 - d)} = \frac{1}{f_2}$

$$v = \frac{f_2(f_1 - d)}{(f_1 + f_2 - d)}$$

Hence, the x-coordinate of final image I_2 is

$$x = d + v = \frac{d + f_2(f_1 - d)}{(f_1 + f_2 - d)}$$

$$= \frac{d(f_1 - d) + f_1 f_2}{(f_1 + f_2 - d)}$$

Imagine an arrow tip is at f_1; its image from lens f_1 is the final image.

Magnification, $m = \frac{v}{u} = \frac{I_2}{O} = \frac{I_2}{\Delta}$

$$I_2 = \left(\frac{v}{u}\right)\Delta = \frac{f_2}{(f_1 + f_2 - d)}\Delta$$

Thus y-coordinate of tip of I_2

$$= \Delta - I_2 = \Delta\left[1 - \frac{f_2}{(f_1 + f_2 - d)}\right]$$

$$= \frac{(f_1 - d)\Delta}{(f_1 + f_2 - d)}$$

Example 185

A thin biconvex lens of focal length $f = 25\,cm$ forms a real image of a real object on a screen. The object is separated by 5 cm from the lens. The screen is drawn closer to the lens by a distance 18 cm. Through what distance must the object be shifted so that its image is again formed on the screen ?

Solution: From lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$u = \frac{fv}{f - v} \quad \ldots (1)$$

Let the screen be shifted by Δv and the corresponding shift in object position be Δu.

From eqn. (1), we have

$$u + \Delta u = \frac{f(v - \Delta v)}{f - (v - \Delta v)}$$

$$\Delta u = \frac{f(v - \Delta v)}{f - (v - \Delta v)} - \frac{fv}{f - v}$$

$$= \frac{-f^2 \Delta v}{(v - \Delta v - f)(v - f)}$$

$$= -\frac{f^2 \Delta v}{(v - f)^2}\left[1 - \frac{\Delta v}{(v - f)}\right]^{-1}$$

$$= -\frac{f^2 \Delta v}{(v - f)^2}\left[1 + \frac{\Delta v}{(v - f)}\right]$$

$$\simeq -\frac{f^2 \Delta v}{(v - f)^2} = -\frac{(25)^2 \times 18}{(475)^2} \simeq -0.5 \text{ mm}$$

We have neglected higher orders of Δv.

Example 186

(a) Fig. 1E.186 (a) shows the optical axis of a lens, the point source of light A and its virtual image A'. Trace the rays to find the position of the lens and of its focuses. What type of lens is it ?

(b) Solve the problem similar to the previous one using Fig. 1E.186 (b).

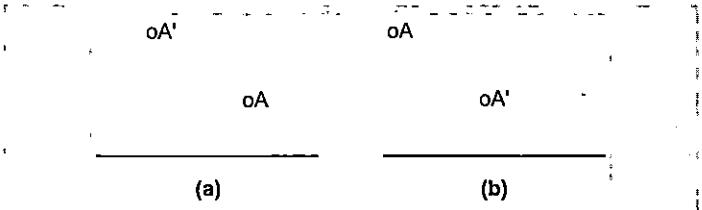


Fig. 1E.186

Solution: (a) The paths of the rays are shown in Fig. 1E.186 (c). First draw the ray *AA'* until it intersects with the principal optical axis and find the centre of the lens *C*. Since the virtual image is magnified, the lens is convex. Draw the ray *AB* parallel to the principal optical axis. It is refracted by the lens so that it passes through its focus and its continuation passes through the virtual image. The ray *A'B* intersects the principal optical axis at point *F*, the focus of the lens.

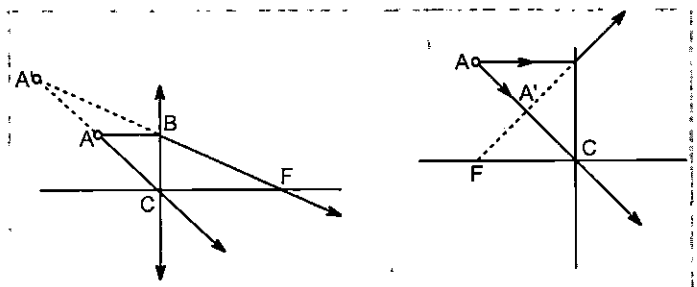


Fig. 1E.186

(b) The paths of the rays are shown in Fig. 1E.186 (d).

Example 187

Fig. 1E.187 (a) shows ray AB that has passed through a divergent lens. Construct the path of the ray up to the lens if the position of its foci F is known.

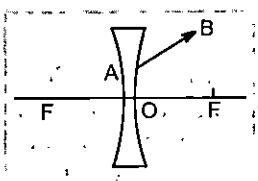


Fig. 1E.187 (a)

Solution: The path of the ray is shown in Fig. 1E.187 (b). Let us continue *AB* up to its intersection with the focal plane of lens *NN*. The beam of parallel rays after refraction in the lens so travels that the continuations of the rays should intersect at *F'*. Ray *F'O* is not refracted as it passes through the optical centre. Thus, ray *CA* passing to point *A* is parallel to *F'O* up to the lens as it passes through the centre of curvature of the lens.

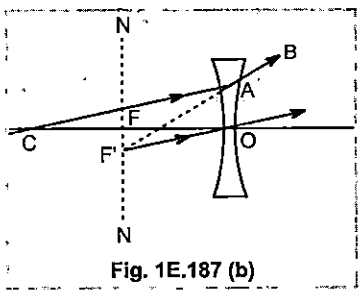


Fig. 1E.187 (b)

Example 188

Fig. 1E.188 (a) shows a luminescent point and its image produced by a lens with an optical axis N_1N_2. Find the position of the lens and its foci.

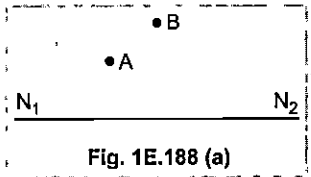


Fig. 1E.188 (a)

Solution: **Case (i).** If *A* is the source and *B* is the image, then the lens will be convergent. The position of the optical centre of the lens *O* and its foci *F* can be found by construction as shown in Fig. 1E.188 (b) (i). Object and image lie on a ray passing through the optical centre. A ray parallel to the principal axis must pass through the focus of the lens.

Case (ii). If *B* is the source and *A* is the image, the lens is divergent. The respective construction is illustrated in Fig. 1E.188 (b) (ii).

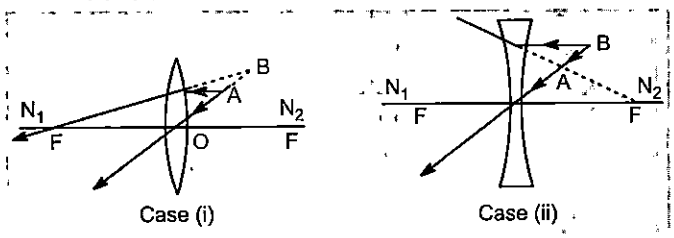


Fig. 1E.188 (b)

Example 189

A convexo-convex lens has a focal length of $f_1 = 10$ cm. One of the lens surfaces having a radius of curvature of $R = 10$ cm is coated with silver. Construct the image of the object produced by the given optical system and determine the position of the image if the object is at a distance of $u = 15$ cm from the lens.

Solution: The image $A'''B'''$ obtained in the system is shown in Fig.1E.189. F_1 and F_2 are the foci of the lens and the mirror, and $A'B'$ is the image produced by the lens if its surface is not coated with silver.

The image $A''B''$ produced by a concave mirror can be plotted, taking into account that ray *BO*, after passing through the lens and being reflected from the mirror surface, will travel along *OB''*; here $\angle BOA = \angle B''OA$. Ray *BC* emerges from the lens parallel to the optical axis of the system and after reflection passes through F_2.

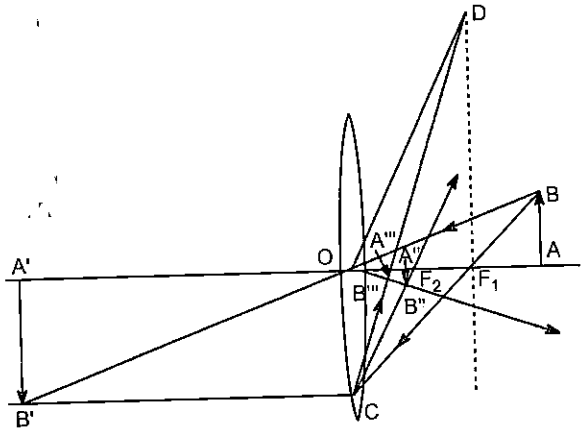


Fig. 1E.189

The rays reflected from the mirror are refracted in the lens once more and produce image $A'''B'''$. Point B''' lies at the intersection of rays OB'' and CD. Ray OB'' passes through the optical centre of the lens after reflection and is thus not refracted. Ray CD can be plotted as follows. After the first refraction in the lens and reflection, ray BC will travel in the direction of F_2 and will be refracted in the lens once more. Ray OD parallel to CF_2 is drawn through the optical centre O until it intersects the focal plane of the lens. The sought ray is obtained by connecting C and D.

Since the rays are refracted in the lens twice, focal length f of the system can be found from

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_1}$$

where $f_2=\frac{R}{2}$ is the focal length of the mirror.

$$f=\frac{f_1 f_2}{f_1+2f_2}=2.5\,\text{cm}$$

Therefore, the distance v to image $A'''B'''$ can be found from the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Hence, $$v=\frac{uf}{u-f}=3\,\text{cm}$$

Example 190

At what distance from a convexo-convex lens with a focal length of $f = 1$ metre should a concave spherical mirror with a radius of curvature of $R = 1$ metre be placed for a beam incident on the lens parallel to the major optical axis of the system to leave the lens, remaining parallel to the optical axis, after being reflected from the mirror? Find the image of the object produced by the given optical system.

Solution: Two cases are possible :

(a) The mirror is at a distance of $d = f + R$ from the lens.

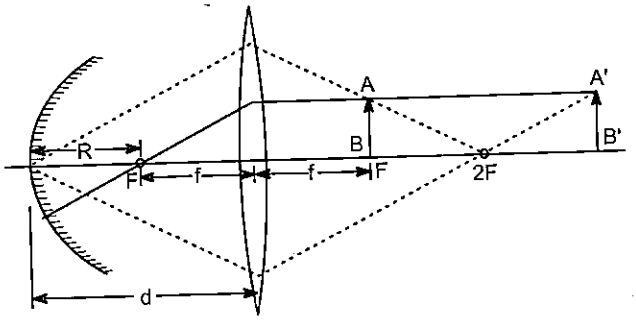


Fig. 1E.190 (a)

The path of the beam parallel to the optical axis of the system and the image of object AB are illustrated in Fig. 1E.190 (a). Image $A'B'$ (direct and real) is obtained to full scale with the object in any position.

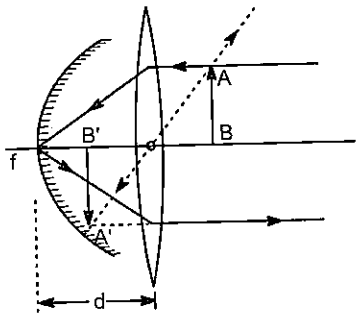


Fig. 1E.190 (b)

(b) The mirror is at a distance of $d = f = R$ from the lens [Fig. 1E.190 (b)]

The image of object $A'B'$, also full scale, will be inversed and virtual with object in any position.

Example 191

A source of light is located at double focal length from a convergent lens. The focal length of the lens is $f = 30$ cm. At what distance from the lens should a flat mirror be placed so that the rays reflected from the mirror are parallel after passing through the lens for the second time ?

Solution: The mirror should be placed between the focus and a point lying on the double focal length. The path of the rays is shown in Fig. 1E.191.

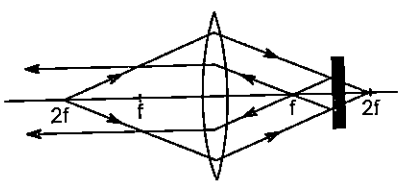


Fig. 1E.191

Example 192

A point object is located at a distance of 100 m from a screen. A lens of focal length 23 cm mounted on a movable frictionless stand is kept between the source and the screen. The stand is attached to a spring of natural length 50 cm and spring constant 800 N/m as shown. Mass of the stand with lens is 2 kg. How much impulse P should be imparted to the stand so that a real image of the object is formed on the screen after a fixed time gap. Also find this time gap. (Neglect width of the stand).

Solution: Let the distance of the lens from the object be l when a real image is formed on the screen. Then

$$\frac{1}{100-l}=\frac{1}{-l}=\frac{1}{23}$$

On solving, we get $l=(50\pm10\sqrt{2})$ cm.

Now, if the lens performs SHM and a real image is formed after a fixed time gap, then this time gap must be one-fourth of the time period.

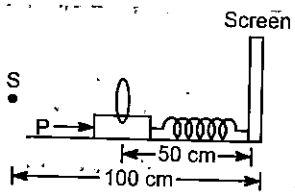


Fig. 1E.192

∴ Phase difference between the two positions of real image must be $\pi/2$. As the two positions are symmetrically located about the origin, phase difference of any of these positions from origin must be $\pi/4$.

$$\Rightarrow\quad 10\sqrt{2}\text{ cm}=A\sin\frac{\pi}{4}\qquad\Rightarrow A=20\text{ cm}$$

To achieve this velocity at the mean position,

$$v_0=A\omega=A\sqrt{\frac{K}{m}}$$

∴ Required impulse $p=mv_0=A\sqrt{Km}=8\text{ kg m/s}$.

Example 193

A point object moves along the principal axis of a convex lens of focal length f, such that its image, also formed on the principal axis at a distance $\frac{4}{3}f$ (at t = 0) moves away from the lens with a uniform velocity, α. Find the velocity of the point source as a function of time t.

Solution: Applying the equation for image formation by a lens, we get

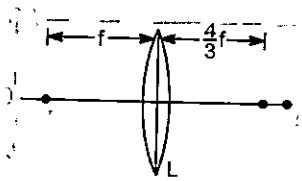


Fig. 1E.193

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{\frac{4}{3}f}-\frac{1}{u}=\frac{1}{f}\qquad\ldots(1)$$

$$u=-4f$$

The position of the image is given by

$$v=\frac{4}{3}f+\alpha t,\text{ at time } t\qquad\ldots(2)$$

∴ The corresponding position of the object is given by (at time t)

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\text{ or }\frac{1}{u}=\frac{1}{v(t)}-\frac{1}{f}$$

On differentiating the equation w.r.t. time, we get

$$-\frac{1}{u^2}\frac{du}{dt}=-\frac{1}{v^2}\frac{dv}{dt}$$

or $$\frac{du}{dt}=\frac{u^2}{v^2}\left(\frac{dv}{dt}\right)=\frac{u^2}{v^2}(\alpha)\qquad\ldots(3)$$

$$\text{Velocity of the source}=\frac{du}{dt}=\left[f\Big/\left(\frac{f}{3}+\alpha t\right)\right]^2\alpha\qquad\ldots(4)$$

Example 194

A convex lens of focal length f is mounted on a stand of total mass m, which is connected through a spring with a fixed surface. A point object is placed on the optical axis at a distance 3f from the pole. If the amplitude of vibration of the lens is 'a', find the distance between the two extreme ends of the vibration of the image.

Solution: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\qquad\ldots(1)$

$v-u=x;\ v=u+x$ putting in (1)

$$\frac{1}{u+x}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow\quad\frac{1}{u+x}=\frac{1}{u}+\frac{1}{f}=\frac{u+f}{uf}$$

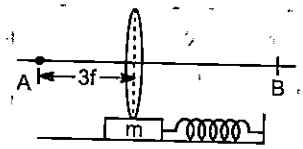


Fig. 1E.194

$$u+x=\frac{uf}{u+f}$$

$$x=\frac{uf}{u+f}-u=u\left[\frac{f}{u+f}-1\right]$$

$$x=\frac{-u^2}{u+f}$$

Differentiating w.r.t. x,

$$\therefore\quad\delta x=\left[\frac{-2u}{u+f}+\frac{u^2}{(u+f)^2}\right]\delta u$$

$$=\frac{-2u(u+f)+u^2}{(u+f)^2}\delta u=-\frac{u^2+2uf}{(u+f)^2}\delta u$$

$$\delta x=\frac{(u+f)^2+f^2}{(u+f)^2}=\left[\left(\frac{f}{u+f}\right)^2-1\right]\delta u$$

$$\delta u=a<<|u|=|3f|$$

$$\delta x = \left[\left(\frac{f}{-3f+f}\right)^2 - 1\right]a = -\frac{3}{4}a$$

$\therefore$ Amplitude $= |\delta x| = \frac{3}{4}a$

Example 195

A convex lens is held 45 cm above the bottom of an empty tank. The image of a point at the bottom of the tank is formed 36 cm above the lens. Now a liquid is poured into the tank to a depth of 40 cm. It is found that the distance of the image of the same point on the bottom of the tank is 48 cm above the lens. Find the refractive index of the liquid.

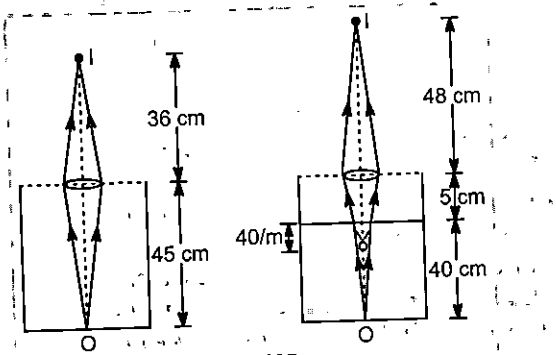


Fig. 1E.195

Solution: From the given condition,

$u = -45\,\text{cm}, v = 36\,\text{cm}$

$\therefore \quad \frac{1}{f} = \frac{1}{36} + \frac{1}{45}$ or $f = 20\,\text{cm}$

Now let μ be the refractive index of the liquid. Then in the present case,

$$u = -\left(5 + \frac{40}{\mu}\right); v = +48\,\text{cm}; f = 20\,\text{cm}$$

$$\therefore \quad \frac{1}{20} = \frac{1}{48} + \frac{1}{5+\frac{40}{\mu}} \quad \text{or} \quad \frac{1}{5+\frac{40}{\mu}} = 0.029$$

or $\quad 5 + \frac{40}{\mu} = 34.29$

or $\quad \frac{40}{\mu} = 29.28$

or $\quad \mu = 1.365$

Example 196

In the given figure there are two thin lenses of same focal length f arranged with their principal axes inclined at an angle α. The separation between the optical centres of the lenses is 2f. A point object lies on the principal axis of the convex lens at a large distance to the left of the convex lens.

(a) Find the coordinates of the final image formed by the system of lenses taking O as the origin of coordinate axes.

(b) Draw the ray diagram.

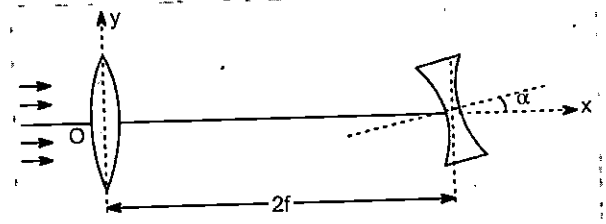


Fig. 1E.196 (a)

Solution: The image will be formed at the focus of the convex lens after refraction through the convex lens (at I).

Now for the concave lens, I acts as an object.

Since the distance is always taken from the pole on the principal axis.

If it can be assumed that the point I is the tip of an object erected on the principal axis of the concave lens, the image of point I will be the tip of the final image (let it be I')

$\therefore$ Distance of object $= u = f\cos\alpha$

Applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

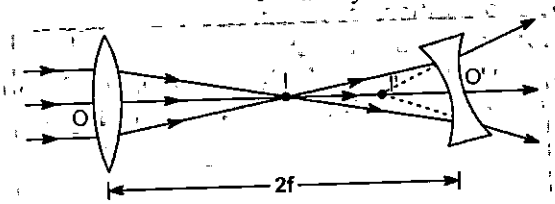


Fig. 1E.196 (b)

or $\quad \frac{1}{v} - \frac{1}{-f\cos\alpha} = \frac{1}{-f}$

$$\frac{1}{v} = \frac{-1}{f}\left(1 + \frac{1}{\cos\alpha}\right)$$

$$v = \frac{-f\cos\alpha}{1+\cos\alpha}$$

Now distance $O'I' = \frac{f\cos\alpha}{1+\cos\alpha} \times \frac{1}{\cos\alpha}$

So x-coordinate of

$$I' = OO' - O'I' = 2f - \frac{f}{1+\cos\alpha}$$

$$f\left(\frac{2+2\cos\alpha-1}{1+\cos\alpha}\right) = f\left(\frac{1+2\cos\alpha}{1+\cos\alpha}\right)$$

y-coordinate $= 0$

So coordinates of image are $\left(f\left(\frac{1+2\cos\alpha}{1+\cos\alpha}\right), 0\right)$.

We can verify the result by putting $\alpha = 0$.

Coordinates after putting $\alpha = 0$

$$\left\{f\left(\frac{1+2}{2}\right),0\right\}=\left\{\frac{3f}{2},0\right\}$$

This result is right if the principal axes of both the lenses coincide, then the image will be formed (finally) at $x=\frac{3f}{2}$.

Example 197

In Fig. 1E.197, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. It is placed at a distance $b = 2$ m from a screen Σ. A thin lens of focal length $f = 16$ cm is cut into two identical half. They are placed at a distances 20 cm and 80 cm from S. The part L_1 is shifted $\Delta = 0.40$ mm while L_2 is shifted 3Δ transverse to the line SC. The gaps between the line SC and the lens parts are filled by an opaque material as shown in the figure. Calculate :

(i) where the image of S will form,

(ii) the refractive index μ of a transparent sheet of thickness $t = 1$ mm to be placed in the path of the rays emerging from one of the parts so that a white spot is formed at point C.

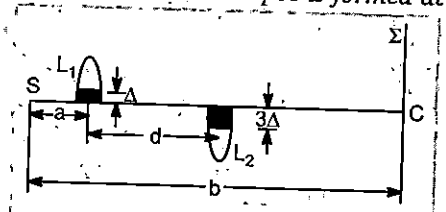


Fig. 1E.197 (a)

Solution: Both the lenses will form images of S. From lens equation,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

For lens L_1, $\frac{1}{v_1}=\frac{1}{16}-\frac{1}{20}$, $v_1 = 80$ cm

For lens L_2, $\frac{1}{v_2}=\frac{1}{16}-\frac{1}{80}$, $v_2 = 20$ cm

Magnification, $m=\frac{v}{u}$

For L_1, $m_1=\frac{80}{-20}=-4$

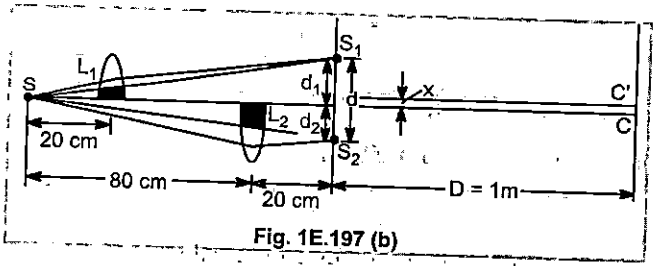


Fig. 1E.197 (b)

Thus, the light rays converge above the principal axis of L_1 at a height 4Δ, *i.e.*, 5Δ above the line *SC*.

Hence, $d_1 = 5\Delta = 5\times 0.40 = 2$ mm

Similarly we can determine d_2.

$$d_2=\frac{3\Delta}{4}+\Delta=\frac{7}{4}\Delta=0.7\text{ mm}$$

Two real images S_1 and S_2 will act as two coherent sources separated by a distance

$$d = d_1 + d_2 = 2.7\text{ mm}$$

The distance of screen from S_1S_2, $D = 2 - 1 = 1$ metre.

Point C is at a height $x=d_1-\frac{d}{2}=0.65$ mm, from perpendicular bisector of S_1S_2.

As the path difference of the rays from S_1 and S_2 reaching at C' is zero, hence a white spot will be formed at C. The white spot is observed at C, *i.e.*, the shifting of fringe pattern is equal to $x = 0.65$ mm. Hence the slab should be placed in the path of rays emerging from the lower part of the lens.

As shifting $x=\frac{(\mu-1)tD}{d}$,

$$0.65\times10^{-3}=\frac{(\mu-1)\times2\times10^{-6}\times1}{2.7\times10^{-3}}$$

which gives $\mu = 1.88$

Example 198

A parallel beam of light falls from vacuum on a surface enclosing a medium with refractive index n. Find the shape of that surface $x(y)$, if the beam is brought into focus at the point F at a distance f from the crest O. What is the maximum radius of a beam that can still be focused ?

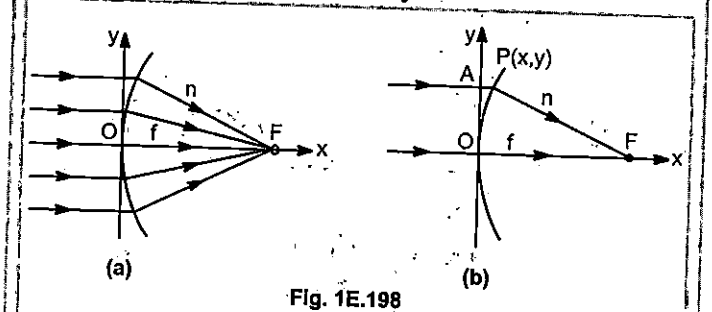


Fig. 1E.198

Solution: The optical path for *OF* ray must be equal to the optical path for *APF* ray.

Hence, $nf = n\sqrt{(f-x)^2+y^2}+x$

$$(n^2-1)x^2+n^2y^2-2n(n-1)fx=0$$

$$x=\frac{+2n(n-1)f\pm\sqrt{4n^2(n-1)^2f^2-4(n^2-1)n^2y^2}}{2(n^2-1)}$$

$$x=\frac{nf}{(n+1)}\left[1-\sqrt{\left(1-\frac{(n+1)^2y^2}{(n-1)^2f^2}\right)}\right]$$

is the required equation

For real value of x,

$$1-\frac{(n+1)^2y^2}{(n-1)^2f^2}\geq 0 \Rightarrow \frac{(n+1)^2y^2}{(n-1)^2f^2}\leq 1$$

$$\Rightarrow \quad y \leq f\sqrt{\frac{(n-1)}{(n+1)}}$$

So, $y_{max} = f\sqrt{\frac{(n-1)}{(n+1)}}$

Example 199

A point object is kept at a distance $d = 1m$ from a parabolic reflecting surface $x = 8y^2$. An equiconvex lens is kept at a distance $s = 80$ cm from the parabolic surface (see Fig.1E.199). The focal length of the lens is 20 cm. Find the position of the image after reflection from the surface.

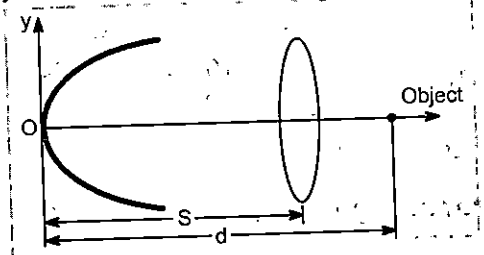


Fig. 1E.199 (a)

Solution: The only action of convex lens is to make the ray after passing through the lens parallel to the x-axis. Let the ray be incident at a point $P(x_1, y_1)$.

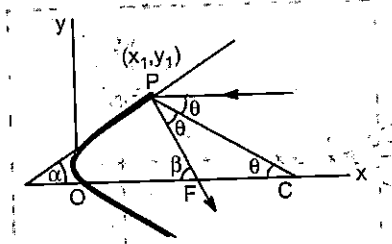


Fig. 1E.199 (b)

$$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \tan\alpha = \frac{1}{16y_1}$$

$$\alpha = 90°-\theta \text{ and } \beta = 2\theta$$

The reflected ray is passing through $F(X, O)$ and has slope $-\tan\beta$.

$$\frac{(y_1-0)}{(x_1-X)} = -\tan\beta$$

$$= -\tan 2\theta = -\frac{2\tan\theta}{1-\tan^2\theta}$$

$$\frac{y_1}{(x_1-X)} = -\frac{2\cot\alpha}{1-\cot^2\alpha}$$

$$= -\frac{2\times 16y_1}{1-16^2y_1^2}$$

$$-1+16^2y_1^2 = 32(x_1-X) = 32(8y_1^2-X)$$

$$X = \frac{1}{32}\text{m} = 3.16\,\text{cm} \qquad \text{(which is unique)}$$

Hence, the position of the image will be 3.16 cm from O.

Example 200

Consider Fig. 1E.200 shown a hemispherical cavity of radius R is curved out from a sphere ($\mu = 1.5$) of radius $2R$ such that the principal axis of the cavity coincides with that of the sphere. One side of the sphere is silvered as shown. Find the value of x for which the image of an object at O is formed at O itself.

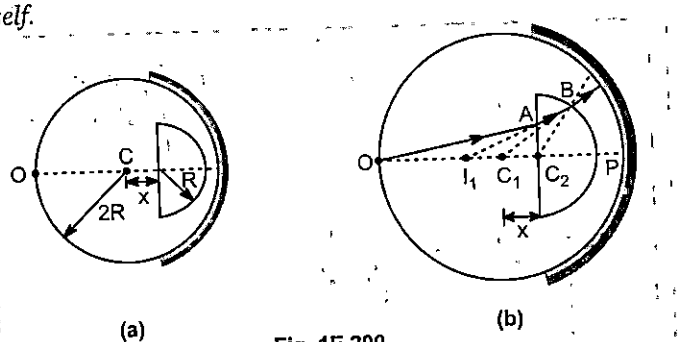


Fig. 1E.200

Solution: Refraction at plane surface (at A),

$$u = -(2R+x)$$

$$\therefore \quad \frac{1}{v}-\frac{\mu}{-(2R+x)} = 0$$

or $$v = \frac{-(2R+x)}{\mu} = -C_2I_1$$

For refraction at B, image I_1 acts as an object.

$$\therefore \quad u = PI_1 = -\left(R+\frac{2R+x}{\mu}\right)$$

$$\therefore \quad \frac{\mu}{v'}-\frac{1}{-\left(R+\frac{2R+x}{\mu}\right)} = \frac{\mu-1}{-R}$$

$$v' = \frac{-\mu}{\left[\frac{\mu-1}{R}+\frac{\mu}{(2R+x+\mu R)}\right]} \qquad \ldots (1)$$

Final image is formed at O, when the rays are incident normally on the silvered surface, i.e., the object is placed at C_1.

∵ The image formed by the hemispherical surface of the cavity acts as an object for the silvered surface, the image should be formed by the hemispherical surface at C_1.

i.e., $$v' = -(R+x) \qquad \ldots (2)$$

From eqns. (1) and (2), we get

$$2x^2 + 9Rx - 8R^2 = 0 \qquad (\text{putting } \mu = 1.5)$$

or $$x = \frac{-9R + \sqrt{145R^2}}{4} \approx \frac{-9R + 12R}{4}$$

or $$x \approx \frac{3}{4}R$$

Example 201

A parallel beam of light falls successively on a thin convex lens of focal length 40 cm and then on a thin convex lens of focal length 10 cm as shown in Fig. 1E.201 (a).

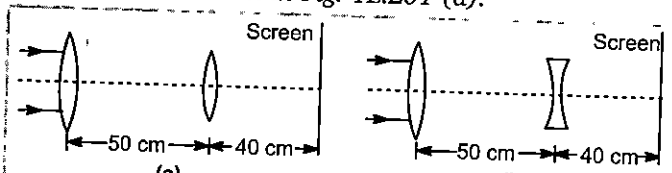


Fig. 1E.201

In Fig. 1E.201 (b) the second lens is an equiconcave lens of focal length 10 cm and made of material of refractive index 1.5. In both the cases, the second lens has an aperture equal to 1 cm.

(a) Compare the area illuminated by the beam of light on the screen which passes through the second lens in the two cases.

Now a liquid of refractive index μ is filled to the right of the second lens in case B such that the area illuminated in both the cases is the same.

(b) Determine the refractive index of the liquid.

Solution: In case (*A*), the incident parallel beam emerges as a parallel beam.

$$\therefore \qquad A_1 = \pi(1)^2 = \pi\,\text{cm}^2$$

In case (*B*), Let x be the radius of the area illuminated. Then

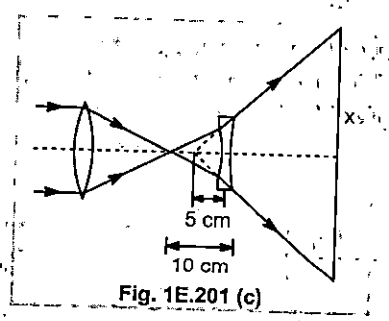


Fig. 1E.201 (c)

$$\frac{x}{45} = \frac{1}{5} \Rightarrow x = 9\,\text{cm}$$

$$\therefore \qquad A_2 = \pi(9)^2 = 81\pi\,\text{cm}^2$$

Thus, $$\frac{A_2}{A_1} = 81$$

When liquid of refractive index μ is filled to the right of this lens, the first surface of the lens (radius of curvature = 10 cm) forms the image at the object only. Considering the refraction at the second surface,

$$\frac{\mu}{\infty} - \frac{1.5}{-10} = \frac{\mu - 1.5}{10} \qquad (\because \text{same area} \Rightarrow v \to \infty)$$

$$\Rightarrow \qquad \mu = 3$$

Example 202

An equiconvex lens of focal length 20 cm and refractive index 3/2, is cut into two equal parts and one part L_2 is silvered and placed as shown. Another concave-convex lens L_1 (radii 10 cm and 20 cm, refractive index = 3/2) is placed with its principal axis parallel to that of L_2 as shown in the figure. A point object is placed at a distance of 80 cm from the lens on its principal axis as shown in the figure. Find the position of the final image.

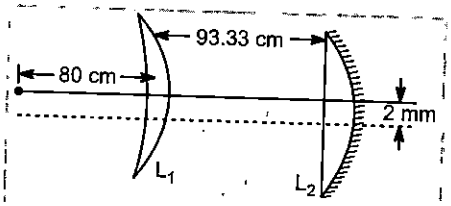


Fig. 1E.202 (a)

Solution: Due to refraction from the first lens, the image will be at 80 cm right and on the principal axis of the lens (since its focal length is 40 cm).

A silvered lens will behave as an effective mirror of power

$$p = 2P_l + P_m$$

$$= 2\frac{1}{f_l} - \frac{1}{f_m} = \frac{2}{f_l} - \frac{2}{R}$$

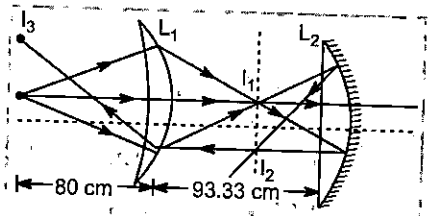


Fig. 1E.202 (b)

For R (using lens formula),

$$\frac{1}{20} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \qquad R = 20\,\text{cm}$$

$$\therefore \qquad P = \frac{2}{40} - \frac{2}{-20} = \frac{3}{20}$$

$$\therefore \qquad F_m = -\frac{1}{P} = -\frac{20}{3} = -6.67$$

Now image due to first lens is at I_1 which is in the plane passing through the centre of the effective mirror. Hence, due to reflection from the effective mirror, position of the

image will be at the same position as I_1 but 4 mm below the principal axis of the lens L_1. Again, due to refraction from the lens, position of the image is above the object at 4 mm.

Hence, I_1 is the image due to refraction through lens L_1 and I_2 is the image due to reflection through L_2, and I_3 is final image due to refraction through lens L_1.

Example 203

A convex lens of focal length 15 cm is split into two halves and the two halves are placed at a separation of 120 cm. Between the two halves of the convex lens a plane mirror is placed horizontally and at a distance of 4 mm below the principal axis of the lens halves. An object of length 2 mm is placed at a distance of 20 cm from one half lens as shown in Fig. 1E.203 (a).

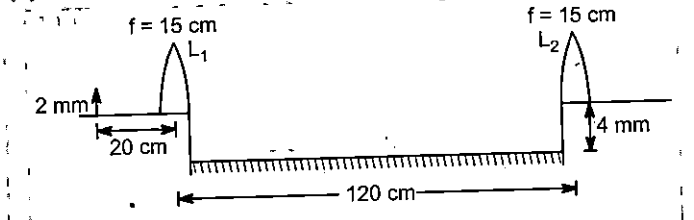


Fig. 1E.203 (a)

(a) Find the position and size of the final image.

(b) Trace the path of the rays forming the image.

Solution: (a) For refraction at lens L_1,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-20}=\frac{1}{+15}$$

$$v = 60\,\text{cm}$$

Magnification, $m=\frac{v}{u}=\frac{60}{-20}=-3$

Hence, the image is inverted.

$AC = 2\,\text{mm}$, $AB = (4/3)\,\text{mm}$, $BC = (2/3)\,\text{mm}$

$$\therefore \quad A'C' = \frac{60}{20}\times 2 = 6\,\text{mm}$$

$$A'B' = 4\,\text{mm},\quad B'C' = 2\,\text{mm}$$

The image is formed by the first half lens as shown in Fig. 1E.203 (b).

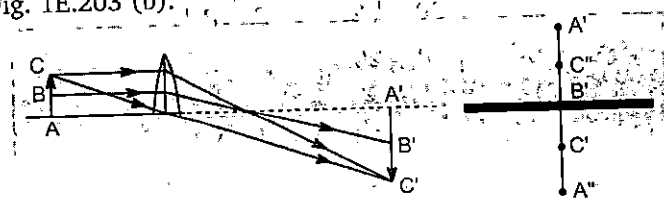


Fig. 1E.203 (b)

The image formed by the plane mirror is $A''C''$ as shown in Fig. 1E.203 (c).

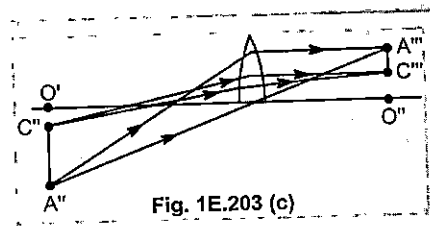


Fig. 1E.203 (c)

For the lens L_2, $u = -60$, $f = +15$

$$\therefore \quad \frac{1}{v}-\frac{1}{-60}=\frac{1}{15}$$

$$\Rightarrow \quad v = +20\,\text{cm}$$

$$\therefore \quad m=\frac{v}{u}=\frac{20}{-60}=\frac{-1}{3}$$

Negative magnification shows that the image $A'''C'''$ of $A''C''$ by the lens is inverted as compared to $A''C''$

$$A'''C''' = \frac{1}{3}.A''C'' = 2\,\text{mm}$$

$$\frac{O''C'''}{O'C''}=\frac{-1}{3}\Rightarrow O''C'''=\frac{-1}{3}O'C''=\frac{2}{3}\,\text{mm}$$

$\therefore$ Final image is at a distance of 20 cm behind the second half lens and at a distance of (2/3) mm above the principal axis. The size of the image is 2 mm and is inverted as compared to the given object.

(b) Ray diagram for the formation of image:

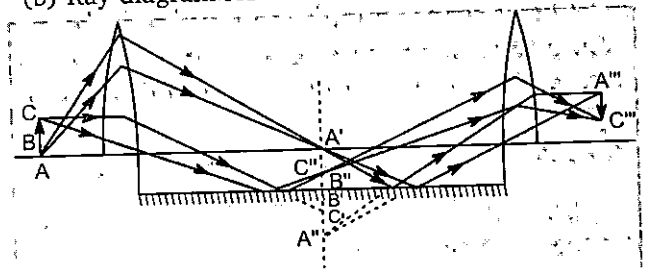


Fig. 1E.203 (d)

Example 204

Two identical plano-convex lenses L_1 ($\mu_1 = 1.4$) and L_2 ($\mu_2 = 1.5$) of radii of curvature $R = 20$ cm are placed as shown in fig. 1E.204 [a(i)].

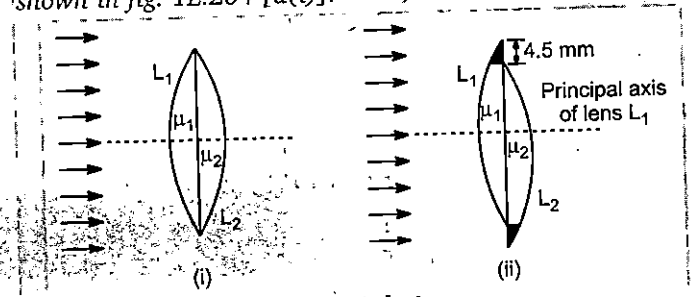


Fig. 1E.204 (a)

(a) Find the position of the image of the parallel beam of light relative to the common principal axis.

(b) Now the second lens is shifted vertically downward by a small distance 4.5 mm and the extended parts of L_1 and L_2 are blackened as shown in Fig.1E.204 [a(ii)]. Find the new position of the image of the parallel beam of light relative to the principal axis of L_1.

Solution: (a) Focal lengths of lenses L_1 and L_2 are respectively given by

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \qquad \Rightarrow f_1 = 50\,\text{cm}$$

$$\frac{1}{f_2}(\mu_2 - 1)\left[\frac{1}{\infty} - \left(-\frac{1}{R}\right)\right] \qquad \Rightarrow f_2 = 40\,\text{cm}$$

The equivalent focal length f of the combination is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow f = \frac{200}{9}\,\text{cm}$$

Hence, the image of the parallel beam is formed on the common principal axis at a distance of 22.22 cm from the combination on the right side.

(b) Image formed by L_1 is at a distance of 50 cm behind the lens. This image lies on the principal axis of L_1 and will act as an object for L_2.

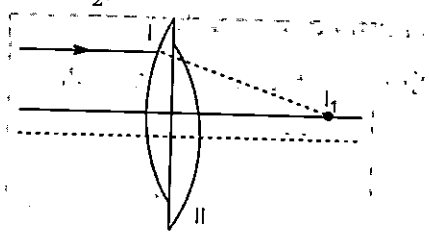


Fig. 1E.204 (b)

For L_2, object distance, $u = +50\,\text{cm}$

$$f_2 = +40\,\text{cm}$$

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{200}{9}\,\text{cm}$$

Magnification caused by L_2,

$$m = \frac{v}{u} = \frac{4}{9}$$

For L_2, object I_1 is at a distance of 4.5 mm above its principal axis.

Hence, distance of image I_2 of the object (virtual) I_2 is at a distance $\frac{4}{9} \times 4.5 = 2\,\text{mm}$ above the principal axis of L_2

[$\because$ height of image $= m \times$ height of object].

Hence, final image is at a distance of 22.22 cm behind the combination at a distance of 2.5 mm below the principal axis of L_1.

Example 205

Near a prism, a convex lens of focal length 12 cm and an object O have been placed as shown in the figure. The lens aperture has a radius 1 cm. The refractive index of the material of the prism is $\sqrt{3}$. The object is placed such that its image formed by the lens is virtual. The rays of light, after getting refracted through the lens, pass through the prism and fall on face PR of the prism.

What is the greatest distance from the lens at which the object should be placed such that all the rays which are getting refracted by the lens and falling on the face PR of the prism get totally internally reflected ? It is given that $\angle QPR = \frac{\pi}{6} + \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and the lens is thin.

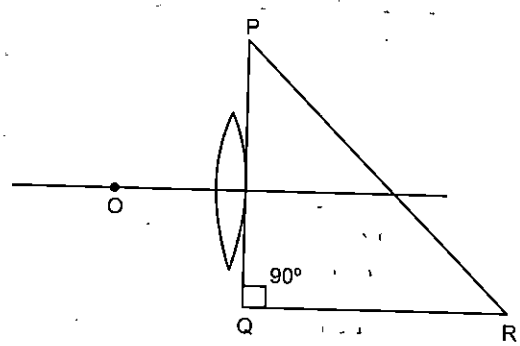


Fig. 1E.205 (a)

Solution: The critical ray is shown. The lens forms the image of O at I. From the figure we see

$$r + C = P \qquad \ldots(1)$$

Critical angle for the prism is

$$C = \sin^{-1}\frac{1}{\sqrt{3}}$$

and $$P = \frac{\pi}{6} + \sin^{-1}\frac{1}{\sqrt{3}}$$

From eqn. (1), we get

$$r = \frac{\pi}{6}$$

Now $$\frac{\sin i}{\sin r} = \sqrt{3}$$

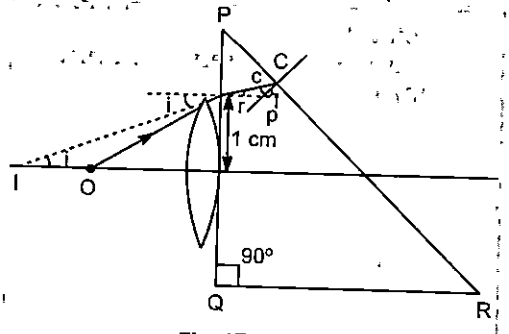


Fig. 1E.205 (b)

Hence, $\sin i = \sqrt{3}\cdot\sin\frac{\pi}{6} \Rightarrow i = 60°$

Image distance $v = 1\cdot\cot 60° = \frac{1}{\sqrt{3}}$ cm

Now $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow -\sqrt{3}-\frac{1}{u}=\frac{1}{12}$

$\Rightarrow u = \frac{-12}{1+12\sqrt{3}}$ cm

$\Rightarrow |u| = \frac{12}{1+12\sqrt{3}}$ cm

Example 206

A thin equiconvex lens of glass ($\mu = 1.5$) having a focal length of 30 cm in air is placed at a distance of 10 cm from a plane mirror, which, in turn, is placed with its plane perpendicular to the optic axis of the lens. Water ($\mu = 4/3$) fills the space between the lens and the mirror. A parallel beam of light is incident on to the lens parallel to the principal axis.

(a) Find the position of the final image w.r.t. the optical centre of the lens.

(b) If the mirror is rotated by 1°, as shown in the figure, find the displacement of the image.

Solution: (a) The focal length of the glass lens is 30 cm.

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \frac{2(\mu-1)}{R}$$

The radius of curvature, $R = 30$ cm.

The object distance, $u = \infty$.

Now we apply Gauss's law at surface S_1 and S_2.

$$\frac{3/2}{v_1}-\frac{1}{u}=\frac{\frac{3}{2}-1}{30} \quad \ldots(1)$$

$$\frac{\frac{4}{3}}{v}-\frac{\frac{3}{2}}{v_1}=\frac{-1/6}{-30} \quad \ldots(2)$$

$\Rightarrow v = 60$ cm

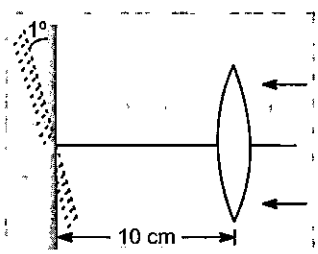


Fig. 1E.206

After reflection from the mirror, the light rays appear to converge to a point 40 cm to the right of the convex lens. This serves as a virtual object for the lens : $u = +40$ cm

$$\frac{\frac{3}{2}}{v_1}-\frac{\frac{4}{3}}{+40}=\frac{\frac{1}{6}}{30}$$

$$\frac{\frac{3}{2}}{v_1}-\frac{1}{v}=\frac{-\frac{1}{2}}{-30}$$

$\Rightarrow v = 18$ cm to the right of the convex lens.

(b) If the mirror is rotated by 1°, the reflected ray rotates by 2°. The virtual object for the lens formed by the reflection from the mirror is displaced by :

$$\Delta y_1 = 50\times\frac{2\pi}{180}\text{ cm}$$

The magnification due to the refraction at the two surfaces of the lens is

$$m = m_1 m_2 = \left(\frac{v}{\mu_3}\Big/\frac{v_1}{\mu_2}\right)\times\left(\frac{v_1}{\mu_2}\Big/\frac{u}{\mu_1}\right)$$

$$= \left(\frac{v}{\mu_3}\Big/\frac{u}{\mu_1}\right) = \frac{18}{1}\Big/\frac{40}{(4/3)} = \frac{18}{30}$$

The displacement of the final image is

$$\frac{18}{30}\times 50\times\frac{2\pi}{180}\text{ cm} = \frac{\pi}{3}\text{ cm}$$

Example 207

A hollow sphere of glass of inner and outer radii R and 2R respectively has a small mark on its inner surface. This mark is observed from a point outside the sphere such that the centre of the sphere lies in between. Prove that the mark will appear nearer than it really is, by a distance $\frac{(\mu-1)R}{(3\mu-1)}$, where R is the radius of the inner surface.

Solution: Refraction at surface 2,

$$\frac{\mu}{v}+\frac{1}{2R}=\frac{\mu-1}{-R}$$

or $$\frac{\mu}{v} = -\frac{1}{R}\left[(\mu-1)+\frac{1}{2}\right]$$

$$= -\frac{1}{R}\left[\frac{2\mu-1}{2}\right] = \frac{-(2\mu-1)}{2R}$$

or $$v = -\left[\frac{2\mu R}{2\mu-1}\right]$$

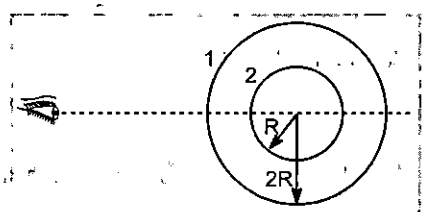


Fig. 1E.207

For surface 1

$$u = -\left(R+\frac{2\mu R}{2\mu-1}\right) = -\left(\frac{4\mu-1}{2\mu-1}\right)R$$

$$\frac{1}{v}+\frac{\mu(2\mu-1)}{(4\mu-1)R}=\frac{1-\mu}{-2R}$$

$$\frac{1}{v} = -\frac{1}{R}\left[\frac{1-\mu}{2} + \frac{\mu(2\mu-1)}{(4\mu-1)}\right]$$

$$= -\frac{1}{R}\left[\frac{4\mu - 1 - 4\mu^2 + \mu + 4\mu^2 - 2\mu}{2(4\mu-1)}\right]$$

$$= -\frac{1}{R}\left[\frac{3\mu-1}{2(4\mu-1)}\right]$$

or $$v = -\frac{2R(4\mu-1)}{(3\mu-1)}$$

∴ Distance between the final image and object,

$$3R - \frac{2R(4\mu-1)}{(3\mu-1)} = R\left[\frac{9\mu - 3 - 8\mu + 2}{3\mu - 1}\right] = \frac{(\mu-1)R}{(3\mu-1)}$$

Example 208

A glass wedge with a small angle of refraction α is placed at a certain distance from a convergent lens with a focal length f, one surface of the wedge being perpendicular to the optical axis of the lens. A point source of light is on the other side of the lens in its focus. The rays reflected from the wedge produce, after refraction in the lens, two images of the source displaced with respect to each other by d. Find the refraction index of the wedge glass.

Solution: Two cases are possible :

(1) The optical axis of the lens is perpendicular to the front face of the wedge. The rays reflected from the front face pass through the lens and produce an image of the point source that coincides with the source itself. The rays reflected from the rear face will be deflected by an angle ϕ [Fig. 1E.208 (a)] determined by the equality $\frac{\sin\phi}{\sin 2\alpha} = n$. Since the angles are small, $\phi \cong 2\alpha n$.

The second image of the source will be obtained at a distance d from the first image, namely, $d = f\phi = f \cdot 2\alpha n$.

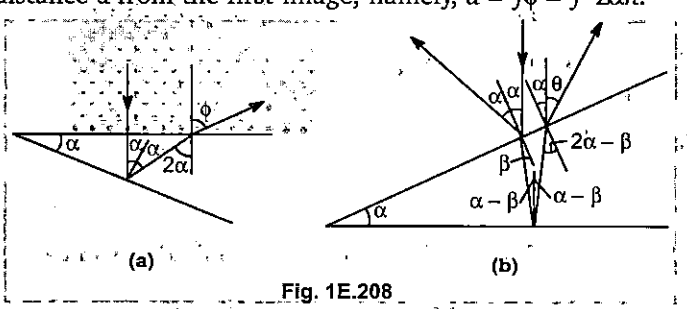


Fig. 1E.208

Hence, $$n = \frac{d}{2\alpha f}$$

(2) The optical axis of the lens is perpendicular to the rear face of the wedge. The rays reflected from the front face will be deflected by an angle 2α [Fig. 1E.208 (b)] and produce an image at a distance of $d_1 = 2\alpha f$ from the source.

The rays reflected from the rear face will be deflected by an angle θ determined from the equations :

$$\frac{\sin\alpha}{\sin\beta} = n \text{ and } \frac{\sin(\alpha+\theta)}{\sin(2\alpha-\beta)} = n$$

When the angles are small, $\theta = 2\alpha(n-1)$. For this reason the second image will be at a distance $d_2 = 2\alpha(n-1)f$ from the source. The total distance between the images is $d = d_1 + d_2 = 2\alpha n f$.

Hence, $n = \frac{d}{2\alpha f}$ as in the first case.

Example 209

A concave mirror has the form of a hemisphere with a radius of $R = 55\,cm$. A thin layer of an unknown transparent liquid is poured into this mirror, and it was found that the given optical system produces, with the source in a certain position, two real images, one of which coincides with the source and the other is at a distance of $l = 30\,cm$ from it. Find the refraction index n of the liquid.

Solution: Since the image that coincides with the source is formed owing to reflection from the part of the mirror not covered by the liquid, the source is obviously arranged at centre O of the hemisphere. Let us find the position of the other image (point A in Fig. 1E.209). According to the law of refraction,

$$\frac{\sin\alpha}{\sin\beta} = n \cong \frac{\alpha}{\beta} \text{ and } \frac{\sin\phi}{\sin\theta} = n \cong \frac{\phi}{\theta}$$

As can be seen from the drawing, $\theta = \beta + 2\gamma$, where $\gamma = \alpha - \beta$ is the angle of incidence of the refracted ray on the mirror and $(R - l - h)\tan\phi \cong (R - h)\tan\alpha$.

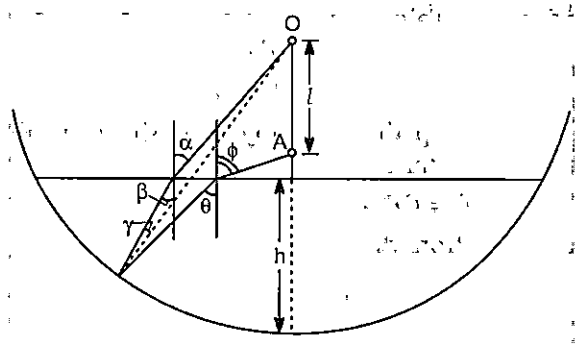


Fig. 1E.209

Neglecting h as compared with R, we can find from this system of equations that

$$n = \frac{2R - l}{2(R - l)} = 1.6$$

Example 210

A narrow parallel beam of light rays is incident on a transparent sphere with a radius R and a refraction index n in the direction of one of the diameters. At what distance f from the centre of the sphere will the rays be focussed ?

Solution: Two cases are possible: the focus is outside the sphere, and inside it. Let us consider the first case. The path of the ray incident on the sphere at an angle i is shown in Fig. 1E.210.

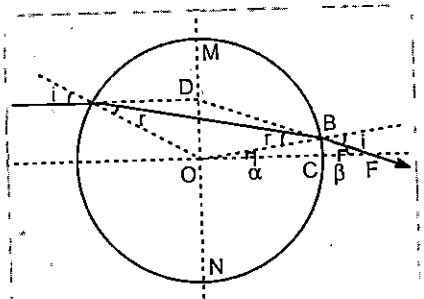


Fig. 1E.210

Bearing in mind that the angles i and r are small, we have, in accordance with the condition of the problem,

$$BC = R\sin\alpha = R\sin(2r - i)$$
$$\cong R(2r - i) \cong \frac{Ri}{n}(2 - n)$$

The focus obviously lies outside the sphere when $n < 2$ and on the surface of the sphere if $n = 2$.

The distance

$$CF = BC\cot\beta \cong \frac{BC}{\beta}$$

and $$\beta \cong 2(i - r) \approx \frac{2i(n-1)}{n}$$

as can easily be found from Fig. 1E.205.

The sought distance is

$$f = R + CF = \frac{Rn}{2(n-1)}$$

When $n > 2$, the path of the ray is as shown in the figure. The sought distance is

$$f' = OF \cong CF - R$$

As can be seen from the figure,

$$CF = AC\cot(i - r) \cong \frac{AC}{i - r} \cong \frac{Ri}{i - r}$$

Hence, $$f' = \frac{R}{n-1}$$

Example 211

Find the position of the main planes of a transparent sphere used as a lens.

Solution: Let us extend ray BF until it intersects the continuation of the ray incident on the sphere parallel to the optical axis (Fig. 1E.211). It can easily be seen that section DO that connects the point of intersection with the centre of the sphere forms a right angle with the direction of the incident ray. Triangle ODF is a right one, since

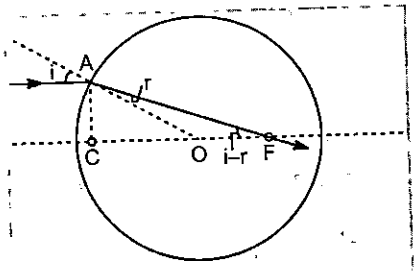


Fig. 1E.211

$$OFB \cong \frac{R}{2}\frac{n}{n-1}\frac{2i(n-1)}{n} = Ri$$

For this reason the main planes of sphere MN coincide and pass through its centre.

Example 212

A camera with an objective of 8 dioptres focal power is used to take photographs of an object lying on the bottom of a pond 1.2 m deep. What is the distance of the film from the objective lens ? The lens is placed close to the surface of the water.

Solution: First one should estimate the apparent depth of the pond d_1 (Fig. 1E.212).

$$d_1 = l\cot\beta,\ d = l\cot\alpha$$

and $$\frac{d_1}{d} = \frac{\tan\alpha}{\tan\beta}$$

For small angles $\tan\alpha \approx \sin\alpha$,

So $$d_1 = \frac{d\sin\alpha}{\sin\beta} = \frac{d}{n}.$$

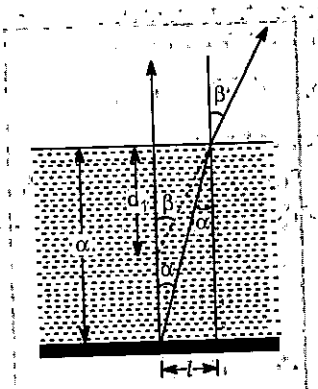


Fig. 1E.212

From the thin lens formula, we obtain $\frac{1}{d_1} + \frac{1}{d'} = \frac{1}{f}$. Therefore the required distance between the objective lens and the film is $d' = df/(d - nf)$.

Example 213

An object AB is at a distance of a = 36 cm from a lens with a focal length of f = 30 cm. A flat mirror turned through 45° with respect to the optical axis of the lens is placed behind it at a distance of l = 1 metre (Fig. 1E.213)

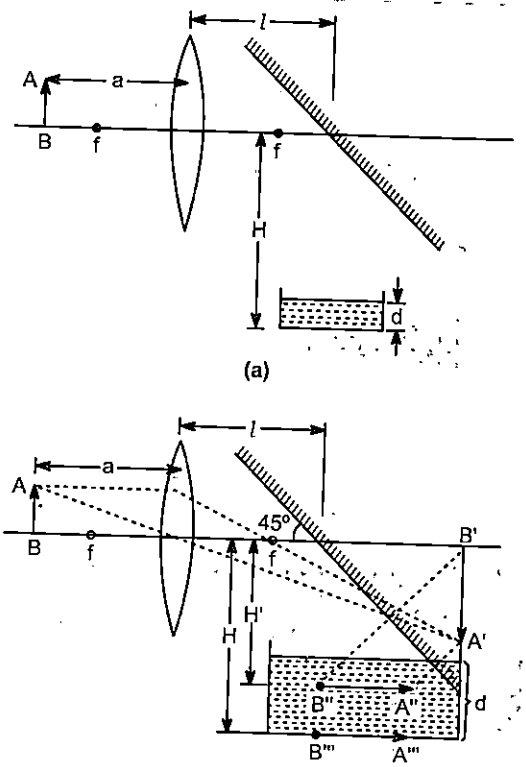


Fig. 1E.213

At what distance H from the optical axis should the bottom of a tray with water be placed to obtain a sharp image of the object on the bottom ? The thickness of the water layer in the tray is d = 20 cm.

Solution: In the absence of the mirror, the image $A'B'$ of object AB [Fig. 1E.213(b)] will be obtained at a distance of $b = \frac{af}{a-f} = 180\,\text{cm}$ from the lens. After being reflected from the mirror, the image will occupy position $A''B''$ and will be at a distance of $H' = b - l = 80\,\text{cm}$ from the optical axis. The layer of water with a thickness of d will shift the image over a distance of $H - H' = d\left(1 - \frac{1}{n}\right)$, where n is the refraction index of water.

Hence, $H = H' + d\left(1 - \frac{1}{n}\right) = 85\,\text{cm}$

Example 214

A converging bundle of light rays in the shape of a cone with the vertex angle of 40° falls on a circular diaphragm of 20 cm diameter. A lens with a focal power of 5 dioptres is fixed in the diaphragm. What will be the new cone angle ?

Solution: The path of the rays is shown in Fig.1E.214. To find x and h one should make use of the similarity of triangles.

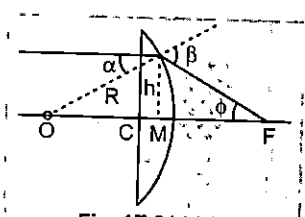


Fig. 1E.214

We pass the ray $ON \| AB$ through the centre of the lens until it intersects with the focal plane. In this case the ray AN is the continuation of the ray MA after its refraction in the lens. The points D and B are conjugate; for instance, B is the virtual image of point D. Hence, denoting $OB = a_1$, $OD = a_2$, we obtain $-\frac{1}{a_1} + \frac{1}{a_2} = \frac{1}{f}$. But $a_1 = R\cot\alpha$, $a_2 = R\cot\beta$, where R is the semidiameter of the lens and we obtain

$$\tan\beta = \tan\alpha + \frac{R}{f}$$

Example 215

The martian year is k = 1.9 times longer than Earth's. A collecting lens on Earth produces an image of Sun on Earth with diameter d_1 = 4 mm. Find the diameter of Sun's image produced by the same lens on Mars. Assume planets' orbiot are circular.

Solution: $\frac{GM_sM_e}{r^2} = \frac{M_e v^2}{r}$

r = distance between Sun and Earth

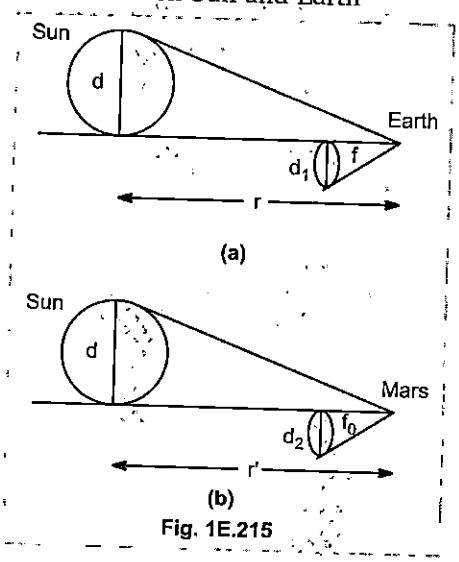


Fig. 1E.215

$$v = \sqrt{\frac{GM_s}{r}} \qquad \therefore \quad v \propto \frac{1}{\sqrt{r}}$$

$$T = \frac{2\pi r}{v} \propto (r)^{\frac{3}{2}} \qquad ...(1)$$

for Martin $\quad 1.9T \propto (r')^{\frac{3}{2}} \qquad ...(2)$

$$1.9 \propto \left(\frac{r'}{r}\right)^{\frac{3}{2}}$$

Earth $\quad \frac{d}{d_1} = \frac{r}{f} \qquad (A)$

$$\frac{d}{d_2} = \frac{r'}{f} \qquad (B)$$

From eqn. (A) and (B)

$$\frac{d_2}{d_1} = \frac{r}{r'}$$

$$\therefore \quad d_2 = d_1\left(\frac{r}{r'}\right) = 4 \times \left(\frac{1}{1.9}\right)^{2/3}$$

$$= 4 \times 0.652 = 2.6 \text{ mm}$$

Example 216

(a) Compare the longitudinal and the lateral magnifications of a thin lens. Consider the case of small longitudinal dimensions of the object.

(b) A ball is placed at a distance double the focal length from the lens on its axis. What will be the shape of its image ?

Solution: (a) The lateral magnification is

$$\beta = \frac{h'}{h} = \frac{a'}{a} = \frac{f}{a-f}$$

The longitudinal magnification is $\alpha = x'/x$. To calculate it, write the thin lens formula in the form

$$\frac{1}{a-x} + \frac{1}{a'+x'} = \frac{1}{f},$$

hence $\quad a' + x' = \frac{f(a-x)}{a-f-x}$

But $\quad a' = af/(a-f)$; therefore

$$x' = \frac{f(a-x)}{a-f-x} - \frac{af}{a-f} = \frac{xf^2}{(a-f)(a-f-x)}$$

Hence the longitudinal magnification is

$$\alpha = \frac{x'}{x} = \frac{f^2}{(a-f)(a-f-x)}$$

$$= \frac{\beta^2(a-f)}{a-f-x} = \frac{\beta^2}{1-x/(a-f)}$$

For small longitudinal dimensions of the body ($x << a-f$) the longitudinal magnification is $\alpha = \beta^2$.

(b) For $a = 2f$ the lateral magnification is $\beta = 1$ and the longitudinal magnification is $\alpha = \frac{1}{1-r/f}$, where r is the radius of the ball.

As can be seen, the longitudinal dimension of the image is greater than the lateral dimension, so the ball will look like an elongated rotational ellipsoid.

Example 217

At present the best sprinters run 100 m in 10 s. What is the appropriate exposure in making snapshots, if the blurring on the negative must not exceed $x = 0.5$ mm ? The snapshots are made at a distance of $d = 6$m, the focal power of the photographic objective is $\phi = 20$ dioptres.

Solution: The ratio of the velocity of the ray across the negative to the velocity of the runner is the same as the ratio of the dimensions of the image to that of the object. Denoting the lateral magnification by β, we obtain

$$v_{neg} = \beta v = v\frac{d'}{d} = v\frac{f}{d} = \frac{v}{\phi d},$$

since when the focal power of the objective is high, the image will lie practically in the focal plane. The exposure time is found by dividing the blurring of the image by the velocity obtained :

$$\tau = \frac{x}{v_{neg}} = \frac{x\phi d}{v}$$

Example 218

Landscape shots are made with a camera having an objective of focal power 7.7 dioptres. The camera is focussed on objects 12 m away from it. It is desired to obtain a sufficiently clear image of objects within a distance of 3m infront and behind–their blurring on the negative should not exceed 0.2 mm. What should be the setting (i.e. diameter) of the diaphragm ? What will be the aperture of the objective at this setting ?

Solution: A schematic diagram of the path of the rays is shown in Fig. 1E.218. Suppose the image of a point A on the negative is sharp.

Then the image on the negative of point B which is closer to the lens will be a small dot of width x. Let the distance from point A to the lens be a, and the distance from point B to the lens be b.

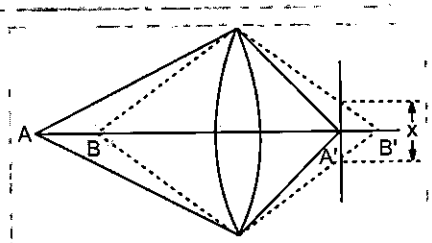


Fig. 1E.218

Then $a' = \dfrac{af}{a-f}$ and $b' = \dfrac{bf}{b-f}$. Obviously $\dfrac{x}{\phi} = \dfrac{b'-a'}{b'}$, where ϕ is the diameter of the diaphragm.

Hence $$\phi = \frac{xb'}{b'-a'} = \frac{bx(a-f)}{f(a-b)} = \frac{bx(a\phi-1)}{a-b}$$

CHROMATIC ABERRATION

Consider a parallel beam of white light incident on a thin convex lens as shown in Fig. 1.226. Since blue light gets refracted more than red light, the point at which the blue light would focus is nearer the lens than the point at which the red light would focus. Thus, the image will appear to be coloured.

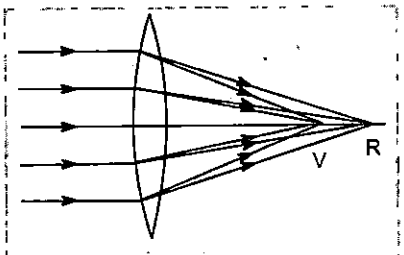


Fig. 1.226

The focal length of a thin lens is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(1)$$

If a change of n by δn (the change of n is due to the change in the wavelength of the light) results in a change of f by δf then we obtain by differentiating Eq. (1)

$$-\frac{\delta f}{f^2} = \delta n\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\delta n}{n-1}\frac{1}{f}$$

i.e., $$\delta f = -f\frac{\delta n}{n-1} \qquad ...(2)$$

which represents the chromatic aberration of a thin lens. If n_V and n_R represent the refractive indices for the blue and red colours respectively, then

$$f_R - f_V = f\left(\frac{n_V - n_R}{n-1}\right) \qquad ...(3)$$

would represent the chromatic aberration.

From the definition of dispersive power,

$$\omega = \frac{\delta n}{n-1}$$

Thus $$\delta f = -\omega f$$

Condition of Achromatism

In case of two thin lenses in contact,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad i.e., \quad -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if $dF = 0$

i.e., $$\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

which in the light of eqn. (5) reduces to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0, \; i.e., \; \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

❒ Note that for an achromatic combination two lenses must be of different material, for

$$\omega_1 = \omega_2, \frac{1}{f_1} + \frac{1}{f_2} = 0$$

or $$\frac{1}{F} = 0$$

or $$F = \infty$$

The combination will behave as a glass slab.

❒ Dispersive powers ω_1 and ω_2 are positive numbers, the condition for achromatism will hold when f_1 and f_2 have opposite sign. That means if one of the lenses is convex, the other must be concave.

Removal of Chromatic Aberration of a Separated Doublet

Let us consider two thin lenses of focal lengths f and f' and separated by a distance t (see Fig. 1.227). The focal length of the combination F would be

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'} - \frac{t}{ff'} \qquad ...(1)$$

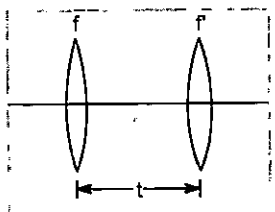


Fig. 1.227

The focal length of the first lens would be given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(2)$$

With a similar expression for $1/f'$. If Δf and Δn represent the changes in the focal length and in the refractive index due to a change $\Delta\lambda$ in the wavelength, then by differentiating eqn. (2), we obtain

$$-\frac{\Delta f}{f^2} = \Delta n\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\Delta n}{(n-1)f}$$

Thus, differentiating Eq. (1), we obtain

$$-\frac{\Delta F}{F^2} = \frac{\Delta f}{f^2} - \frac{\Delta f'}{f'^2} + \frac{t}{f}\frac{\Delta f'}{f'^2} + \frac{t}{f'}\frac{\Delta f}{f^2}$$

$$= \frac{\Delta n}{(n-1)f} + \frac{\Delta n'}{(n'-1)f'} - \frac{t}{f}\frac{\Delta n'}{(n'-1)f'} - \frac{t}{f'}\frac{\Delta n}{(n-1)f}$$

$$= \frac{\omega}{f} + \frac{\omega'}{f'} - \frac{t}{ff'}(\omega + \omega')$$

where, as before, ω and ω' represent the dispersive powers. Consequently, for the combination to have the same focal length for blue and red colours we should have

$$\frac{t(\omega+\omega')}{ff'} = \frac{\omega}{f} + \frac{\omega'}{f'}$$

or $$t = \frac{\omega f' + \omega' f}{\omega + \omega'}$$

If both the lenses are made of the same material, then $\omega = \omega'$ and the above equation simplifies to

$$t = \frac{f + f'}{2}$$

implying that the chromatic aberration is very small if the distance between the two lenses is equal to the mean of the focal lengths. This is indeed the case for the Huygens' eyepiece.

Example 219

A point object is placed infront of a plane refracting surface at a distance x_0 from it. If a light ray from the objects meets the refracting surface at a height h as shown in Fig. 1E.219, find the spherical aberration produced by the plane surface.

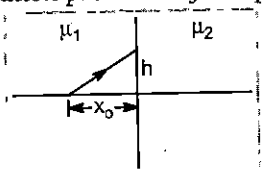


Fig. 1E.219 (a)

Solution: Using Snell's law,

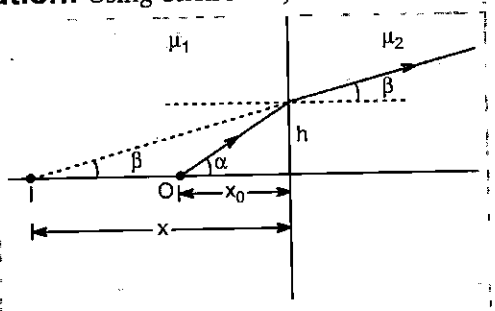


Fig. 1E.219 (b)

$$\mu_1 \sin\alpha = \mu_2 \sin\beta$$

Also, $$x = h\cot\beta = \frac{h}{\sin\beta}\sqrt{1-\sin^2\beta}$$

or $$x = \frac{\mu_2 h}{\mu_1 \sin\alpha}\sqrt{1-\left(\frac{\mu_1}{\mu_2}\right)^2 \sin^2\alpha}$$

From the figure, $$\sin\alpha = \frac{h}{\sqrt{h^2 + x_0^2}}$$

$$\therefore \quad x = \frac{\mu_2\sqrt{h^2+x_0^2}}{\mu_1}\left[1-\left(\frac{\mu_1}{\mu_2}\right)^2 \frac{h^2}{h^2+x_0^2}\right]^{1/2}$$

or $$x = \frac{\mu_2}{\mu_1}x_0\left(1+\frac{h^2}{x_0^2}\right)^{1/2}\left[1-\left(\frac{\mu_1}{\mu_2}\right)^2\left(\frac{h}{x_0}\right)^2\left(\frac{1}{\frac{h^2}{x_0^2}+1}\right)\right]^{1/2}$$

$$x = \frac{\mu_2}{\mu_1}x_0\left[1+\frac{h^2}{x_0^2}-\left(\frac{\mu_1}{\mu_2}\right)^2\frac{h^2}{x_0^2}\right]^{1/2}$$

$$= \frac{\mu_2}{\mu_1}x_0\left[1+\frac{h^2}{2x_0^2}\left(1-\frac{\mu_1^2}{\mu_2^2}\right)\right]$$

For paraxial ray approximation, $h \to 0$

$$x' = \frac{\mu_2}{\mu_1}x_0$$

Thus, $$x = x'\left[1+\frac{h^2}{2x_0^2}\left(1-\frac{\mu_1^2}{\mu_2^2}\right)\right]$$

Spherical aberration $= x - x' = \frac{1}{2}\left(\frac{\mu_2}{\mu_1}\right)\frac{h^2}{x_0}\left[1-\left(\frac{\mu_1}{\mu_2}\right)^2\right]$

*OPTICAL INSTRUMENTS

Human Eye; Corrective Lenses

The human eye behaves like an optical instrument. It has a diaphragm, called **iris**, that adjusts automatically to control the amount of light entering the eye. The eye produces an inverted real image of objects on the retina. The brain then performs inversion of the image to produce the image that we perceive in its proper orientation. The **retina** of eye plays the role of a camera film. It has two types of cells : **rods** which detect low levels of light and **cones** which detect colour and bright light conditions in night.

The light is refracted at the front surface of the **cornea.** The **ciliary muscles** accomplish fine adjustment for focussing at different distances. Ciliary muscles adjust the curvature of the lens, thus changing of the focal length.

To focus on a distant object the muscles are relaxed and the lens is thin [Fig. 1.228(a)], the parallel rays focus at the focal point (on the retina). To focus on a near object, the muscles contract, the eye lens thickens, thus shortening the focal length so that the image can be focussed on the retina.

Myopia (Nearsightedness)

A myopic eye can focus only on nearby objects, there is inability to see distant objects clearly. Even with a relaxed ciliary muscles the focal length of the eye lens is not large enough to image distant object on the retina. For a myopic eye the image of distant object is located between the eye

* Note that this article is not IIT-JEE syllabus

lens and the retina. A corrective lens makes the final image of the lens and optical system appear on retina.

The farthest object that can be seen clearly by a myopic person is called **far point** d_{far}, for a normal person the far point is taken as infinity.

A corrective lens will place the image of distant objects at the far point of the eye. The image of the lens becomes an object for the eye. If the object lies within the far point, it will be clearly visible. The corrective lens creates a *virtual image* of a distant object that acts as a *virtual object* for the eye.

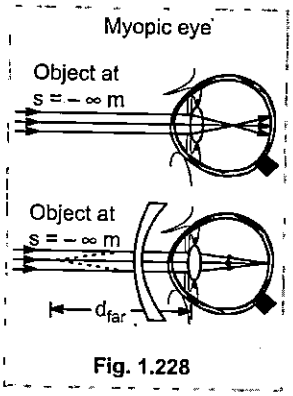


Fig. 1.228

Applying the lens equation

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}, \qquad \ldots (1)$$

$$u=-\infty, \quad v=-d_{far}$$

Negative sign implies that the image is virtual.

Thus $$\frac{1}{-d_{far}}-\frac{1}{-\infty}=\frac{1}{f}$$

or $$f=-d_{far}$$

Hence a diverging lens ($f < 0$) whose focal length is equal in magnitude to the far point is needed to correct myopia. Opthalmolegists quote the focal length in *dioptre* (dp).

$$1\text{dioptre}=\frac{1}{f\,(\text{in metre})}$$

Hyperopia (Farsightedness)

Hyperopia refers to the inability to focus on nearby objects although distant objects are seen clearly. Even the simple task of reading can only be accomplished by placing the book far from the eye. In this case the eye produces an image of a nearby object behind the retina.

The closest distance at which the eye can focus clearly is called the **near point.** A normal eye has a near point of 25 cm.

Let us denote the near point by d_{near}, it is the closest distance from the unaided eye that can be seen clearly. The corrective lens creates a virtual image of the reading book at $-d_{near}$, this virtual image then becomes a virtual object for the eye. For the eye the virtual object is far away and the image produced by the eye is sharp on the retina.

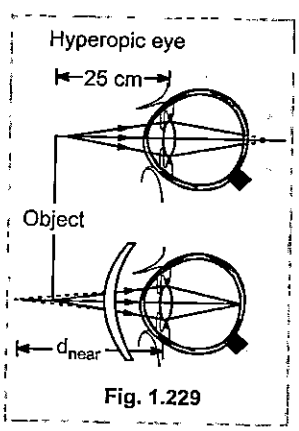


Fig. 1.229

From the lens equation

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}, \quad \frac{1}{-d_{near}}-\frac{1}{-d}=\frac{1}{f}$$

Here d is the convenient distance of object.

As $d < d_{near}$, the focal length is positive. Thus a converging lens is used to correct for hyperopia.

Visual angle : The angle subtended by an object at the eye is called visual angle. How large an object appears depends on the size of the image it makes on the retina. When an object is viewed at a shorter distance, the image on the retina is greater, so the object appears larger.

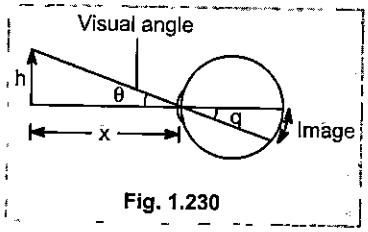


Fig. 1.230

From the figure, $$\tan\theta \simeq \theta = \frac{h}{x}$$

An optical instrument improves the visual angle of an object.

Astronomical Telescope

(i) Refracting astronomical telescope: In Fig. 1.231 shown the objective forms a real, inverted, diminished image of a distant object in its focal plane. The eyepiece then forms a virtual, erect and magnified image of the intermediate image formed by the objective.The intermediate image lies between the focus and the optical centre of the eyepiece.

Magnification of a telescope (M)

$$= \frac{\text{Visual angle subtended by final image } A''B''}{\text{Visual angle subtended by object at unaided eye}}$$

$$M=\frac{\beta}{\alpha}\simeq\frac{\tan\beta}{\tan\alpha}=\frac{A'B'/C_2A'}{A'B'/C_1A'}$$

$$=\frac{C_1A'}{C_2A'}=-\frac{f_o}{u_e} \qquad \ldots (1)$$

Case (i) : *Final image at least distance of distinct vision* ($D = 25\,cm$).

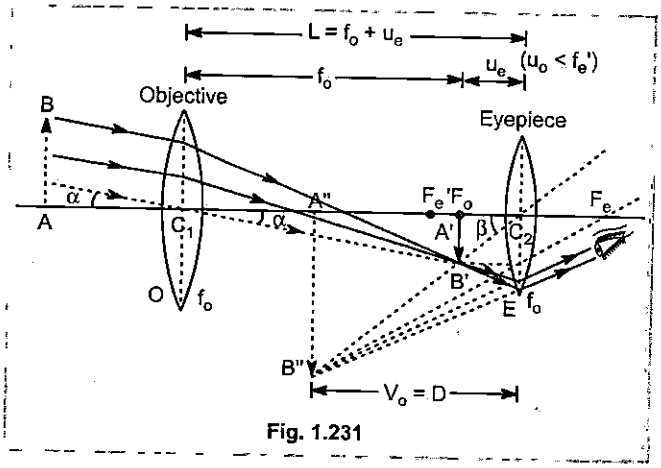


Fig. 1.231

From the lens equation,

$$\frac{1}{-D}-\frac{1}{(-u_e)}=\frac{1}{+f_e}$$

or $$\frac{1}{u_e}=\frac{1}{f_e}+\frac{1}{D}=\frac{1}{f_e}\left(1+\frac{f_e}{D}\right)$$

Thus, $$M=-\frac{f_o}{u_e}=-\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right) \quad ...(2)$$

In this formula only absolute value of f_o, f_e and D are substituted.

Case (ii) : *Final image at infinity (normal use)*

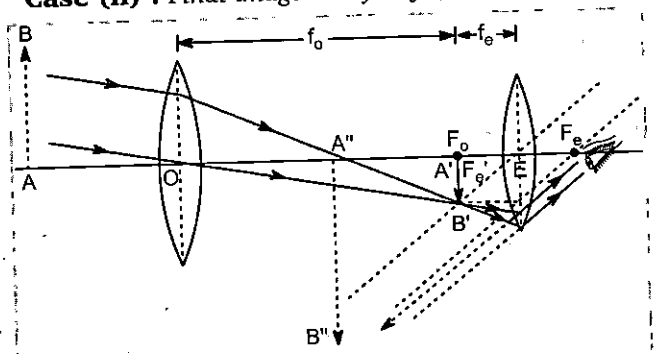


Fig. 1.232

In this case the eye is relaxed (unaccommodated), hence referred to as normal use.

In this case an intermediate image is formed at the focus of the eyepiece.

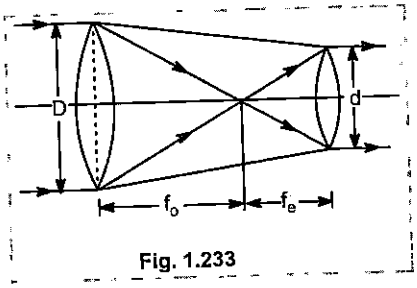


Fig. 1.233

From eqn. (1), we have

$$M=-\frac{f_o}{f_e}$$

Length of telescope in this case,

$$L=f_o+f_e$$

In this case the emerging light from the telescope is parallel to the axis of the telescope, then

$$\frac{f_o}{f_e}=\frac{\text{Aperture of objective}}{\text{Aperture of eyepiece}}=\frac{D}{d}$$

Hence magnification, $$M=-\frac{f_o}{f_e}=-\frac{D}{d}$$

- In order to increase the magnification, focal length of objective should be larger and eyepiece smaller.
- Minus sign with magnification shows that final image is inverted.
- When final image is formed at least distance of distinct vision, magnification obtained is greater and length of telescope is smaller.

(ii) Reflecting astronomical telescope : (Newtonian reflecting telescope)

In this telescope objective is a concave or parabolic mirror, the image formation is by reflection. Fig. 1.234 shows its image formation.

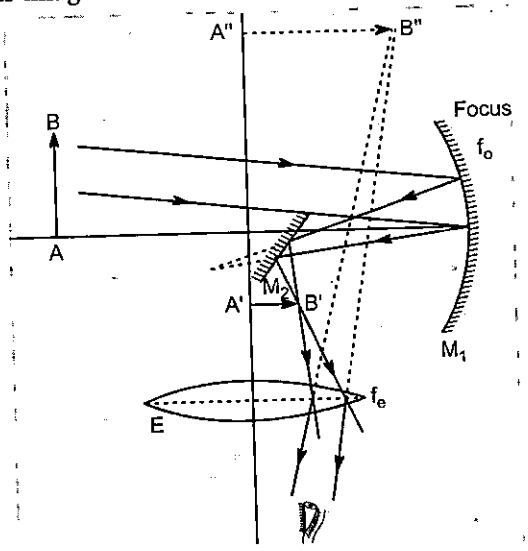


Fig. 1.234

The formula for magnification is same as eqns.(1), (2) and (3).

Final image at D, $$M=-\frac{f_o}{f_e}\left(1+\frac{f_o}{D}\right)$$

Final image at infinity, $$M=-\frac{f_o}{f_e}$$

In a reflecting telescope, absorption of light is negligible, therefore a bright image is formed. Lenses of large aperture are difficult to mould, because during solidification of hot glass distortion occurs. Due to large aperture of a reflecting telescope, its resolving power is greater in comparison to a refracting telescope. Reflecting telescope is free of chromatic aberration and spherical aberration.

Terrestrial Telescope

It is similar to a refracting telescope, having an additional convex lens at a distance (f_o+2f) from the objective. This lens erects the final image. The intermediate image $A'B'$ from the objective is at a distance $2f$ from the erecting lens that inverts it to $A''B''$ which is now at $2f$ from the erecting lens. Now $A''B''$ is an object for the eyepiece that forms a virtual, erect and magnified image. The formula for magnification is the same as obtained earlier accept with a positive sign.

$$M=+\frac{f_o}{u_e}$$

Case (i) : *When the final image is at least distance of distinct vision.*

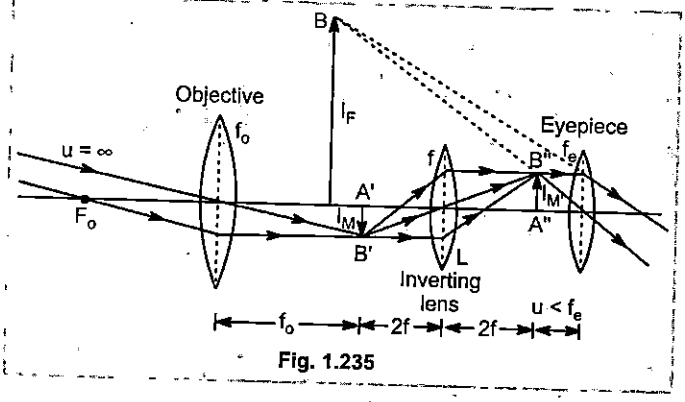


Fig. 1.235

Magnification, $M = \frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$

Length of telescope, $L = f_o + 4f + u_e$

and $u_e = \frac{f_e D}{f_e + D}$

Case (ii) : *When the final image is formed at infinity.*

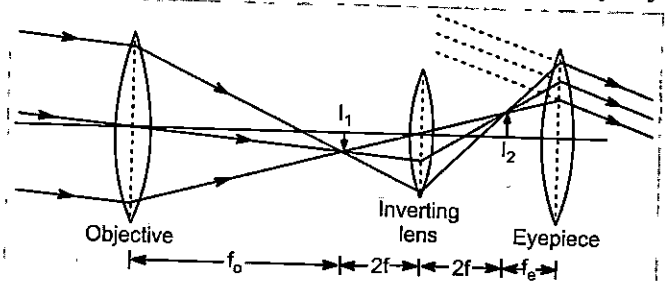


Fig. 1.236

Magnification, $M = \frac{f_o}{f_e}$

Length of telescope, $L = f_o + 4f + f_e$

Galilean Telescope

In this case the eyepiece is a concave lens, kept in such a way that the rays from the objective converge at the first focus or away from it. The first image $A'B'$ is inverted and it acts as a virtual object for the eyepiece that forms an inverted image of $A'B'$. The final image $A''B''$ is erect and magnified. Formula for magnification can be obtained by substituting $-f_e$ in place of f_e in the formula for magnification obtained earlier. Thus,

$$M = \frac{f_o}{u_e} = \frac{f_o}{f_e}\left(1-\frac{f_e}{v_e}\right)$$

Case (i) : *Final image at least distance of distinct vision (D).*

Magnification, $M = \frac{f_o}{f_e}\left(1-\frac{f_e}{D}\right)$

Length of telescope, $L = f_o - u_e$ (where $u_e > f_e$)

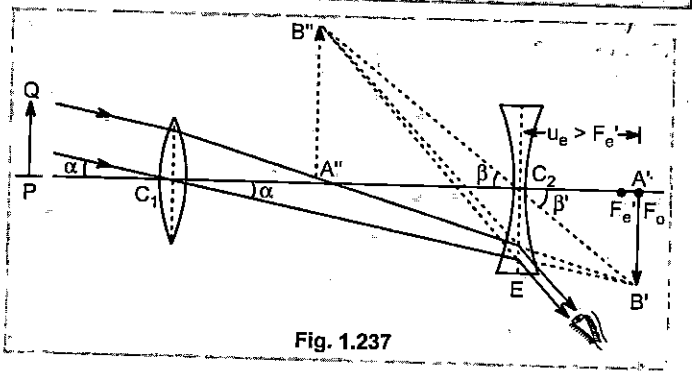


Fig. 1.237

Case (ii) : *Final image at infinity.*

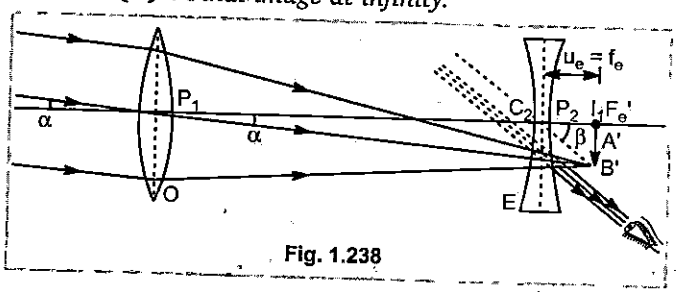


Fig. 1.238

Magnification, $M = \frac{f_o}{f_e}$

Length of telescope, $L = f_o - f_e$

SIMPLE MICROSCOPE

A simple magnifying glass is a convex lens of short focal length. A magnifying glass allows to place the object closer to our eye so that it subtends a greater visual angle.

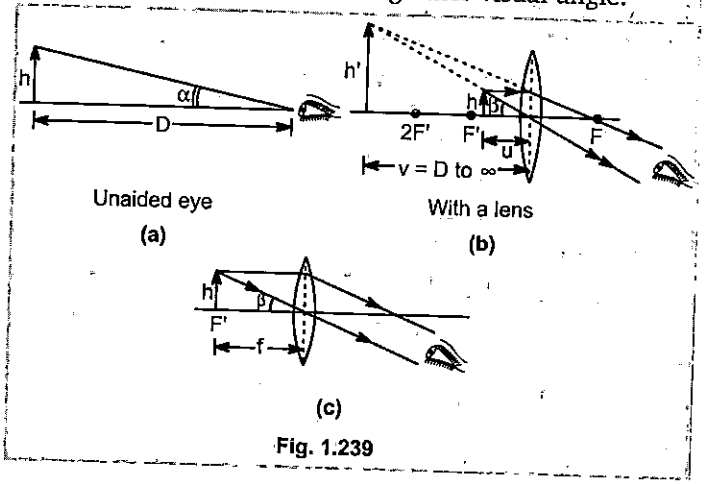


Fig. 1.239

Fig. 1.239(b) shows that the object is placed at the focal point or just within it so that a virtual image is formed. If the eye is relaxed, the final image is formed at infinity.

Magnification of a microscope is defined as

$$M = \frac{\text{Visual angle subtended by final image}}{\text{Maximum visual angle of object subtended at unaided eye}}$$

$$= \frac{\beta}{\alpha_{max}}$$

From figure, $\beta = \frac{h'}{v} = \frac{h}{u}$

and $\alpha_{max} = \frac{h}{D}$

Thus $M = \frac{(h/u)}{(h/D)} = \frac{D}{u}$

Case (i) : *Final image at least distance of distinct vision.*

From lens equation, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-D} - \frac{1}{(-u)} = \frac{1}{f}$$

or $$\frac{1}{u} = \left(\frac{1}{f} + \frac{1}{D}\right)$$

or $$M = \frac{D}{u} = D\left(\frac{1}{f} + \frac{1}{D}\right) = \left(1 + \frac{D}{f}\right)$$

In this case image distance v is minimum, so u will also be minimum, hence magnification is maximum. Eye is in maximum strain position.

Case (ii) : *Final image at infinity.*

As $v = -\infty$, hence $u = f$

$$M = \frac{D}{f}$$

In this case magnification obtained is least. In this position the eye is relaxed, unstrained.

- If the eye is at a distance x from the eyepiece, the magnification in case (i), $M = 1 + \frac{D - x}{f}$ and in case (ii), $M = \frac{D - x}{f}$.
- If higher wavelength is used, focal length of the lens increases, resulting in reduced magnification.

COMPOUND MICROSCOPE

It is a two lens system, both the lenses objective and eyepiece are convex lenses of short focal length and small aperture. Due to small aperture, spherical aberration is nullified.

The object is placed between F_o' and $2F_o'$. The objective forms a real, inverted, magnified image $A'B'$ is formed between $2F$ and ∞, at v_o. Intermediate image $A'B'$ is a real object for the eyepiece and lies between the eyepiece and its focus. The final image $A''B''$ is erect (relative to $A'B'$) and magnified.

Magnifying power $= \frac{\text{Angle subtended by final image } A''B''}{\text{Maximum visual angle at unaided eye}}$

$$= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \left(\frac{A'B'}{EA'}\right) \Big/ \left(\frac{AB}{D}\right)$$

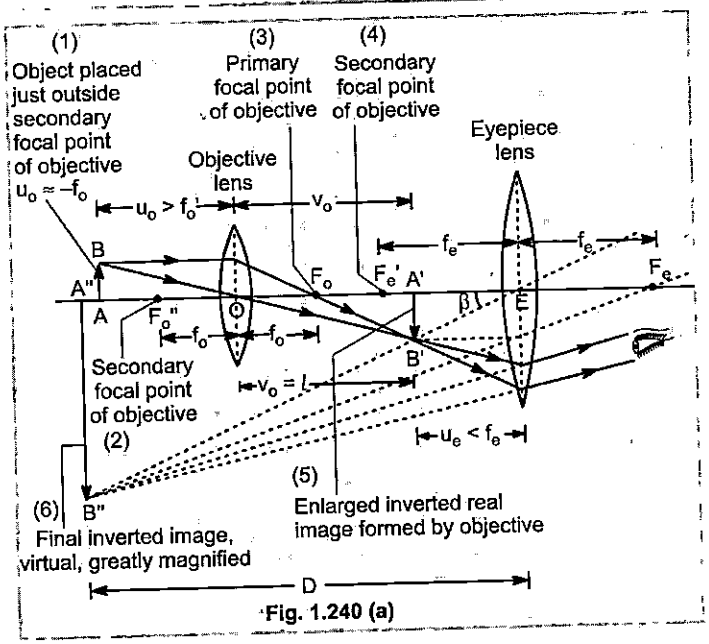


Fig. 1.240 (a)

$$= \frac{A'B'}{AB}\left(\frac{D}{EA'}\right)$$

As $\frac{A'B'}{AB} = \frac{+v_0}{-u_e}$ and $EA' = -u_e$

then $$M = \frac{v_o}{-u_e}\left(\frac{-D}{-u_e}\right)$$

$$= -\frac{v_o}{u_e}\left(\frac{D}{u_e}\right)$$

and length of microscope, $L = v_o + u_e$

Case (i) : *Final image at least distance of distinct vision (D).* From lens equation, we have

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

or $$\frac{D}{u_e} = \left(1 + \frac{D}{f_e}\right)$$

or $$M = -\frac{v_o}{u_o}\frac{D}{u_e} = -\frac{v_o}{u_e}\left(1 + \frac{D}{f_e}\right)$$

and $$L = v_o + u_e = v_o + \left(\frac{f_e D}{f_e + D}\right)$$

In this case u_e is minimum because v_e is minimum, therefore maximum magnification is obtained.

Case (ii) : *Final image at infinity.*

From lens equation,

$$\frac{1}{-\infty} - \frac{1}{(-u_e)} = \frac{1}{f_e}$$

or $$u_e = f_e$$

or $$M = -\frac{v_o}{u_e}\left(\frac{D}{f_e}\right)$$

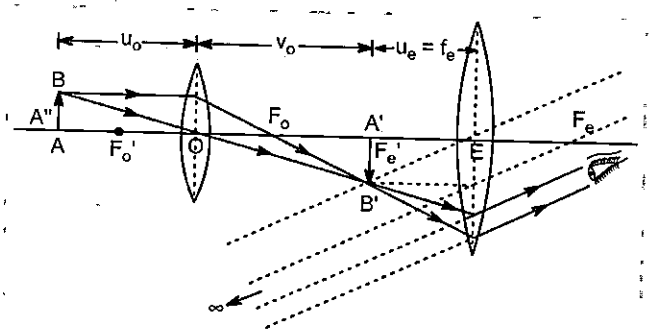


Fig. 1.240 (b)

and $$L = v_o + f_e \text{ (maximum)}$$

In this case u_e is maximum therefore magnification is minimum.

Example 220

An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eyepiece is 36 cm and the final image is formed at infinity. Determine the focal length of objective and eyepiece.

Solution: In case of astronomical telescope if object and final image both are at infinity,

$$MP = -(f_o/f_e) \quad \text{and} \quad L = f_o + f_e$$

So here $-(f_o/f_e) = -5$ and $f_o + f_e = 36$

Solving these for f_o and f_e, we get

$$f_o = 30\,\text{cm} \text{ and } f_e = 6\,\text{cm}$$

Example 221

The diameter of the moon is 3.5×10^3 km and its distance from the earth is 3.8×10^5 km. It is seen through a telescope having focal lengths of objective and eyepiece as 4 m and 10 cm respectively. Calculate (a) magnifying power of telescope (b) length of telescope tube and (c) angular size of image of moon.

Solution: For normal adjustments,

(a) $$|MP| = \frac{f_o}{f_e} = \frac{4 \times 100}{10} = 40$$

(b) $L = f_o + f_e = 400 + 10 = 410\,\text{cm} = 4.10\,\text{m}$

(c) As the angle subtended by moon on the objective of telescope.

$$\theta_o = \frac{3.5 \times 10^3}{3.8 \times 10^5} = \frac{3.5}{3.8} \times 10^{-2}\ \text{rad}$$

and as $|MP| = |(\theta/\theta_o)|$, the angular size of final image,

$$|\theta| = |MP| \times \theta_o = 40 \times \frac{3.5}{3.8} \times 10^{-2} = 0.3684\,\text{rad}$$

i.e., $$|\theta| = 0.368 \times \frac{180}{\pi} \simeq 21^\circ$$

Example 222

An astronomical telescope consisting of an objective of focal length 60 cm and eyepiece of focal length 3 cm is focussed on the moon so that the final image is formed at least distance of distinct vision, i.e., 25 cm from the eyepiece. Assuming the angular diameter of moon as $(1/2)^\circ$ at the objective, calculate (a) angular size and (b) linear size of image seen through the telescope.

Solution: As final image is at least distance of distinct vision,

$$|MP| = \frac{f_o}{f_e}\left[1 + \frac{f_e}{D}\right] = \frac{60}{3}\left[1 + \frac{3}{25}\right] = 22.4$$

Now as by definition $MP = (\theta/\theta_o)$, so the angular size of image,

$$\theta = MP \times \theta_o = 22.4 \times \left[\frac{1}{2}\right]^\circ$$

$$= 11.2^\circ = \frac{\pi}{180} \times 11.2 \simeq 0.2\,\text{rad}$$

and if I is the size of final image which is at least distance of distinct vision,

$$\theta = (I/25),$$

i.e., $$I = 25 \times \theta = 25 \times 0.2 = 5\,\text{cm}$$

Example 223

A Galilean telescope consists of an objective of focal length 12 cm and an eyepiece of focal length 4 cm. What should be the separation of the two lenses when the virtual image of a distant object is formed at a distance of 24 cm from the eyepiece? What is the magnifying power of the telescope under this condition ?

Solution: As the object is distant, *i.e.* $u = -\infty$, so

$$\frac{1}{v} - \frac{1}{-\infty} = \frac{1}{f_o} \text{ i.e., } v = f_o = 12\,\text{cm}$$

i.e., the objective will form the image I_M at its focus which is at a distance of 12 cm from O.

Now as the eyepiece of focal length -4 cm forms the image I at a distance 24 cm from it,

$$\frac{1}{-24} - \frac{1}{u_e} = \frac{1}{-4}, \qquad \text{i.e., } u_e = \frac{24}{5} = 4.8\,\text{cm}$$

i.e., the distance of I_M from the eye lens EA is 4.8 cm. So the length of tube,

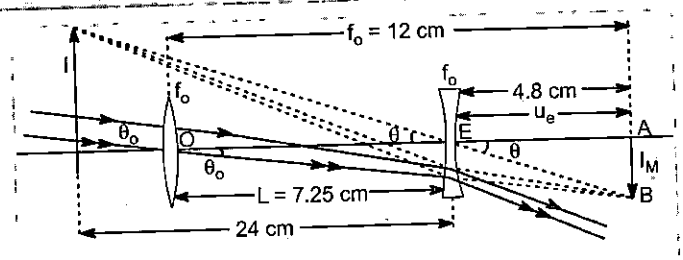


Fig. 1E.223

$$L = OA - EA = 12 - 4.8 = 7.2\,\text{cm}$$

Now by definition,

$$MP = \frac{\theta}{\theta_o} \simeq \frac{\tan\theta}{\tan\theta_o} = \frac{(AB/EA)}{(AB/OA)} = \frac{OA}{EA}$$

So, $$MP = \frac{f_o}{u_e} = \frac{12}{4.8} = \frac{10}{4} = 2.5$$

i.e., the image is erect, virtual and is at a distance of $24 - 7.2 = 16.8$ cm infront of the objective.

Example 224

The aperture of the largest telescope in the world is 5 m. If the separation between the moon and the earth is 4×10^5 km and the wavelength of visible light is 5000 Å, calculate (a) resolving power of telescope and (b) minimum separation between objects on the surface of moon so that they are just resolved.

Solution: (a) definition,

$$RP = \frac{D}{1.22\lambda} = \frac{5}{1.22 \times 5 \times 10^{-7}} = \frac{1}{1.22} \times 10^7$$

(b) As limit of resolution $\Delta\theta = (1/RP)$ and if d is the distance between objects on the surface of moon which is at a distance r from the telescope, $\Delta\theta = (d/r)$,

So, $$\Delta\theta = \frac{1}{RP} = \frac{d}{r}, \text{ i.e., } d = \frac{r}{RP}$$

$\therefore$ $$d = \frac{(4 \times 10^5) \times 10^3 \times 1.22}{10^7} = 48.8\,\text{m}$$

Example 225

A compound microscope is used to enlarge an object kept at a distance 0.03 m from its objective which consists of several convex lenses in contact and has focal length 0.02 m. If a lens of focal length 0.1 m is removed from the objective, find out the distance by which the eyepiece of the microscope must be moved to refocus the image.

Solution: If initially the objective forms the image at distance v_1,

$$\frac{1}{v_1} - \frac{1}{-3} = \frac{1}{2}, \text{ i.e., } v_1 = 6\,\text{m}$$

Now as in case of lenses in contact,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots \text{ or } \frac{1}{F} = \frac{1}{f_1} + \frac{1}{F'} \text{ with } \frac{1}{F'} = \frac{1}{f_2} + \frac{1}{f_3} + \ldots$$

So if one of the lenses is removed, the focal length of the remaining lens system,

$$\frac{1}{F'} = \frac{1}{F} - \frac{1}{f_1} = \frac{1}{2} - \frac{1}{10} \text{ i.e., } F' = 2.5\,\text{cm}$$

This lens will form the image of same object at a distance v_2 such that

$$\frac{1}{v_2} - \frac{1}{-3} = \frac{1}{2.5} \qquad \text{i.e., } v_2 = 15\,\text{cm}$$

So to refocus the image, the eyepiece must be moved by the same distance through which the image formed by the objective has shifted, *i.e.*, $15 - 6 = 9$ cm away from the objective.

Example 226

The focal lengths of the objective and the eyepiece of a compound microscope are 2.0 cm and 3.0 cm respectively. the distance between the objective and the eyepiece is 15.0 cm. The final image formed by the eyepiece is at infinity. Find the distance of object and image produced by the objective, from the objective lens.

Solution: As the final image is at infinity, the distance of intermediate image from eye lens u_e will be given by

$$\frac{1}{-\infty} - \frac{1}{u_e} = \frac{1}{f_e} \qquad \text{i.e., } u_e = -f_e = -3\,\text{cm}$$

and as the distance between the lenses is 15.0 cm, the distance of intermediate image (formed by objective) from the objective will be

$$v = L - u_e = L - f_e = 15 - 3 = 12\,\text{cm}$$

and if u is the distance of object from objective,

$$\frac{1}{12} - \frac{1}{u} = \frac{1}{2} \quad \text{i.e., } u = -2.4\,\text{cm}$$

So the object is at a distance of 2.4 cm infront of the objective.

Example 227

(a) An astronomical telescope consists of an eyepiece having two lenses of focal lengths 5 cm and 4 cm. The gap t between the eyepiece lenses is adjusted such that the longitudinal spherical aberration caused by it is minimum. The telescope is adjusted for final image at infinity. Determine the magnification of the telescope.

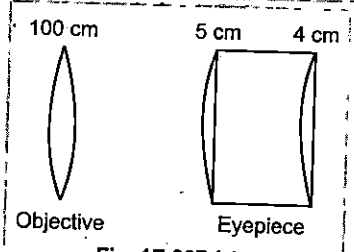


Fig. 1E.227 (a)

(b) An astronomical telescope is being used in normal adjustment (i.e. the final image is formed at infinity). If the length of the telescope is reduced by $\frac{1}{11}$th of the focal length of the eyepiece, then the final image is found to be formed at the near point of the eye, the least distance of distinct vision is 25 cm and the angular magnification of the telescope in the normal use is 20. Determine the angular magnification after changing the tube length. Also draw the ray diagrams in both cases.

Solution: (a) The least spherical aberration is due to separated doublet. The separation of t must be equal to the difference of focal length of lenses.

$$t = f_1 - f_2 = 5 - 4 = 1\,\text{cm}$$

Now the focal length of the eyepiece is given by

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2} = \frac{1}{5} + \frac{1}{4} - \frac{1}{5\times 4} = \frac{8}{20}$$

$$f_e = (20/8)\,\text{cm}$$

For final image at infinity,

$$M = \frac{f_o}{f_e} = \frac{100}{(20/8)} = 40$$

(b) When the final image is formed at infinity, the first image is formed at the foci of both the lenses and tube length = $f_o + f_e$.

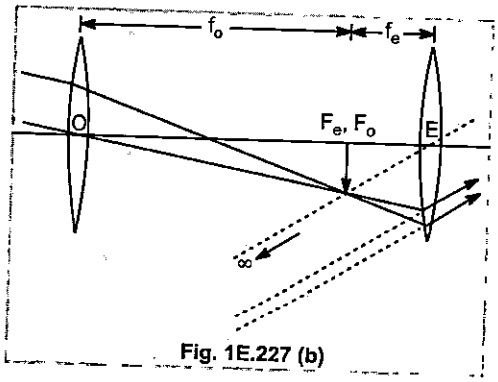


Fig. 1E.227 (b)

When the final image is at least distance of distinct vision, the first image will be lying between focus F_o and optical centre E of the eye lens.

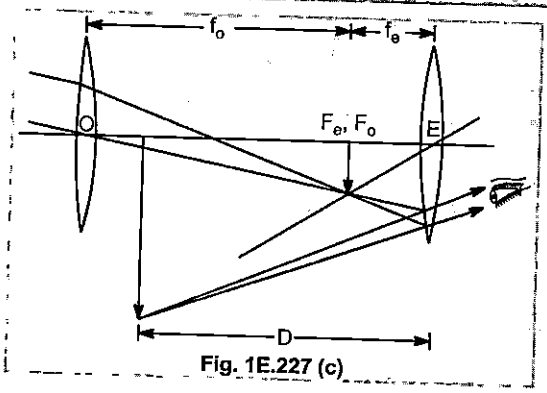


Fig. 1E.227 (c)

$\therefore$ In the second case $u_e = -\left[f_e - \frac{f_e}{11}\right] = -\left[+\frac{10}{11} f_e\right]$

$$v_e = -D = -25$$

Using $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$ we get $f_e = 2.5\,\text{cm}$

In normal adjustment, $|M_1| = \frac{f_o}{f_e}$

where $M_1 = +20,\ f_e = 2.5$

$\Rightarrow$ $f_o = 2.5 \times 20 = 50\,\text{cm}$

When the final image is formed at near point,

$$|M_2| = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

$$= 20\left(1 + \frac{2.5}{25}\right) = 20\ (1.1) = 22$$

Example 228

In a compound microscope the objective and the eyepiece have focal lengths of 0.95 cm and 5 cm respectively, and are kept at a distance of 20 cm. The last image is formed at a distance of 25 cm from the eyepiece. Calculate the position of the object and the total magnification.

Solution: As the final image is at 25 cm infront of the eyepiece,

$$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

i.e., $u_e = -\frac{25}{6}$

and so, $m_e = \frac{v_e}{u_e} = \frac{-25}{(-25/6)} = 6$ ...(1)

Now for the objective,

$$v = L - u_e = 20 - (25/6) = (95/6)$$

So if the object is at a distance u from the objective,

$$\frac{6}{95} - \frac{1}{u} = \frac{1}{0.95}$$

i.e., $u = -\frac{95}{94}$ cm

i.e., the object is at a distance (95/94) cm infront of the field lens.

Also, $m = \frac{v}{u} = \frac{(95/6)}{(-95/94)} = -\left[\frac{94}{6}\right]$ ...(2)

So total magnification,

$$M = m \times m_e = -\left[\frac{94}{6}\right] \times (6) = -94$$

i.e., **the final image is inverted, virtual and 94 times that of object.**

Remark : *In this example as final image is at least distance of distinct vision D (= 25 cm),*

$$m_\theta = \left[1 + \frac{D}{f_e}\right] = 1 + \frac{25}{5} = 6 = m_e$$

So $MP = m \times m_\theta = m \times m_e =$ *Linear magnification*

Solid Angle

Figure shows a differential area element $d\vec{\mathbf{s}}$ at a position vector $\vec{\mathbf{r}}$ from point *O*, that subtends solid angle $d\omega$ at *O*.

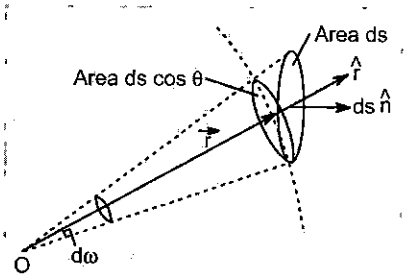


Fig. 1.241

$$d\omega = \frac{d\vec{\mathbf{s}} \cdot \hat{\mathbf{r}}}{r^2} = \frac{ds}{r^2}\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}$$

$$d\omega = \frac{ds\cos\theta}{r^2}$$

as $\hat{\mathbf{n}} \cdot \hat{\mathbf{r}} = \cos\theta$.

Here vector $d\vec{\mathbf{s}}$ has magnitude *ds* and direction outward normal represented by unit vector $\hat{\mathbf{n}}$. Angle between $\hat{\mathbf{r}}$ and $\hat{\mathbf{n}}$ is θ. Total solid angle subtended by a finite area can be obtained from

$$\omega = \int \frac{ds\cos\theta}{r^2}$$

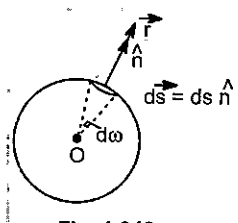


Fig. 1.242

Solid angle subtended by a spherical surface at its centre is

$$\omega = \frac{(4\pi r^2)\cos\theta}{r^2}$$

$$= 4\pi \text{ steradian}$$

Any closed surface subtends 4π steradian at any inside point and zero at outside point.

PHOTOMETRY

In this branch of optics we study the factors that are responsible for the sensation of brightness, light emitting capacity of a source.

Basic definitions and principles

1. Radiant flux *(R)* : The total energy radiated by a source per sec is called radiant flux, measured in **watt.**

2. Luminous flux (ϕ) : Total light energy emitted by a source per sec. Its unit is **lumen** (lm). In the visible light all the colours and wavelength do not produce the same sensation at our eye. Only the wavelengths between 4000 Å to 7800 Å produce visual sensation at our eye, out of which green-yellow light (λ = 5550 Å) gives maximum sensation.

Radiant flux expresses radiated energy of all the wavelengths whereas luminous flux expresses capacity to generate total brightness.

Unit of luminous flux, lumen, is arbitrary, defined as : If monochromatic light of wavelength 5550 Å has radiant flux $\frac{1}{685}$ watt, then luminous flux is one lumen.

3. Luminous intensity *(L)* : Luminous flux per unit solid angle,

$$L = \frac{\Delta\phi}{\Delta\omega}$$

It represents light energy emitted by a source per sec per unit solid angle. SI unit is lumen per steradian, expressed as **candela** (cd).

For an isotropic point source,

$$\omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ steradian}$$

$$L = \frac{\phi}{\omega} = \frac{\phi}{4\pi}$$

or $\phi = 4\pi L$

Luminous flux = $4\pi \times$ (luminous intensity)

4. Illuminance *(I)* : The luminous flux incident per unit area of a surface,

$$\text{Illuminance } (I) = \frac{\Delta\phi}{\Delta s}$$

Its unit is **lux** (lx).

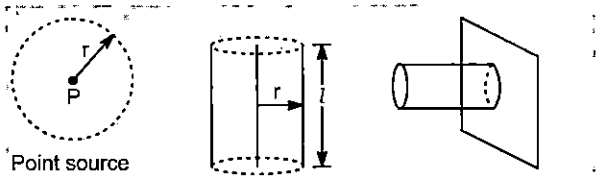


Fig. 1.243

Relation between luminous intensity and illuminance (Lambert's cosine law)

Illuminance at an angle θ on an area *ds* is

$$I = \frac{d\phi}{ds}$$

and $$L = \frac{d\phi}{d\omega}$$

$$I = L\frac{d\omega}{ds}$$

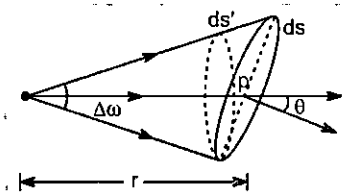


Fig. 1.244

For an isotropic point source,

$$d\omega = \frac{ds'}{r^2} = \frac{ds\cos\theta}{r^2}$$

$$\boldsymbol{I = \frac{L\cos\theta}{r^2}}$$

This law is called Lambert's cosine law.

For a given source and plane of illumination L and h = const.

or $r\cos\theta = h$ = const.

or $$I = \frac{L}{h^2}\cos^3\theta$$

or $$I = \frac{Lh}{r^3}$$

or $I \propto \cos^3\theta$

and $I \propto (1/r^3)$

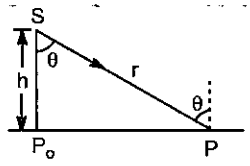


Fig. 1.245

Example 229

A flat mirror M is arranged parallel to a wall and light from a point source S on the wall is reflected back to the wall and forms a spot of light. With what velocity will the spot move along the wall if the mirror is brought up to the wall with a velocity v ? How will the dimensions of the spot of light and its illumination change ?

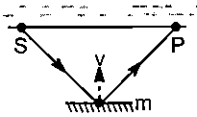


Fig. 1E.229 (a)

Solution: By the law of reflection, the spot of light *AB* on the wall will remain stationary and hence the velocity of the spot along the wall is **zero.**

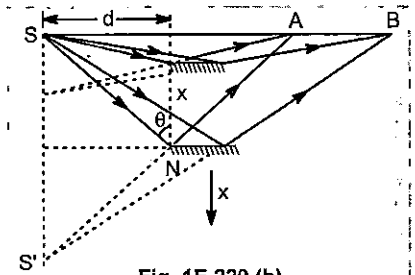


Fig. 1E.229 (b)

The size of the mirror is always half the size of the spot as this is seen from the similar triangles *S'AB* and *S'NM*.

Here area of *AB* is four times the area of *MN*. Hence intensity of spot of light is one-fourth of the illumination of the mirror. At a large distance illumination of the mirror is zero because of distance and again when the mirror is closest to the wall, the illumination is zero on account of oblique incidence of light. Hence in some intermediate position illumination is maximum.

Let *d* be the distance of the normal to the mirror from *S* along the plane. Then

$$I \text{ (illumination of the mirror)} = \frac{P\cos\theta}{(\sqrt{a^2 + x^2})^2}$$

where P = illuminating power

$$\Rightarrow \quad I = \frac{Px}{\sqrt{(a^2 + x^2)}(d^2 + x^2)} = \frac{Px}{(d^2 + x^2)^{3/2}}$$

$$\left(\text{Formula for illumination is } I = \frac{P\cos\theta}{r^2}\right)$$

$$\frac{dI}{dx} = P\frac{(d^2 + x^2)^{3/2}\cdot 1 - x\cdot 3/2(d^2 + x^2)^{1/2}\cdot 2x}{(d^2 + x^2)^3}$$

When I is maximum,

$$\frac{dI}{dx} = 0 \Rightarrow (d^2 + x^2)^{3/2} - 3x^2(d^2 + x^2)^{1/2} = 0$$

$$\Rightarrow \quad d^2 + x^2 - 3x^2 = 0$$

$$\Rightarrow \quad d^2 = 2x^2 \Rightarrow x = \frac{d}{\sqrt{2}}$$

Example 230

At what distance should the posts for street lamps be installed so that the illumination on the ground at the point lying halfway between two posts is not less than 4/15 lx ? The height of the posts is h = 12 m. The luminous intensity of the lamps is I = 300 cd. Assume that a noticeable illumination is provided only by the two lamps on either side.

Solution: The illumination provided by one lamp on the ground at a distance $a = \frac{l}{2}$ will be

$$\frac{E}{2} = \frac{Ih}{(h^2 + a^2)^{3/2}}$$

Hence, $$a = \sqrt{\left(\frac{2Ih}{E}\right)^{2/3} - h^2} = 27.5 \text{ m}$$

Example 231

During fitting jobs in a subway an electric lamp is secured at the top point of the tunnel A [Fig.1E.231 (a)]. What is the ratio between the illuminations produced by the lamp at the lowest point B and those at the point C lying at the level with the horizontal cross-section of the tunnel ? The luminous intensity of the lamp is the same in all directions.

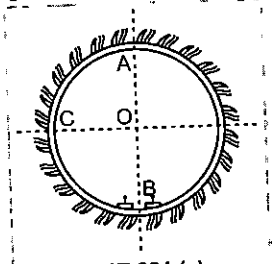


Fig. 1E.231 (a)

Solution: The illumination at the point B is

$$E_0 = \frac{I}{4R^2}$$

and at the point C,

$$E_1 = \frac{I\cos\alpha}{r^2}$$

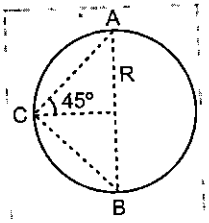


Fig. 1E.231 (b)

From triangle ABC [Fig. 1E.231 (b)],

$$r = AC = 2R\cos\beta$$

where β is the angle at the vertex A. Therefore

$$E_1 = \frac{E_0\cos\alpha}{\cos^2\beta}$$

When $\alpha = \beta = 45°$, $E_1 = E_0\sqrt{2}$

Example 232

A round table of radius r is illuminated by a lamp of luminous intensity (candle power) I hanging above its centre. What should be the height of the lamp above the table for the illuminance of the table fringe to be maximum ? What is its value ? What is the illuminance at the centre of the table in these conditions ?

Solution: The illuminance near the edge of the table is $E = \frac{I\cos\alpha}{R^2} = \frac{Ih}{(r^2 + h^2)^{3/2}}$, where h is the elevation of the lamp above the centre of the table. Let us find the maximum of the function obtained. The condition for a maximum is

$$\frac{d}{dh}\left[\frac{h}{(r^2 + h^2)^{3/2}}\right] = 0$$

We have $(r^2 + h^2)^{-3/2} - \frac{3}{2}h(r^2 + h^2)^{-5/2} \cdot 2h = 0$

Multiplying by $(r^2 + h^2)^{5/2}$ we obtain $-3h^2 + r^2 + h^2 = 0$, so $h = r/\sqrt{2}$. From this we can then find the illuminance at the edge and at the centre of the table.

Example 233

A point light source of 10 cd luminous intensity is in the centre of a concave mirror with a radius of curvature of 40 cm and of 20 cm diameter. It illuminates a screen at a distance of 2 m from the source. What is the maximum illuminance of the screen ? How will the illuminance change, if the mirror is withdrawn ?

Solution: The maximum illuminance will be at a point on the principal axis of the optical system (Fig. 1E.233). In this case of a sufficiently narrow beam of light, the role of the concave mirror is to double the luminous flux falling on the screen. Therefore, if the mirror is withdrawn, the illuminance will decrease by one-half.

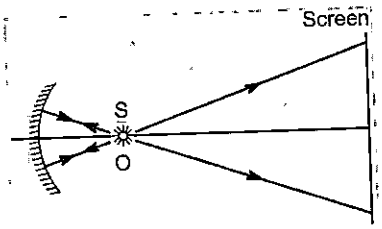


Fig. 1E.233

Example 234

A lens of diameter D and a focal length f projects a small object place at a great distance from the lens on to a screen. Show that the illuminance of the image on the screen will be proportional to the luminance and to the aperture ratio of the lens. (The aperture ratio is the square of the ratio of the lens' diameter to its focal length)

Solution: If the object is removed far from the lens, its image lies practically in the focal plane. The magnification is $\beta = h'/h = d'/d = f/d$.

The illuminance of the image is equal to the luminous flux $\phi = I\Omega$ divided by the area of the image $S' = \pi h'^2/4$. Noting that the solid angle is $\Omega = \pi D^2/4d^2$, where D is the diameter of the lens, we obtain

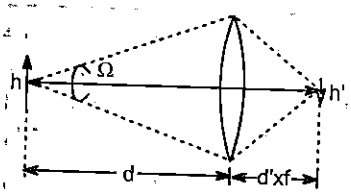


Fig. 1E.234

$$E = \frac{\phi}{S'} = \frac{4\pi D^2 I}{4\pi d^2 h'^2}$$

$$= \frac{D^2 I d^2}{d^2 h^2 f^2} = \frac{I}{h^2} \cdot \frac{D^2}{f^2}$$

But $B = \dfrac{4I}{\pi h^2}$ is the brightness of the object and $R = \pi\beta = \dfrac{4I}{h^2}$ is its luminance. We have $E = \dfrac{R}{4} \cdot \dfrac{D^2}{f^2}$, where $\dfrac{D^2}{f^2}$ is the focal power of the lens. Note that we have neglected the light losses in the lens.

Example 235

A screen is 1 m away from a light source. A diverging lens of − 5 dioptre focal power is placed between the source and the screen so that the position of the light source coincides with that of the virtual focus. How will the illuminance of the screen on the optical axis of the system change ?

Solution: According to the sign convention formulated in the focal length f of the diverging lens and the distance d' from the virtual image are both negative. Without the lens the luminous flux is distributed over an area $S_0 = \pi D_0^2/4$. With the lens the same flux is distributed over an area $S = \pi D^2/4$ (Fig. 1E.235).

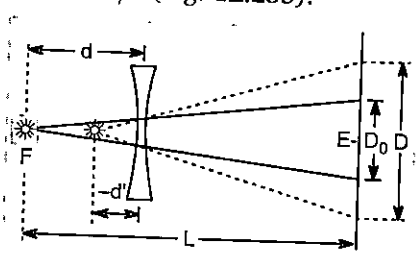


Fig. 1E.235

Therefore $\dfrac{E}{E_0} = \dfrac{D_0^2}{D^2}$. But $D_0 = \dfrac{\phi L}{d}$ and $D = \dfrac{\phi(L - d - d')}{-d'}$, where ϕ is the lens diameter. In our problem $d = -f$, so $-\dfrac{1}{f} + \dfrac{1}{d'} = \dfrac{1}{f}$, *i.e.*, $d' = \dfrac{f}{2}$. We have

$$\frac{D_0}{D} = \frac{L}{2L + f} \quad \text{and} \quad \frac{E}{E_0} = \frac{L^2}{(2L + f)^2}$$

Example 236

The sunrays which reach the earth surface bring in every minute an energy approximately equal to 1.94 cal per 1 cm² of the terrestrial surface. (with perpendicular incidence). Determine the total amount of energy received by the entire terrestrial surface. What fraction in this of the total energy of light emission from the Sun ? What planet receives more energy from the Sun—the Earth of Jupiter ? The distance from the Earth to the Sun is $R_1 = 1.5 \times 10^8$ km, the distance to Jupiter R_2 is 5.20 times larger than to the Earth , the radius of the Earth is 6.3×10^3 km and the radius of Jupiter is 11.14 times of the Earth.

Solution: $Q = 2.4 \times 10^{15}$ kcal, 0.441×10^{-9} of all the energy of light emission from the Sun; the total amount of energy received by the Earth is 0.22 of the energy received by Jupiter.

The amount of energy received by the Earth is

$$Q_1 = \pi r_1^2 q_1$$

where πr_1^2 is the cross-sectional area of the Earth.

The ratio of Q to the total energy of emission E is

$$\frac{Q_1}{E} = \frac{\omega_1}{4\pi} = \frac{r_1^2}{4R_1^2}$$

where ω_1 is the solid angle which the Earth makes with the Sun. The ratio of the energies received by the Earth and Jupiter is

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} = \frac{r_1^2 R_2^2}{r_2^2 R_1^2}$$

Example 237

In constant conditions of illumination a certain object is photographed first from a large distance and then from a small distance.

How will the illuminations of the photographic plates in the camera differ in these two cases? Which of the two requires a longer exposure ?

Solution: The illumination of a plate in a camera depends on the quantity of light passed through the lens and on the ratio of the area of the object being photographed to the area of its image on the plate.

The quantity of light Q passing through the lens is proportional to the solid angle which the lens makes with the point of the object, *i.e.* it is directly proportional to the area of the aperture of the lens S and is inversely proportional to the square of the distance a_1 from the camera to the object [Fig 1E.237 (a)] or $Q \sim \dfrac{S}{a_1^2}$. The ratio of the linear dimensions of the object to those of the image is

equal to the ratio of the distances of the object and the image to the camera lens [Fig. 1E.237 (b)]

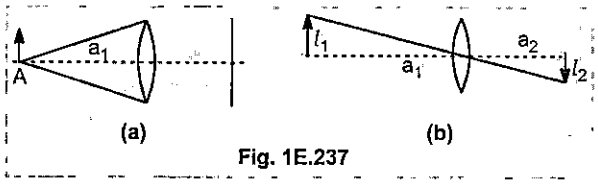


Fig. 1E.237

Therefore, the area of the object σ_1 is related to the area of the image σ_2 as the square of the distances of the object and the image from the lens, *i.e.*,

$$\frac{\sigma_1}{\sigma_2} = \frac{a_1^2}{a_2^2}$$

Comparing the results obtained, we find the illumination of the image

$$E \sim \frac{Q\sigma_1}{\sigma_2} = \frac{S}{a_1^2}\frac{a_1^2}{a_2^2} = \frac{S}{a_2^2}$$

i.e.,the illumination of the image in the camera is inversely proportional to the square of the distance from the lens to the image.

The image of a distant object is closer to the lens than that of an object located nearby. Hence, the illumination of the image of a distant object will always be greater than that of the image of a near object. In the first case (near object), a longer exposure is required than in the second case.

Example 238

A point source of light placed at a distance from a screen creates an illumination of 2.25 lx at the centre of the screen. How will this illumination change if on the other side of the source and at the same distance from it we place:

(a) An infinite flat mirror parallel to the screen ?

(b) A concave mirror whose centre coincides with the centre of the screen ?

(c) A convex mirror with the same radius of curvature as the concave mirror ?

Solution: (a) The rays reflected from the flat mirror increase the illumination at the centre of the screen. The presence of the mirror is equivalent to the appearance of a new source (with the same luminous intensity) arranged at a distance from the screen three times greater than that of the first source. For this reason the illumination should increase by one-ninth of the previous illumination, *i.e.*,

$$E_a = 2.5\,\text{lx}$$

(b) The concave mirror is so arranged that the source is in its focus. The rays reflected from the mirror travel in a parallel beam. The illumination along the axis of the beam of parallel rays is everywhere the same and equal to the illumination created by the point source at the point of the mirror closest to it. The total illumination at the centre of the screen is equal to the sum of the illuminations produced by the source at the centre of the screen and reflected by the rays :

$$E_b = 2 \times 2.25\,\text{lx} = 4.5\,\text{lx}$$

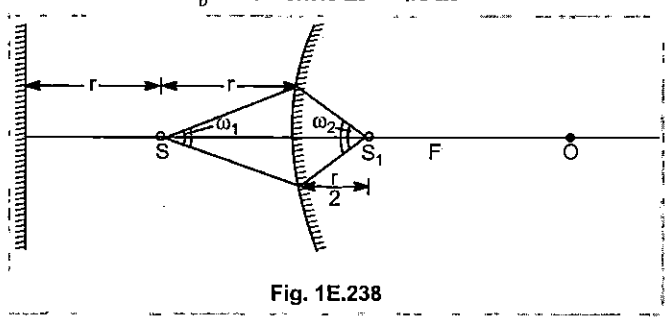


Fig. 1E.238

(c) The virtual image of the point source in the convex mirror is at a distance of 2.5 r from the screen (r is the distance from the screen to the source). The luminous flux ϕ emitted by the virtual source is equal to that of the real source incident on the mirror :

$$I_1\omega_1 = I_2\omega_2$$

Since the solid angle ω_1 of the flux incident on the mirror from source S (Fig. 1E.238) is one-fourth of the solid angle ω_2 inside which the rays from the virtual source S_1 propagate, the luminous intensity I_2 of the virtual source is one-fourth of the intensity of source S. For this reason the virtual source creates at the centre of the screen an illumination of $4 \times (2.5)^2 = 25$ times smaller that the real source. Hence $E_c = 2.34$ lx.

Subjective Problems

1. When the Sun is either rising or setting and appears to be just on the horizon, it is infact below the horizon. Assuming that the atmosphere has uniform density and hence uniform index of refraction n, and extends to a height h above the surface of Earth at which point it abruptly stops, show that its apparent position is an angle δ above the true position, and δ is given by

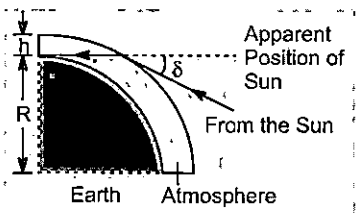


Fig. 1.1

$$\delta = \sin^{-1}\left(\frac{nR}{R+h}\right) - \sin^{-1}\left(\frac{R}{R+h}\right)$$

2. A mountaineer on a peak observes a rainbow in a valley, caused by water droplets in the air 8 km away. The valley is 2 km below the mountain peak and entirely flat. What fraction of the complete circular arc of the rainbow is visible to him ?

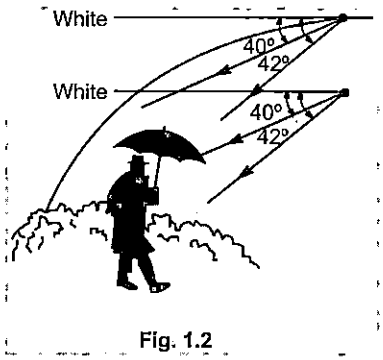


Fig. 1.2

[Ans. 62.2%**]**

3. A large transparent cube, refractive index $n = 1.59$, has a small air bubble below one surface. When a coin of diameter 1.90 cm is placed directly over the bubble on the outside surface of the cube, one cannot see the bubble by looking down into the cube. What is the range of the possible depths of the air bubble beneath the surface ?

[Ans. $1.08\text{ cm} < d < 1.17\text{cm}$**]**

4. One face of a slab of thickness t is painted white and a small hole scraped clear at point P serves as a source of diverging rays when the slab is illuminated from below. Ray PBB' strikes the clear face at a critical angle and is totally reflected, as are rays PCC'. On the painted surface there appears a dark circle of diameter d, surrounded by an illuminated region or halo. Derive an expression for n in terms of measured quantities d and t.

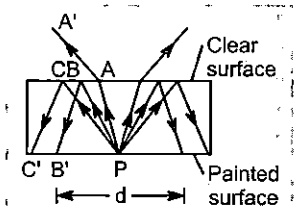


Fig. 1.4

[Ans. $n = [1 + (4t/d)^2]^{1/2}$**]**

5. A dynamite explosion at S surves as a source of sound on the surface of Earth. The velocity of sound in an unconsolidated region immediately below the Earth's surface is much lower than the sound velocity v_2 in the rock layer below. Two rays reach the detector SRD (direct reflection) and $SABD$ that encounters a hard rock layer at a critical angle. (a) What is the time taken for the path $SABD$. (b) Determine the condition under which the critically refracted ray reaches D before the directly reflected ray.

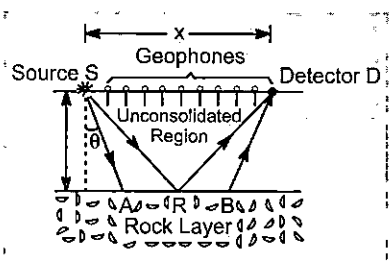


Fig. 1.5

[Ans. (a) $t = \frac{x}{v_2} + \frac{2d}{v_1}\cos C$, (b) $x > 2d\sqrt{\frac{v_1+v_2}{v_2-v_1}}$ **]**

6. An optical fibre has a cylindrical cross-section of diameter d and index of refraction n bent sharply. What is the smallest radius of curvature at a short bent section for which the total internal reflection will be assured for light initially travelling parallel to the axis of the fibre ?

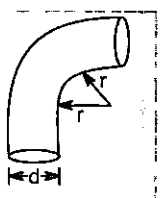


Fig. 1.6

[Ans. $(R_1+R_2)^2 + [\omega(L_1+L_2) - (C_1+C_2)/\omega C_1 C_2]^2$**]**

7. A layer of oil of refractive index n_0 floats on water of refractive index n_w. A beam of light is directed upward from a source in the water. Show that the critical angle for total internal reflection is independent of n_0, even though, for angles close to C (critical angle), the total internal reflection always takes place at the oil-air surface.

8. A prism having a 120° apex angle is made of glass for which $n = 1.53$. (a) Is there any range of angles of incidence i_1 for light rays striking one face of the prism that will result in total reflection at the second face ? (b) If so, what is the value of i_1 that corresponds to the critical angle for total reflection ?

[Ans. (a) yes, for $40.81\ v < i_1 < 90°$, (b) no value of i_1 exists]

9. Three plane mirrors are arranged so that they are mutually perpendicular as shown in Fig.1.9. Let the line of intersection of the mirrors be the coordinate axes. (a) A ray OP represented by vector $\vec{r}_1 = (x_1, y_1, z_1)$ is reflected from the mirror in the x-y plane. What are the components of the reflected vector? (b) If $\vec{r}_2$ is now reflected at the y-z plane, what are the components of the reflected vector $\vec{r}_3$? (c) If $\vec{r}_3$ is reflected at the z-x plane, what are the components of the reflected vector $\vec{r}_4$? (d) Is there any significant difference for an incident ray that is reflected from the mirrors in some other order ?

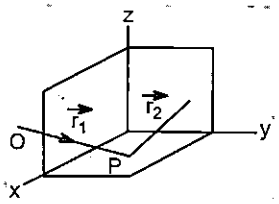


Fig. 1.9

[Ans. (a) $\vec{r}_2 = (x_1, y_1, -z_1)$, (b) $\vec{r}_3 = (-x_1, y_1, -z_1)$, (c) $\vec{r}_4 = (-x_1, -y_1, -z_1)$ (d) No]

10. A material having an index of refraction n is surrounded by a vacuum and is in the shape of a quarter circle of radius R. A light parallel to base of the material is incident from the left at a distance of L above the base and emerges out of the material at the angle θ. Determine an expression for θ.

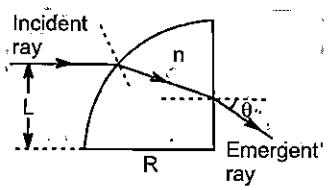


Fig. 1.10

[Ans. $\theta = \sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{\mu^2 R^2 - L^2} - \sqrt{R^2 - L^2}\right)\right]$]

11. A ray of light is refracted through a sphere (refractive index μ) in such a way that it passes through the extremities of two radii which make an angle β with each other. Prove that if α is the deviation of the ray caused by its passage through the sphere,

$$\cos\left(\frac{\beta - \alpha}{2}\right) = \mu \cos\left(\frac{\beta}{2}\right)$$

12. In Fig. 1.12 shown the light beam strikes surface 2 at the critical angle. Determine the angle of incidence i_1.

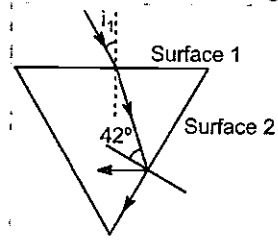


Fig. 1.12

[Ans. 27.5°]

13. A transparent cylinder of radius $R = 2$ m has a mirrored surface on its right half as shown in figure. A light ray travelling in air is incident on the left surface of the cylinder. The incident light ray and emergent ray are parallel and $d = 2$ m. Determine the refractive index of the material.

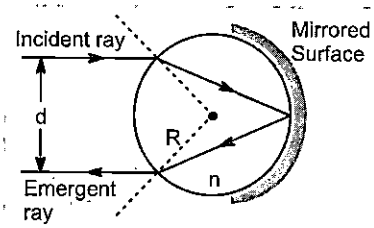


Fig. 1.13

14. A light ray travelling in air is incident on one face of a right angled prism with a refractive index $n = 1.5$ as shown in the figure and the ray follows the path shown in Fig. 1.14. If θ = 60° and the base of the prism is mirrored, what is the angle φ made by the emergent ray with the normal to the right face of the prism ?

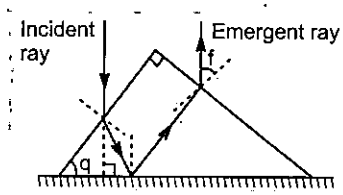


Fig. 1.14

15. An optical fibre of a glass core (refractive index n_1) is surrounded by a cladding (refractive index $n_2 < n_1$). Suppose a beam of light enters the fibre from air at an angle θ with the fibre axis as shown in the figure. (a) Show that the greatest possible value of θ for which a ray can be propagated down the fibre is given by

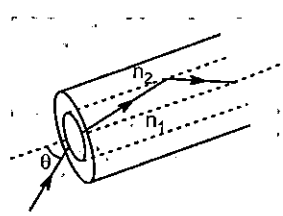


Fig. 1.15

$$\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

(b) In an optical fibre different rays travel different paths along the fibre, leading to different travel times. This causes a light pulse to spread out as it travels

along the fibre, resulting in information loss. The delay time should be minimized in designing a fibre. Consider a ray that travels a distance L along fibre axis and another that is reflected at the critical angle as it travels to the same destination as the first. Show that the difference Δt in the times of arrival is given by

$$\Delta t = \frac{L}{c}\frac{n_1}{n_2}(n_1 - n_2)$$

16. A light ray enters a rectangular glass slab at an angle $i_1 = 45°$ and emerges at an angle $i_2 = 76°$, as shown in Fig. 1.16. (a) Determine the refractive index of refraction for the slab. (b) If the light ray enters the plastic at a point $L = 50$ cm from the bottom edge, how long does it take the light ray to travel through the slab ?

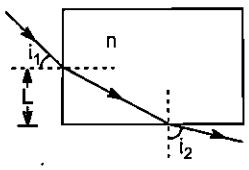


Fig. 1.16

[Ans. (a) 1.20, (b) 3.40 ns]

17. Fig. 1.17 shows an arrangement of mirrors forming a range finder for a camera. Mirror M_1 is partially silvered and fixed in position while M_2 is totally silvered and rotates about a vertical axis. Looking into M_1, you would see two images and by rotating M_2, they are made to exactly overlap. Since d is known accurately and the angle through which the second mirror is turned can be easily measured, a scale can be computed that displays the distance L directly on the camera. Write an expression for L in terms of ϕ and d, taking the tilt of the fixed mirror to be 45°.

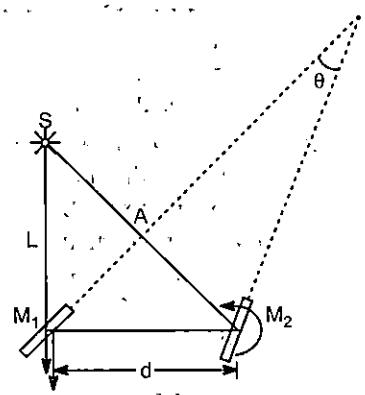


Fig. 1.17

[Ans. $L = d/\tan 2\phi$]

18. Fig. 1.18 shows an arrangement to determine the speed of light in air to that in water. When mirror M^*, which can be rotates, is at rest in position 1, light travels from the source through a narrow slit to M^*, then to M_1, then back to M^*, and finally to a ground glass scale where an image of slit is cast at S_0. When M^* is at rest in position 2, the light reflected from M^* passes through a water filled tube, is reflected at M_2, passes back through the tube to M^*, and again falls on S_0. (a) What happens when M^* is neither in position 1 nor in position 2 ? When M^* is rotating rapidly with angular speed ω, light travels alternately over both paths. However, the light reflected from M_1 falls on the scale at S_1, and the light reflected from S_2. Explain qualitatively why (b) the light beams are

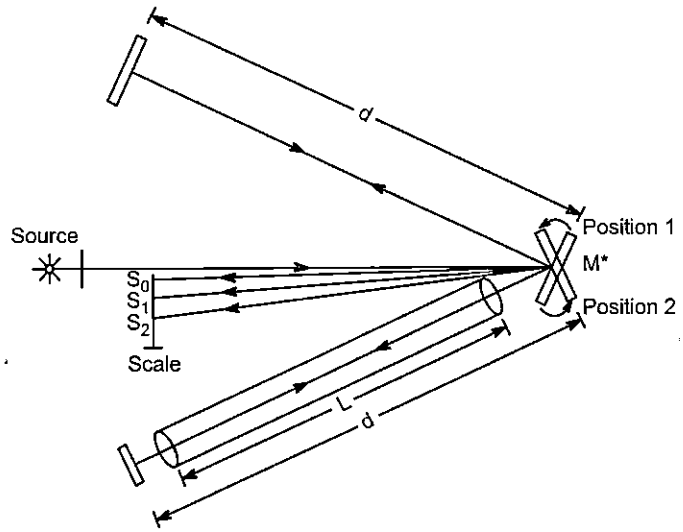


Fig. 1.18

displaced from S_0 and (c) the two displacements are not same. (d) Let δ_1 be the distance between S_0 and S_1. Neglecting the difference between the speeds of light in air and c, the speed in vacuum, show that

$$c = \frac{4R\omega d}{\delta_1}$$

(e) Let δ_2 be the distance between S_0 and S_2, show that

$$\frac{c}{v_{water}} = 1 + \left(\frac{\delta_2}{\delta_1} - 1\right)\frac{d}{L}$$

which is independent of ω and c.

19. A fish is at a depth d under water. Take the index of refraction of water at 4/3. Show that when the fish is viewed at an angle of refraction i, the apparent depth of the fish is

$$h = \frac{3d\cos i}{\sqrt{7 + 9\cos^2 i}}$$

20. If light travelling in a medium for which the index of refraction is n_1 is incident at an angle θ on the surface of a medium of index n_2 so that the angle between the reflected and refracted rays is β, show that

$$\tan\theta = \frac{n_2 \sin\beta}{n_1 - n_2\cos\beta}$$

21. A concave mirror forms the real image of a point source lying on the optical axis at a distance of 50 cm from the mirror. The focal length of the mirror is 25 cm. The mirror is cut in two and its halves are drawn a distance of 1 cm apart in a direction perpendicular to the optical axis (Fig. 1.21). How will the images formed by the halves of the mirror be arranged ?

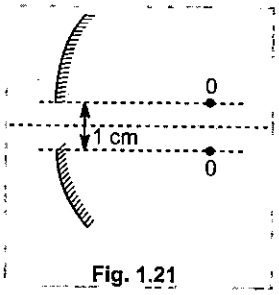


Fig. 1.21

[**Ans.** At a distance of 50 cm from the mirror and 2 cm from each other.]

22. A thin flat glass plate is placed infront of a convex mirror. At what distance b from the plate (Fig. 1.22) should a point source of light S be placed so that its image produced by the rays reflected from the front surface of the plate coincides with the image formed by the rays reflected from the mirror ? The focal length of the mirror is $F = 20$ cm and the distance from the plate to the mirror $a = 5$ cm. How can the coincidence of the images be established by direct observation ?

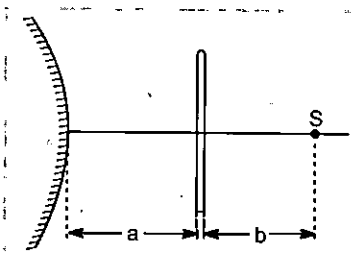


Fig. 1.22

[**Ans.** $b = \sqrt{2aF + a^2} = 15$ cm]

23. The focal length of a concave mirror can roughly be determined by the following method: place a needle A at a distance d from the mirror (Fig. 1.23), then place a flat mirror P at a distance a from the concave mirror and a second needle B at a distance b from the flat mirror. Move the mirror P to match the virtual images A and B of both needles formed by the concave and flat mirrors. Knowing the values of a, b and d corresponding to the coincidence of the images, determine the focal length of the mirror. Can these images be observed by the eye at the same time ?

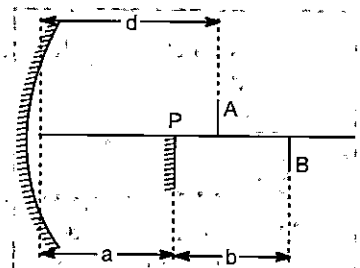


Fig. 1.23

[**Ans.** $F = \dfrac{d(b-a)}{b-(a+d)}$]

24. A screen S is placed a distance $b = 5$ cm from a circular convex mirror as shown in Fig. 1.24. An object KP of height $h = 3$ cm is arranged a distance $a = 5$ cm from the screen. Where must an observer position himself to see the image of the entire object ? What are the maximum dimensions of the object (with the given arrangement of the object, the mirror and the screen) for the mirror to reproduce an image of the entire object ? The diameter of the mirror is $d = 10$ cm.

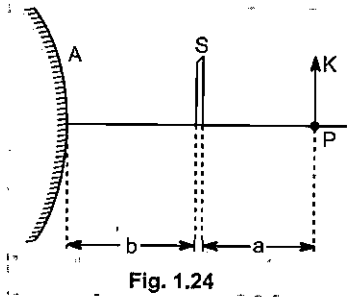


Fig. 1.24

[**Ans.** $h_1 = 5$ cm]

25. Photographs of the ground are taken from an aircraft flying at an altitude of 2,000 m by a camera with a focal length of 50 cm. (a) What will be the scale of the photographs ? How will the scale change if the aircraft flies at an altitude of 1,000 m? (b) The size of the film in the camera in the previous problem is 18 × 18 cm. What area can be photographed by this camera at any one time ?

[**Ans.** (a) 1 : 4000 and 1 : 2000,
(b) $S_1 \simeq 0.52\,km^2$, $S \simeq 0.13\,km^2$]

26. A convergent lens forms on a screen an image of a lamp magnified to twice its normal size. After the lens has been moved 36 cm closer to the screen, it gives an image diminished by a factor of two. Find the focal length of the lens.

[**Ans.** $f = 24$ cm]

27. What are the smallest details of an object that can be observed separately with the naked eye at a distance of 2 km ? The minimum angle of vision of eye is $\phi = 1'$.

[**Ans.** 60 cm]

28. A thin convexo-convex lens is placed on a flat mirror. Where should a point source of light be arranged so that its image produced by this system is real and coincides with the source itself ?

[**Ans.** At the principal focus of the lens]

29. An optical system consists of a convergent lens with a focal length of 30 cm and a flat mirror placed at a distance $b = 15$ cm from the lens. Determine the position of the image formed by this system if an object is at a distance $a_1 = 15$cm infront of the lens. Plot the path of the rays in this case.

[**Ans.** At 60 cm on the same side as the object]

30. Plot the image of an object in an optical system consisting of a convergent lens and a flat mirror arranged in the focal plane of the lens. The object is infront of the lens and between the focus and the double focal length of the lens. What will be the size of the image if the object is positioned arbitrarily ?

[**Ans.** The image will be real and will lie between the lens and the focus]

31. Determine the position of the image produced by an optical system consisting of a concave mirror with a focal length of 10 cm and a convergent lens with a focal length of 20 cm. The distance from the mirror to the lens is 30 cm and from the lens to the object 40 cm. Plot the image.
[Ans. 100 cm**]**

32. A convergent and a diverging lenses having focal lengths of 30 and 10 cm, respectively, are arranged at a distance of 20 cm from each other. Where should a source of light be placed for this system to emit a beam of parallel rays ?
[Ans. The source should be at infinity**]**

33. Plot the image of an object formed by a system of two convergent lenses. The focal length of the first lens is 9 cm and of the second 15 cm. The second lens is in the focal plane of the first lens.The object is at a distance of 36 cm from the first lens. Calculate the distance a_2 of the image from the second lens.
[Ans. 2.5 cm**]**

34. A plano-parallel plate is cut as shown in Fig. 1.34 and the lenses thus obtained are slightly drawn apart. How will a beam of parallel rays change after passing through this system if the beam is incident: (a) from the side of the convex lens? (b) from the side of the concave lens ? How will the behaviour of the beam depend on the distance between the lenses ?

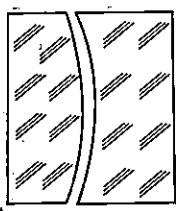


Fig. 1.34

35. The distance between two point sources of light is $l = 24$ cm. Where should a convergent lens with a focal length of $f = 9$ cm be placed between them to obtain the images of both sources at the same point ?
[Ans. 6 cm from one source and 18 cm from the other**]**

36. A thin convergent lens produces the image of a certain object on a screen. The height of the image is h_1. Without changing the distance between the object and the screen, the lens is shifted, and it is found that the height of the second sharp image is h_2. Determine the height of the object H.
[Ans. $H = \sqrt{h_1 h_2}$**]**

37. What is the radius R of a concave spherical mirror at a distance of $a = 2$ metre from the face of a man if he sees in it his image that is one and a half times greater than on a flat mirror placed at the same distance from the face ?
[Ans. $R = 6$ m**]**

38. A man wishing to get a picture of a zebra photographed a white donkey after fitting a glass with black streaks on to the objective of his camera. What will be on the photograph ?
[Ans. Each section of the lens produces a full image irrespective of the other sections. Therefore, there will be no strips on the photograph, and the image will simply be less bright.**]**

39. The layered lens shown in Fig. 1.39 (a) is made of two kinds of glass. What image will be produced by this lens with a point source arranged on the optical axis ? Disregard the reflection of light on the boundary between layers.

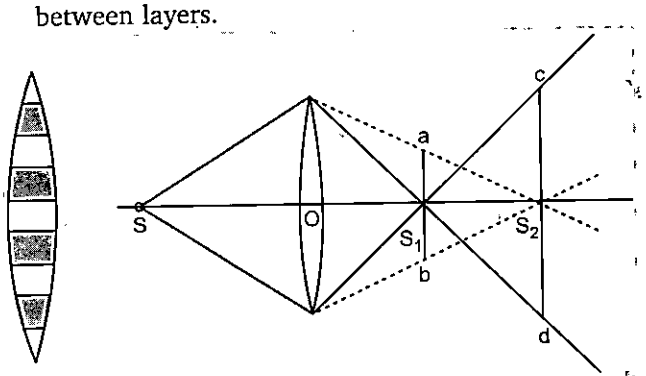


Fig. 1.39

[Ans. Any section of the lens gives an image identical in shape to that produced by the entire lens. The layered lens can be therefore regarded as two lenses with different focal lengths, but with a common optical centre. Accordingly this lens will produce two images: at point S_1 and at point S_2 [Fig. 1.39 (b)]. The image of the source will be surrounded by a bright halo with a diameter of ab or, respectively, cd on a screen arranged perpendicular to the optical axis at point S_1 or S_2.**]**

40. A converging beam of rays is incident on to a divergent lens so that the continuations of all the rays intersect at a point lying on the optical axis of the lens at a distance of $b = 15$ cm from it. Find the focal length of the lens in two cases : (a) After being refracted in the lens, the rays are assembled at a point at a distance of $a_1 = 60$ cm from the lens. (b) The continuations of the refracted rays intersect at a point at a distance of $a_2 = 60$ cm infront of the lens.
[Ans. (a) 20 cm, (b) 12 cm**]**

41. The distance between an electric lamp and a screen is $d = 1$ metre. In what positions of a convergent lens with a focal length of $f = 21$ cm will the image of the lamp filament be sharp? Can an image be obtained if the focal length is $f' = 26$ cm ?
[Ans. 70 cm and 30 cm from the lamp, no**]**

42. Find by construction the optical centre of a lens and its main foci on the given optical axis N_1N_2 if the positions of the source S and the image S' are known [Fig. 1.42 (a)]

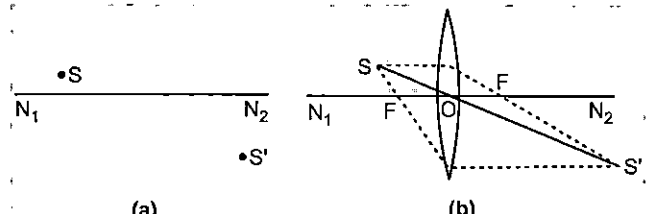


Fig. 1.42

[Ans. The centre of the lens O is the point of intersection of straight lines SS' and N_1N_2. The foci can easily be found by constructing the rays parallel to the major optical axis [Fig. 1.42 (b)]

43. The position of the optical axis N_1N_2, the path of ray AB incident upon a lens and the refracted ray BC are known [Fig. 1.43 (a)]. Find by construction the position of the main foci of the lens.

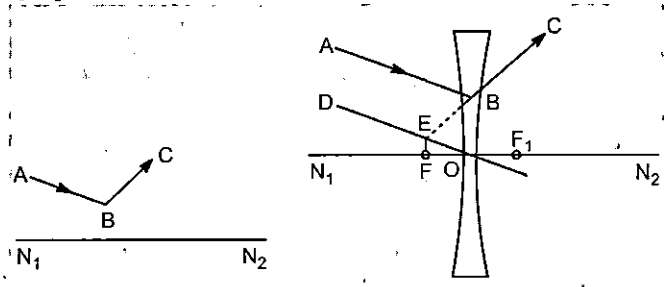


Fig. 1.43

[Ans. Point O, which is the optical centre of the lens, can be found by dropping perpendicular BO onto straight line N_1N_2 [Fig. 1.43 (b)]. Let us draw an auxiliary optical axis DO parallel to ray AB and extend straight line BC until it intersects DO at point E lying in the focal plane. Let us drop a perpendicular from E onto N_1N_2 to find point F, one of the main foci of the lens. By using the property of reversibility of the ray, we can find the other main focus F_1]

44. A parallel beam of rays is incident on a convergent lens with a focal length of 40 cm. Where should a divergent lens with a focal length of 15 cm be placed for the beam of rays to remain parallel after passing through the two lenses ?

45. A parallel beam of light is incident on a system consisting of three thin lenses with a common optical axis. The focal lengths of the lenses are equal to $f_1 = +10$ cm, $f_2 = -20$ cm and $f_3 = +9$ cm, respectively. The distance between the first and the second lenses is 15 cm and between the second and the third 5 cm. Find the position of the point at which the beam converges when it leaves the system of lenses.

[Ans. The rays will travel in a parallel beam.]

46. A lens with a focal length of $f = 30$ cm produces on a screen a sharp image of an object that is at a distance of $a = 40$ cm from the lens. A plane parallel plate with a thickness of $d = 9$ cm is placed between the lens and the object perpendicular to the optical axis of the lens. Through what distance should the screen be shifted for the image of the object to remain distinct ? The refraction index of the glass of the plate is $n = 1.8$.

[Ans. The screen should be shifted by 60 cm]

47. An object is at a distance of $d = 2.5$ cm from the surface of a glass sphere with a radius of $R = 10$ cm. Find the position of the image produced by the sphere. The refraction index of the glass is $n = 1.5$.

[Ans. Virtual image at 15 cm infront of the sphere]

48. A spherical flask is made of glass with a refraction index n. The thickness of the flask walls ΔR is much less than its radius R. Taking this flask as an optical system and considering only the rays close to the straight line passing through the centre of the sphere, determine the position of the foci and the main planes of the system.

[Ans. $\frac{R^2}{2(n-1)\Delta R}$]

49. A beam of light is incident on a spherical drop of water at an angle i. Find the angle θ through which the beam is deflected from the initial direction after a single reflection from the internal surface of the drop.

[Ans. $\theta = \pi + 2i - 4\sin^{-1}\left(\frac{\sin i}{n}\right)$]

50. A glass sphere ($n = 1.50$) with a radius of 15.0 cm has a tiny air bubble 5.00 cm above its centre. The sphere is viewed looking down along the extended radius containing the bubble. What is the apparent depth of the bubble below the surface of the sphere ?

[Ans.8.57 cm]

51. A hollow glass sphere of radius 5.00 cm is filled with glycerine. When shaken, bits of white plastic simulate snow falling on a small figure of Santa Claus inside the sphere. If the Santa figure is 4.26 cm from the outside surface of the sphere, how close to the surface does he appear to be to someone looking in at him ? (Assume the sphere has the same index of refraction as glycerine.)

[Ans. 3.98 cm]

52. One end of a long glass rod ($n = 1.50$) is formed into a convex surface of radius 6.00 cm. An object is positioned in air along the axis of the rod. Find the image positions corresponding to object distances of (a) 20.0 cm, (b) 10.0 cm and (c) 3.00 cm from the end of the rod.

[Ans. (a) 45 cm, (b) –90 cm, (c) – 6 cm]

53. A goldfish is swimming inside a spherical plastic bowl of water, with an index of refraction of 1.33. If the goldfish is 10.0 cm from the wall of the 15.0 radius bowl, where does it appear to an observer outside the bowl ?

[Ans. Inside the bowl at –9.01 cm]

54. A goldfish inside a spherical bowl sees a Cheshire cat through the side of the bowl. If the bowl has a diameter of 25.0 cm and the cat's eye is 10.0 cm from

the bowl, where is the image of the cat's eye as seen by the goldfish ? Ignore the bowl's thin layer of glass.
[Ans. – 18.1 cm]

55. A novelty aquarium is built so that one glass face ($n = 1.50$) is actually a plano-convex lens. The convex face with radius of curvature 10.0 cm is on the outer side of the aquarium, which is filled with water ($n = 1.33$). To a viewer on the outside, what is the apparent length of a 1.00 cm long tetra swimming parallel to and 25.0 cm from the inside face of the aquarium ?
[Hint: When two different media of refractive indices n_1 and n_3 are on opposite sides of a lens of index n_2, the lens-maker's formula becomes

$$\frac{n_1}{u}+\frac{n_3}{v}=\frac{n_2-n_1}{R_1}+\frac{n_3-n_2}{R_2}$$]

56. Two converging lenses with a common optic axis and the secondary focal point of the first coincident with the primary focal point of the second is known as an *afocal* arrangement. A pair of afocal lenses can be used as a *beam expander* to change the diameter of a collimated beam of light entering along the optic axis. If the lenses have focal lengths f_1 and f_2 and the entering beam has diameter d, find the expression for the diameter D of the exiting beam.
[Ans. $D = (f_2/f_1)d$]

57. A biconcave lens ($f_1 = -25.0$ cm) is placed 20.0 cm infront of a concave mirror ($f_2 = 5.00$ cm) as shown in Fig. 1.57. A 2.00 cm high object is placed 15.0 cm to the left of the lens. Using the lens and mirror equations and a ray diagram, find the location of three images : (a) the image formed by the lens as rays travel to the right, (b) the image formed after rays reflect from the mirror, and (c) the final image after the leftward travelling rays once again pass through the lens. Complete the ray diagram and characterize each image and object.

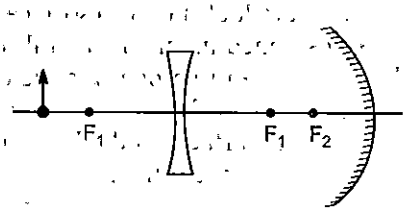


Fig. 1.57

[**Ans.** – 9.375 cm, + 0.625 cm, virtual erect ; 6.026 cm, –0.205, real, inverted; – 8.96 cm, 0.641 ; final image is 8.96 cm to the right of the lens, + 0.0822, virtual, enlarged] .

58. An object 2.00 cm high is placed 40.0 cm to the left of a converging lens with a focal length of 30.0 cm. A diverging lens with a focal length of –20.0 cm is placed 110 cm to the right of the converging lens. (a) Determine the final position and magnification of the final image. (b) Is the image upright or inverted ? (c) Repeat parts (a) and (b) for the case in which the second lens is a converging lens with a focal length of + 20.0 cm.
[**Ans.** (a) 20 cm to the right of second lens; $M = -6$ (b) inverted (c) 6.67 cm to the right of second lens, $M = -2$, inverted]

59. The object in Fig. 1.59 is midway between the lens and the mirror. The mirror's radius of curvature is 20.0 cm and the lens has a focal length of –16.7 cm. Considering only the light that leaves the object and travels first toward the mirror, locate the final image formed by this system. Is this image real or virtual? Is it upright or inverted ? What is the overall magnification ?

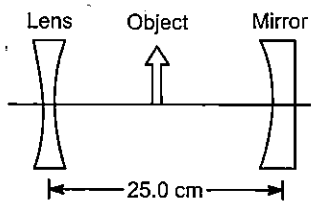


Fig. 1.59

[**Ans.** 25.3 cm to the right of the mirror, virtual, upright, enlarged 8.05 times]

60. A converging lens ($f_1 = 12.0$ cm) is separated by 50.0 cm from a diverging lens ($f_2 = -10.0$ cm) as shown in Fig. 1.60. An object 1.0 cm high is placed 25.0 cm to the left of the first lens. Find the position of the final image and its height.Complete a ray diagram and characterize the image.

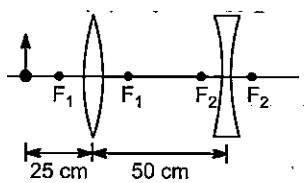


Fig. 1.60

[**Ans.** – 7.29 cm, to the left of the second lens, $m = -0.25$, virtual, inverted, $h_2 = 0.250$ cm]

61. The lens and mirror in Fig. 1.61 have focal lengths of +80.0 cm and – 50.0 cm, respectively. An object is placed 1.00 m to the left of the lens, as shown. Locate the final image, which is formed by light that has gone through the lens twice. State whether the image is upright or inverted and determine the overall magnification.

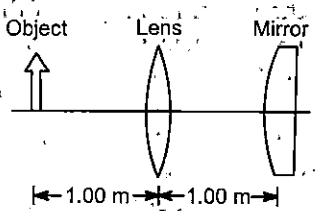


Fig. 1.61

[**Ans.** 160 cm to the left of the lens ; inverted, $m = -0.8$]

62. Fig. 1.62 shows a ray, parallel to the axis but distant from it, falling on a concave spherical mirror of radius R at P. The centre of curvature of the mirror is at C, and the three angles ϕ are equal. The reflected ray misses the focus F. The distances x and y are known, respectively, as the *longitudinal* and *transverse spherical aberration.*

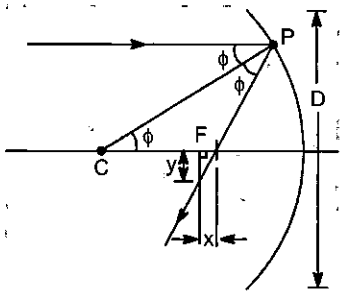


Fig. 1.62

(a) Show that $x = \frac{R}{2}\left(\frac{1}{\cos\phi} - 1\right)$

(b) Show that $y = x\tan 2\phi = \frac{R}{2}\left(\frac{1}{\cos\phi} - 1\right)\tan 2\phi$

63. The spherical aberration shown can be eliminated by using a mirror of paraboloidal form. The figure shows a cross section of such a mirror. Shown also is the geometrical definition of a parabola as the locus of points equidistant from a point called the focus and a line called the directrix. Show that all the rays parallel to the axis are reflected so as to pass through the focus, regardless of their distance from the axis.

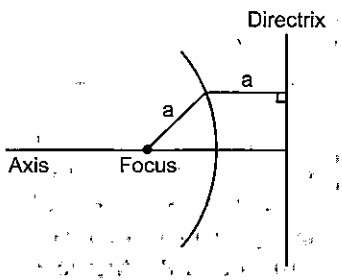


Fig. 1.63

[Hint. You may find it easier to carry out the proof in terms of parallel wave fronts, using Huygen's construction]

64. According to the geometrical definition, an ellipse is the locus of all points for which the sum of the distances from two points called the foci is a constant. Using this definition, show that a mirror in the form of an ellipse, with a point object at one focus, will produce an image at the other focus.

65. Fig. 1.65 shows a thin glass ($n = 1.50$) converging lens for which the radii of curvature are $R_1 = 15.0$ cm and $R_2 = -12.0$ cm. To the left of the lens is a cube with a face area of 100 cm^2. The base of the cube is on the axis of the lens, and the right face is 20.0 cm to the left of the lens. (a) Determine the focal length of the lens. (b) Draw the image of the square face formed by the lens. What type of geometric figure is this ? (c) Determine the area of the image.

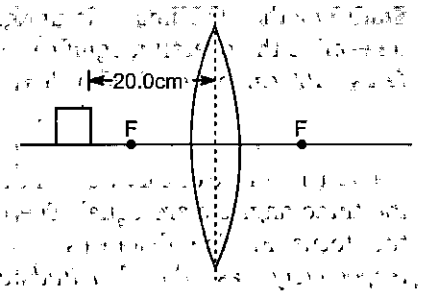


Fig. 1.65

66. A biconvex lens ($f_1 = 10.0$ cm) is placed 40.0 cm infront of a concave mirror ($f_2 = 7.50$ cm) as shown in Fig. 1.66. An object 2.00 cm high is placed 20.0 cm to the left of the lens. Using the lens and mirror equations and a ray diagram, find the locations of three images : (a) the image formed by the lens as rays travel to the right, (b) the image formed after the rays reflect from the mirror, and (c) the final image after the leftward travelling rays once again pass through the lens. Complete the ray diagram and characterize each image and object.

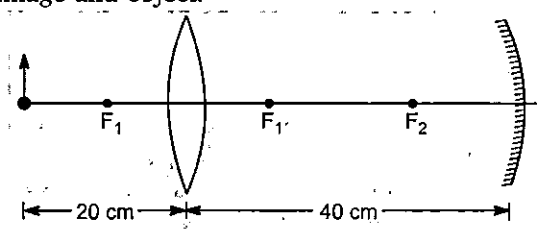


Fig. 1.66

[Ans. 20cm, –1, real, inverted; +12 cm, –0.6, real, erect (that is reinverted); 15.6 cm, –0.556, real, inverted; overall magnification 0.333.]

67. A spherometer is a device used to measure the radius of curvature R of spherical surfaces. It consists of three legs positioned in an equilateral triangle with sides of length y. A centre probe is raised or lowered by means of a micrometer drive. The spherometer is first placed on a flat surface and the zero position of the micrometer noted. The instrument is then placed on a spherical surface and the micrometer is adjusted until the centre probe just touches the surface. The displacement z of the micrometer between the two readings is known as the *sagitta*. (a) Make a simple argument to show that $y^2 + (z - R)^2 = R^2$ so that $R = \frac{z^2 + y^2}{2z}$

(b) A student records the following micrometer positions in millimetre : flat surface 11.222, convex face 8.716, concave face 12.473, when using a spherometer with y given by the manufacturer as 50.0 mm. The student subsequently measures the focal length of the lens to be 186 cm. What is the index of refraction of the material from which the lens is constructed ?

[Ans. (b) 1.538]

68. Two rays travelling parallel to the principal axis strike a large plano-convex lens having a refractive index of 1.60 (Fig. 1.68). If the convex face is spherical, a ray near the edge does not pass through the focal point (spherical aberration occurs). If this face has a radius of curvature of 20.0 cm and the two rays are

$h_1 = 0.500$ cm and $h_2 = 12.0$ cm from the principal axis, find the difference in the positions where they cross the principal axis.

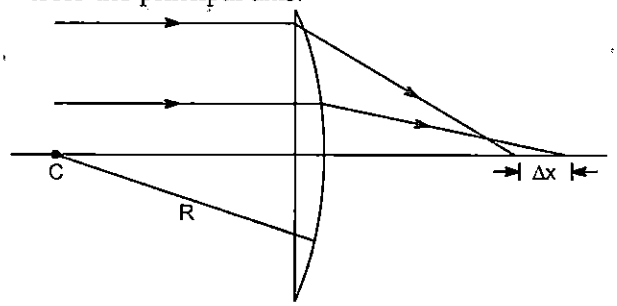


Fig. 1.68

[Ans. 21.3 cm**]**

69. A parallel beam of light enters a glass hemisphere perpendicular to the flat face, as shown in Fig. 1.69. The radius is $R = 6.00$ cm and the index of refraction is $n = 1.560$. Determine the point at which the beam is focussed. (Assume paraxial rays).

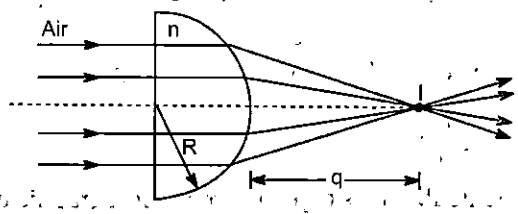


Fig. 1.69

[Ans. 0.107 m to the right of the vertex of the hemispherical face.**]**

70. A biconvex thin lens is made of glass having index of refraction n. The lens has surfaces of equal curvature R. The lens is made into a mirror by silvering one of the surfaces. Parallel light is incident on the lens from the clear (unsilvered) side. Show that the focal length of the lens/mirror is

$$f = \frac{R}{2(2n-1)}$$

***71.** A round hall with a diameter of $D = 30$ metre is illuminated by a lamp secured in the centre of the ceiling. Find the height h of the hall if the minimum illumination of the wall is double that of the floor.

[Ans. $h = 7.5$ m **]**

***72.** A lamp rated at $I_1 = 100$ cd hangs above the middle of a round table with a diameter of $D = 3$ metre at a height of $H = 2$ metre. It is replaced by a lamp with $I_2 = 25$ cd and the distance to the table is changed so that the illumination of the middle of the table remains as before. How will the illumination of the edge of the table change?

[Ans. $\frac{E_1}{E_2} = 3$ **]**

***73.** Sources of light S_1 and S_2 of equal intensity are arranged at the vertices of an isosceles right triangle (Fig. 1.73). How should a small plate A be positioned for its illumination to be maximum ? The sides of the triangle $AS_1 = AS_2 = a$

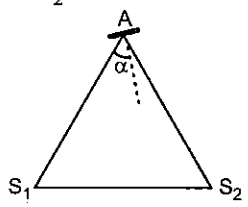


Fig. 1.73

[Ans. $E_{max} = \sqrt{2}\frac{I}{a^2}$**]**

***74.** An attempt to use a photometer to measure the luminous intensity of a certain source of light failed since the luminous intensity was very high and the illumination of the photometer fields with the aid of a standard source could not be equalized even when the source being investigated was placed on the very edge of the photometer bench. Then a third source was employed with a luminous intensity lower than that of the one being investigated. At a distance of $r_1 = 10$ cm from the photometer the standard source produced the same illumination of the fields as the third one that was placed at a distance of $r_2 = 50$ cm. After that, the standard source was replaced by the one being investigated and equal illuminations were obtained at distances from the photometer $r_3 = 40$ cm (source being investigated) and $r_4 = 10$ cm (auxiliary source). Find how many times the luminous intensity of the source being investigated is greater than that of the standard source.

[Ans. $I_x = 400 I_0$**]**

***75.** The ray of a searchlight falls on the wall of a house and produces a bright spot with a radius of $r = 40$ cm. How many times will the illumination of the wall of a remote house be smaller if the radius of the spot on it is 2 metre ?

[Ans. The illumination will be 25 times smaller**]**

***76.** A lamp with a luminous intensity of $I = 100$ cd is fastened to the ceiling of a room. Determine the total luminous flux falling onto all the walls and the floor of the room.

[Ans. $\phi = 628$ lm **]**

***77.** What part of the energy radiated by the Sun reaches the Earth ? The radius of the Earth is 6,400 km and the average distance from the Earth of the Sun is 149,000,000 km.

[Ans. The earth receives $\frac{1}{2.25 \times 10^9}$ of the total energy of the Sun**]**

* Problem based on photometry

*78. A hot glowing wire is placed on the axis of a hollow cylinder with a radius R. The length of the wire is much greater than the height of the cylinder. How many times will the illumination of the internal surface of the cylinder change if its radius is R_2 (assume that $R_2 < R_1$) ?
[Ans. $\frac{E_1}{E_2} = \frac{R_2}{R_1}$]

*79. At what height should a lamp be hung above the centre of a round table to obtain the maximum illumination on its edges ?
[Ans. $h \simeq 0.71R$]

80. An equiconvex lens of crown glass and an equiconcave lens of flint glass make an achromatic system. The radius of curvature of the convex lens is 0.54 m. If the focal length of the combination for mean colour is 1.50 m and the refractive indices for the crown glass are $\mu_R = 1.53$ and $\mu_V = 1.55$, find the dispersive power of flint glass.
[Ans. $\omega_F = 0.055$]

81. Two lenses, one made of crown glass and the other of flint glass, are to be combined so that the combination is achromatic for the blue and red light and acts as a convex lens of focal length 35 cm. Calculate the focal length of the components if for :
Crown glass $\mu_Y = 1.5175$ and $(\mu_B - \mu_R) = 0.00856$
Flint glass $\mu_Y = 1.6214$ and $(\mu_B - \mu_R) = 0.01722$
[Ans. $f_{\text{flint}} = -23.8$ cm, $f_{\text{crown}} = 14.17$ cm]

82. State whether the following statement is true or false giving reason in brief : "A parallel beam of white light is incident on a combination of a concave and a convex lens, both of the same material. Their focal lengths are 15 cm and 30 cm respectively, for the mean wavelength in white light. On the other side of the lens system, one sees coloured patterns with violet colour at the outer edge."

83. A telescope objective of focal length 60 cm is made of two thin lenses, one of crown glass of refractive index 1.52 and other of flint glass of refractive index 1.66. One surface of the flint glass is plane. Calculate the radii of curvature of both the lenses which form the achromatic doublet if dispersive powers of crown and flint glass are 0.0151 and 0.0302 respectively.
[Ans. Crown glass: 25.74 cm and 39.6 cm; Flint glass: 39.6 m and ∞]

84. Why does a swimmer see only hazy contours of objects when he opens his eyes under water, while they are distinctly visible if he is using a mask ?

85. A short-sighted man, the accommodation of whose eye is between $a_1 = 12$ cm and $a_2 = 60$ cm, wears spectacles through which he distinctly sees remote objects. Determine the minimum distance a_3 at which the man can read a book through his spectacles.
[Ans. 15 cm]

86. Two men, a far-sighted and a short-sighted one, see objects through their spectacles as a man with normal eyesight. When the far-sighted man accidentally puts on the spectacles of his short-sighted friend, he found that he could see distinctly only infinitely far objects. At what minimum distance a can the short-sighted man read small type if he wears the spectacles of the far-sighted man ?
[Ans. 12.5 cm]

87. An object is examined by a naked eye from a distance D. What is the angular magnification if the same object is viewed through a magnifying glass held at a distance r from the eye and so arranged that the image is at a distance L from the eye ? The focal length of the lens is f. Consider the cases : (1) $L = \infty$, (2) $L = D$.
[Ans. (1) $\frac{D}{f}$, (2) $\frac{D}{f} + 1 - \frac{r}{f}$]

88. The objective is taken out of a telescope adjusted to infinity and replaced by a diaphragm with a diameter D. A screen shows a real image of the diaphragm having a diameter d at a certain distance from the eyepiece. What was the magnification of the telescope?.
[Ans. $\frac{D}{d}$]

89. The double-lens objective of a photographic camera is made of a divergent lens with a focal length of $f_1 = 5$ cm installed at a distance of $l = 45$ cm from the film.Where should a convergent lens with a focal length of $f_2 = 8$ cm be placed to obtain a sharp image of remote objects on the film ?
[Ans. $d = \frac{l - f_1}{2} \pm \frac{l + f_1}{2}\sqrt{1 - \frac{4f_2}{1 + f_1}}$]

90. Verify the claim in the text that, in order to invert the image of a celestial telescope using a converging lens of focal length f, the length of the telescope must be lengthened by at least $4f$.

91. The main focal length of the objective of a microscope is $f_{\text{obj}} = 3$ mm and of the eyepiece $f_{\text{eye}} = 5$ cm. An object is at a distance of $a = 3.1$ mm from the objective. Find the magnification of the microscope for a normal eye.
[Ans. 180 times]

92. The Yerkes refracting telescope has a 1.00 m diameter objective lens with a focal length of 20.0 m. Assume that it is used with an eyepiece that has a focal length of 2.50 cm. (a) Determine the magnification of the planet Mars as seen through this telescope. (b) Are the Martian polar caps seen right side up or upside down ?
[Ans. (a) – 80.0, (b) upside down]

93. Astronomers often take photographs with the objective lens or the mirror of a telescope alone, without an eyepiece. (a) Show that the image size h' for this telescope is given by $h' = fh/(f - p)$, where h is

the object size, f is the objective focal length, and p is the object distance. (b) Simplify the expression in part (a) for the case in which the object distance is much greater than the objective focal length. (c) The "wingspan" of the International Space Station is 108.6 m, the overall width of its solar-panel configuration.Find the width of the image formed by a telescope objective of focal length 4.00 m when the station is orbiting at an altitude of 407 km.

94. Galileo devised a sample terrestrial telescope that produces an upright image. It consists of a converging objective lens and a diverging eyepiece at opposite ends of the telescope tube. For distant objects, the tube length is the objective focal length less the absolute value of the eyepiece focal length. (a) Does the user of the telescope see a real or the virtual image ? (b) Where is the final image ? (c) If a telescope is to be constructed with a tube 10.0 cm long and a magnification of 3.00, what are the focal lengths of the objective and the eyepiece ?
[Ans. (a) virtual, (b) at infinity]

95. A compound microscope has an objective of focal length 0.300 cm and an eyepiece of focal length 2.50 cm. If an object is 3.40 mm from the objective, what is the magnification ?
[Hint : Use the lens equation for the objective]
[Ans. – 75.0]

96. The distance between the eyepiece and the objective lens in a certain compound microscope is 23.0 cm. The focal length of the eyepiece is 2.50 cm, and that of the objective 0.400 cm. What is the overall magnification of the microscope ?
[Ans. – 575]

97. Galileo's telescope [he called it a *cannocchiale* ('eye-tube')] is sketched in Fig. 1.97. As in the telescope the second focus of the objective coincides with the first focus of the ocular. However, the ocular is a *diverging* lens. Draw a ray diagram for the Galilean telescope. Is the image erect or inverted ? real or virtual ?

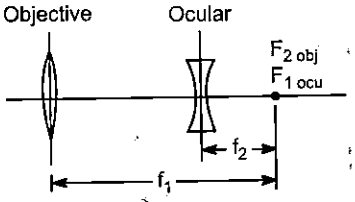


Fig. 1.97

98. A nearsighted person cannot see objects clearly beyond 25.0 cm (her far point). If she has no astigmatism and contact lenses are prescribed for her, what power and type of lens are required to correct her vision ?
[Ans. 4 dioptres ; a diverging lens]

99. Show that, if you place the object so that the image is at your near point instead of at infinity, the angular magnification is increased to the value

$$\alpha = \frac{d}{f} + 1$$

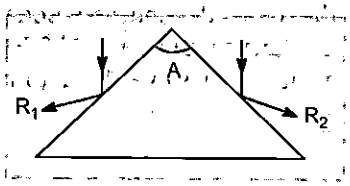
100. A fisherman tying flies desires a magnifier that produces an angular magnification of 2.00 when used with eyes in the relaxed (unaccommodated) state. What focal length lens should be used ? How close to the fly does the sportsman's eyes have to be ?
[Ans. 12.5 cm]

Problems

Level 1 Only One Alternative is Correct

1. For a prism kept in air it is found that for an angle of incidence 60°, the angle of refraction 'A', angle of deviation 'δ' and angle of emergence 'e' become equal. Then the refractive index of the prism is :
(a) 1.73 (b) 1.15
(c) 1.5 (d) 1.33

2. A parallel beam of light is incident on a prism of prism angle A as shown figure. Find the angle θ between the two reflected beams R_1 and R_2 from two faces as shown.

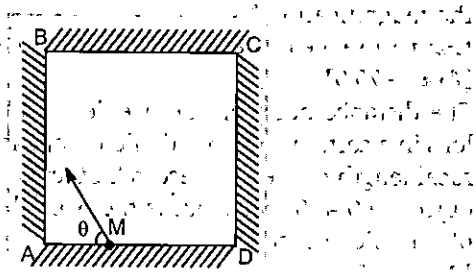


(a) $2A$ (b) $3A$
(c) $4A$ (d) none of these

3. Two thin prisms of dispersive power ω and ω' are combined such that they neither produce average deviation nor do they produce dispersion. If angle of one of the prisms is 2° and the refractive index is 1.5 for yellow light, 1.49 for red light and 1.51 for violet light, find ω upto two digits after decimal:
(a) 0.01 (b) 0.02
(c) 0.03 (d) 0.04

4. A television picture is built up by scanning successive horizontal lines across the screen. It takes about 50 microseconds to scan the screen width. Ghosting refers to a weak horizontally displaced repetition of the main picture. These occur when the TV displays an additional delayed signal, due to reflection from an object such as a hill. If the ghost picture is displaced by 10% of the screen width, what is the additional path length of the reflected signal ?
(a) 15 km (b) 1.5 km
(c) 15 m (d) 1.5 m

5. Four identical mirrors are made to stand vertically to form a square arrangement as shown in a top view. A ray starts from the mid point M of mirror AD and after two reflections reaches corner D. Then, angle θ must be:

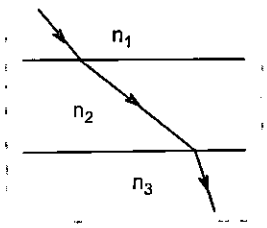


(a) $\tan^{-1}(0.75)$ (b) $\cot^{-1}(0.75)$
(c) $\sin^{-1}(0.75)$ (d) $\cos^{-1}(0.75)$

6. A small object is placed at the bottom of a beaker containing a liquid of refractive index $\frac{4}{3}$ to a depth of 20 cm. A pin is placed horizontally over the beaker and adjusted until there is no parallax between the image of the pin formed by reflection at the liquid surface and the image of the object. The height of the pin above the liquid surface is :
(a) 20 cm (b) 15 cm
(c) 10 cm (d) 30 cm

7. A parallel beam of light passes parallel to the axis and falls on one face of a thin convex lens of focal length f and after two internal reflections emerges from the second face and forms a real image. Find the distance of the image from the lens if μ is the refractive index of the lens?
(a) $f(\mu-1)/3\mu-1$ (b) $(\mu-1)/f(3\mu-1)$
(c) $(3\mu-1)/f(\mu-1)$ (d) $f(\mu-1)$

8. The figure shows the path of a ray of light as it passes through three different materials with refractive indices n_1, n_2 and n_3. The figure is drawn to scale. What can we conclude concerning the indices of these three materials ?



(a) $n_3 > n_2 > n_1$ (b) $n_3 < n_1 < n_2$
(c) $n_2 < n_1 < n_3$ (d) $n_1 < n_3 < n_2$

9. It is desired to produce an achromatic combination of thin lenses having an effective focal length f in air. The two lenses to be used for the combination must be have their dispersive powers ω_1 and ω_2 and focal length for yellow as f_1 and f_2 respectively then : (An achromatic combination means that the combination must have same focal length for all the colours)

(a) $f_1 = \left(1-\frac{\omega_1}{\omega_2}\right)f$ (b) $f_2 = \left(1-\frac{\omega_2}{\omega_1}\right)f$

(c) $f_1 = \left(1+\frac{\omega_2}{\omega_1}\right)f$ (d) $f_2 = \left(1+\frac{\omega_1}{\omega_2}\right)f$

10. A certain large plane mirror has its area normal along $+\hat{\mathbf{i}}$ direction. A particular point P on t mirror has its initial position vector as $5\hat{\mathbf{i}}$. The mirror starts rotating in a counter clockwise direction in the x-y plane without changing its orientation such that the position of point P at a times is given as :

$r_p = 5\{\cos(2\pi t)\hat{\mathbf{i}} + \sin(2\pi t)\hat{\mathbf{j}}\}$

A certain object lies in the x-y plane at the point (10, 0). Now choose the correct statement :

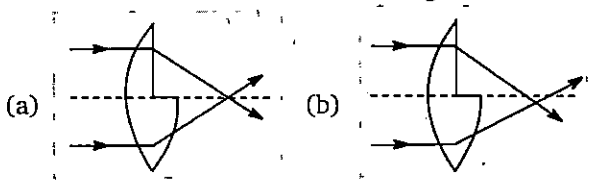
(a) x-coordinate of the image at $t = \frac{1}{4}$ sec is 20π units.

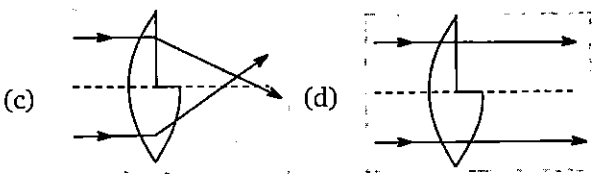
(b) Maximum speed of the image is 10π units.

(c) Maximum acceleration of the image is $40\pi^2$ units.

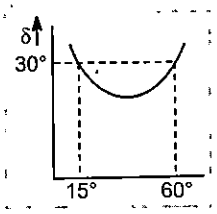
(d) Relative speed of the image w.r.t. mirror is 20π units.

11. Choose the correct ray diagram of a thin equi-convex lens which is cut as shown in the figure:



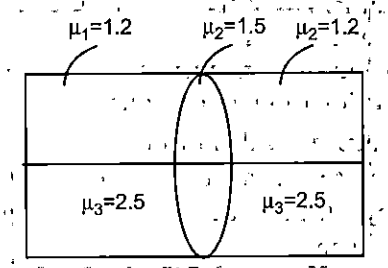


12. Figure shows graph of deviation δ *versus* angle of incidence for a light ray striking a prism. Angle of prism is :



(a) 30° (b) 45°
(c) 60° (d) 75°

13. A thin lens of refractive index 1.5, and focal length in air 30 cm is placed inside, a large container containing two immiscible liquids as shown below. If an object is placed at an infinite distance close to principle axis (common intersection line of the layers), the distance between images :



(a) 25 cm (b) 45 cm
(c) 65 cm (d) 85 cm

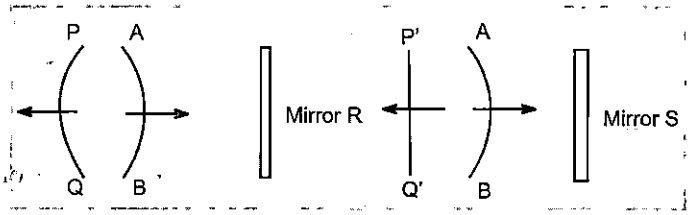
14. The left end of a long glass rod 6 cm in diameter has a convex hemispherical surface 3.00 cm radius. The refractive index of glass is 1.6. This rod is immersed in a liquid. An object 90.0 cm from the vertex of the left end of the rod and on its axis from the image at a point 1.6 inside the rod. What is the refractive index of the liquid ?

(a) 1.22 (b) 1.32
(c) 1.42 (d) 1.52

15. A thin prism P_1 with angle 4° made of glass of refractive index 1.54 is combined with another thin prism P_2 made of glass of refractive index 1.72 to produce no deviation. The angle of prism P_2 is.

(a) 3° (b) 2.6°
(c) 4° (d) 5.33°

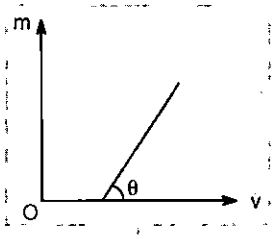
16. *AB* is incident wave front on mirror *R* and mirror *S* whereas *PQ* and *P'Q'* is reflected wave front from mirror *R* and mirror *S* respectively.



Choose the correct statement :

(a) mirror *A* is concave and mirror *B* is convex
(b) mirror *A* is plane and mirror *B* is convex
(c) mirror *A* is plane and mirror *B* is concave
(d) mirror *A* is convex and mirror *B* is plane

17. Variation of magnification (m) produced by a thin convex lens *versus* distance (v) of image from pole of the lens is shown in the graph. Which of the following statements is not correct ?



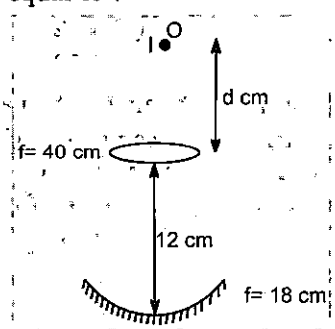
(a) Focal length of the lens is equal to intercept on v-axis
(b) Focal length of the lens is equal to inverse of the slope of the line.
(c) Magnitude of intercept on m-axis is equal to unity.
(d) Magnitude of intercept on v-axis is equal to unity.

18. A ball hits a rigid horizontal surface with angle a incidence 45°. If angle of reflection is 60° then the factor by which vertical component of the velocity of the ball changes during collision by :

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

19. A convex lens of focal length 40 cm is held coaxially above a concave mirror of focal length 18 cm. A luminous point object placed *d* cm above the lens on its axis gives rise to final image coincident with itself. Then *d* is equal to :

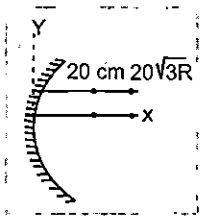


(a) 15 (b) 18
(c) 24 (d) 30

20. The power of a lens having refractive index 1.25 is +3 diopters. When placed in a liquid its power is –2 diopters. The refractive index of the liquid is :

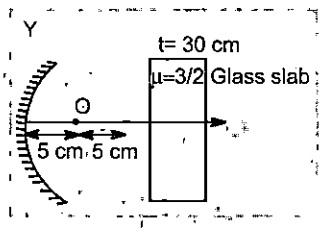
(a) 1.2 (b) 1.4
(c) 1.5 (d) 1.6

21. The coordinates of the image of point object *P* formed by a concave mirror of radius of curvature 20 cm (consider paraxial rays only as shown in the figure is :



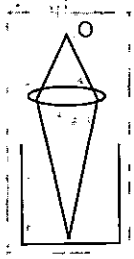
(a) (16 cm, $-4\sqrt{3}$ cm) (b) (16 cm, $+4\sqrt{3}$ cm)
(c) (30 cm, $-10\sqrt{3}$ cm) (d) (30 cm, $-10\sqrt{3}$ cm)

22. A point abject *O* is placed in front of a concave mirror of focal length 20 cm on the principal axis of the concave mirror at a distance of 5 cm from the pole as shown in the figure. A glass slab is also placed in front of the mirror, perpendicular to this principal axis. The image formed by perpendicular to this principle axis. The image formed by the system for first reflection and then refraction is :



(a) virtual behind the mirror
(b) real between the mirror and the slab
(c) virtual between the mirror and the slab
(d) can't be decided by above data

23. A real image of a luminous object is formed by a convex lens at the bottom of an empty beaker. The beaker is filled to a depth of 7 cm with a liquid of refractive index 1.4. Then in order to get the image again at the bottom of the beaker, how much should the beaker be moved vertically?



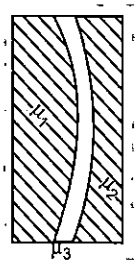
(Assume that lens and object remain fixed)

(a) 2cm upward (b) 2.8cm upward
(c) 2.8cm downward (d) 2cm downward

24. A ray of light passes through and equilateral glass prism in such a manner that the angle of incidence is equal to the angle of emergence and each of these angles is equal to (3/4) of the angle of prism. The angle of deviation is :
(a) 45° (b) 70°
(c) 39° (d) 30°

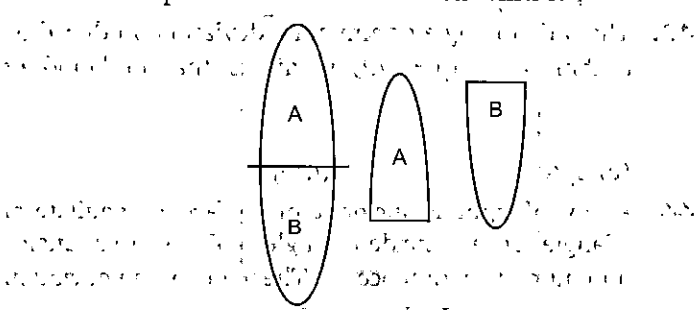
25. A double convex lens is made of glass which has its refractive index 1.55 for violet rays and 1.50 for red rays. If the focal length for violet rays is 20 cm, the focal length for red ray will be:
(a) 9 cm (b) 18 cm
(c) 20 cm (d) 22 cm

26. The adjacent figure shows a thin plano-convex lens of refractive index μ_1 and a thin plano-concave lens of refractive index μ_2, both having same radius of curvature R of their curved surfaces. The thin concavo- convex lens of refraction index μ_3 has radius of curvature R of both its surfaces. The concavo-convex lens is so placed in between the plano-convex and plano-concave lenses that the plane surfaces are parallel to each other. The focal length of the combination is :



(a) $\frac{R}{(\mu_1+\mu_2+\mu_3-1)}$ (b) $\frac{R}{(\mu_1+\mu_2+\mu_3)}$
(c) $\frac{R}{(\mu_1-\mu_2)}$ (d) $\frac{R}{(\mu_1-\mu_2-\mu_3-3)}$

27. An equi-convex lens has power D. It is cut into two symmetrical halves by a plane containing the principal axis and then two pieces are joined as shown in figure. The power of the combination will be:

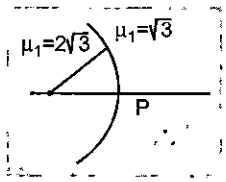


(a) P (b) $2P$
(c) $P/2$ (d) Zero

28. Two coherent monochromatic light beams of intensities I and $4I$ are superposed, the maximum and minimum possible intensities in the resulting beam are:
(a) $5I$ and I (b) $5I$ and $3I$
(c) $9I$ and I (d) $9I$ and $3I$

29. Monochromatic lights of wavelengths 420nm and 540nm are incident simultaneously and normally on a double slit apparatus with slit separation of 0.0756mm and screen is at a distance of 1m. The total number of dark fringes due to both wavelengths on the screen is :
(a) 360 (b) 280
(c) 19 (d) 38

30. A point source O kept at distance of $2R$ form pole P of concave interface having radius of curvature R emits a unidirectional beam at an angle of 30° to the principle axis PO. The distance of the image of point source O from pole P is :

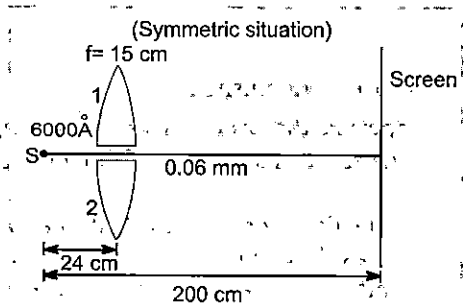


(a) $2R/3$ (b) R
(c) $2R/\sqrt{3}$ (d) none of these

31. If the critical angle for total internal reflection from a medium to vacuum is 30° the velocity of light in the medium is :
(a) 3×10^8 m/s (b) 1.5×10^8 m/s
(c) 6×10^8 m/s (d) $\sqrt{3}\times10^8$ m/s

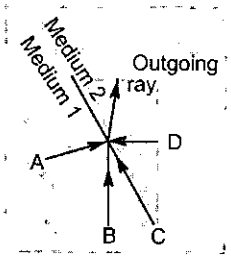
32. A diver looking up through the water in a lake sees the outside world contained in a circular horizon. If the refractive index of water is (4/3) and the eye of the diver is situated $12v$ cm below the surface, what is the radius of the horizontal circle :
(a) $12\times3\times\sqrt{5}$ (b) $12\times(4/3)$
(c) $12\times(3/\sqrt{7})$ (d) $12\times\sqrt{7}\times3$

33. Interference fringes of yellow light of wavelength 6000Å are formed by Billet split lenses. The distance from source to lens is 24cm. The focal length of lens is 15cm. The lens halves are separated by 0.06mm. The distance of source to screen is 200cm. The fringe width (in mm) is :



(a) 6.1 (b) 7.1
(c) 5.1 (d) 8.1

34. A ray of light is incident on a boundary between two substances as shown. The substances have different indices of refraction. Which incident ray could not have produced the outgoing ray ?



(a) A (b) B
(c) C (d) D

35. An object is moving towards a converging lens on its axis. The image is also found to be moving towards the lens. Then, the object distance 'u' must satisfy :
(a) $2f < u < 4f$ (b) $f < u < 2f$
(c) $u > 4f$ (d) $u < f$

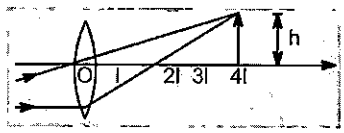
36. For a glass prism of refractive index $\sqrt{3}$, the angle of minimum deviation is equal to the angle of the prism. The angle of the prism is nearly :
(a) 80° (b) 70°
(c) 60° (d) 50°

37. Two plane monochromatic coherent waves produce interference pattern (of alternately bright and dark bands) on a screen. When the angle θ between the two beams is decreased the fringe width.
(a) increases
(b) is unaffected
(c) decreases
(d) increases and then decreases

38. An object was placed upright 25 cm in front of a converging lens with a focal length of 20 cm. A concave mirror with a focal length of 15 cm was placed 120 cm behind the lens. Which of these describes the final image ?
(a) real, enlarged
(b) virtual, upright
(c) virtual, inverted
(d) inverted, diminished

39. In comparing the properties of visible light waves to microwaves, which of the following statements is false ?
(a) Visible light waves travel at the same speed in glass as do microwaves
(b) Visible light waves have a higher frequency in glass than do microwaves
(c) Visible light waves travel at the same speed in vacuum as do microwaves
(d) Both visible light waves and microwaves can be refracted by glass

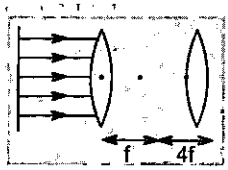
40. A thin converging lens L_1 forms a real image of an object located far away from the lens as shown in the figure. The image is located at a distance $4l$ and has height h. A diverging lens of focal length l is placed $2l$ from lens L_1. Another converging lens of focal length $2l$ is placed $3l$ from lens L_1. The height of final image thus formed is :



(a) h (b) $\frac{h}{2}$
(c) $4h$ (d) $2h$

41. A rod of length L lies along the axis of a concave mirror of focal length f. The near end of the rod is at a distance $L > f$ from the mirror. Its image will have a length :
(a) $\frac{Lf^2}{(L-f)(2L-f)}$ (b) $\frac{L^2f}{(L-f)(2L-f)}$
(c) $\frac{2Lf^2}{(L-f)(2L-f)}$ (d) $\frac{2L^2f}{(L-f)(2L-f)}$

42. Two convex lenses of focal length f and $4f$ are arranged to have their foci at the same point as shown in the figure. A beam of parallel light of diameter 1 cm falls on to the first lens. The beam exiting the second lens is :



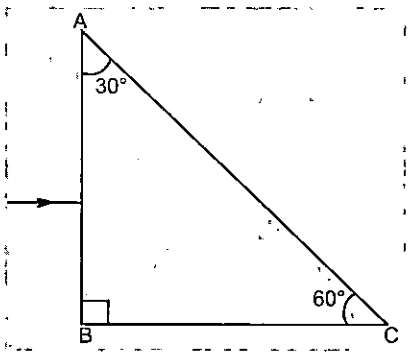
(a) diverging
(b) converging
(c) parallel of size 0.25 cm
(d) parallel of size 4 cm

43. The ratio of angle of minimum deviation produced by a thin prism ($\mu = 3/2$) in air to that in liquid of refractive index 9/7 is :
(a) 1/3 (b) 3
(c) 1/4 (d) 4

44. A ray of light is incident on a face of equilateral triangle at an incident angle 40°. At this angle, minimum deviation occurs. The value of the deviation is :
(a) 60° (b) 10°
(c) 20° (d) 40°

45. A ray of light is incident on a right angle prism of refractive index $\sqrt{3}$ as shown in the figure. The ray of light is reflected and refracted at the surface AC of the

prism. The angle between the reflected and refracted rays at the surface is :



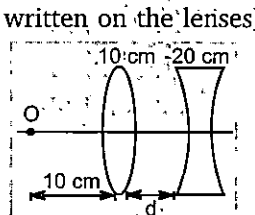
(a) 90° (b) 120°
(c) 150° (d) 45°

46. A small amount of transparent liquid is poured over the shiny convex base of a vessel as shown. The magnitude of focal length of the equivalent mirror thus formed is :

(a) less than that of convex base of the vessel
(b) more than that of convex base of the vessel
(c) equal to that of convex base of the vessel
(d) independent of the refractive index of the liquid

47. A converging lens forms an image of an object on a screen. The image is real and has twice the size of the object. If the positions of the screen and the object are interchanged, leaving the lens in its original position, what is the new image size on the screen ?
(a) Twice the object size
(b) Same as the object size
(c) Half the object size
(d) Can't say as it depends on the focal length of the lens

48. What should be the value of distance d so that final image is formed on the object itself. (focal lengths of the lenses are written on the lenses).

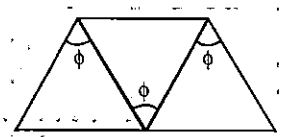


(a) 10 cm (b) 20 cm
(c) 5 cm (d) none of these

49. The passenger side-view mirror on an automobile often has the notation, "Objects seen in mirror are closer than they appear". Is the image really farther away than the object ?
(a) Yes, the image is smaller and farther away than the object
(b) No, the image is smaller and closer than the object
(c) No, the image is larger and closer than the object
(d) Yes, the image is larger and farther away than the object

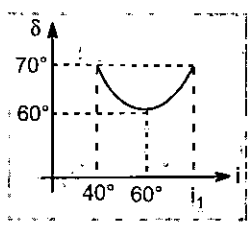
50. Three coherent, equal intensity light rays arrive at point P on a screen to produce an interference minimum of zero intensity. If any two of the rays are blocked, the intensity of the light at P is I_1. What is the intensity of the light at P if only one of the rays is blocked ?
(a) 0 (b) $I_1/2$
(c) I_1 (d) $2I_1$

51. A direct-vision prism is made out of three prisms, each with a refracting angle of 60°, attached to each other as shown in the figure. Light of a certain wavelength is incident on the first prism. The angle of incidence is 30° and the ray leaves the third prism parallel to the direction of incidence. The refractive index of the glass of the first and third prisms is 1.5. Find the refractive index of the material of the middle prism. ($\sqrt{6}$ = 2.45)



(a) 1.95 (b) 2.96
(c) 2.84 (d) 3.95

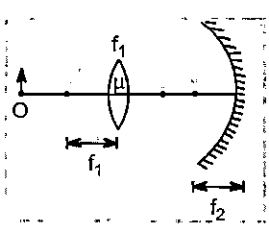
52. The curve of angle of incidence *versus* angle of deviation shown has been plotted for prism. The value of refractive index of the prism used is :



(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{2}{\sqrt{3}}$

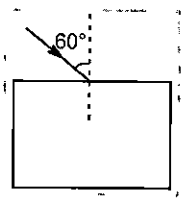
53. In the given curve of above question 52. Find the value of angle i_1 in degrees is :
(a) 40° (b) 60°
(c) 70° (d) 90°

54. An object is placed infront of a converging lens at a distance equal to twice the focal length f_1 of the lens. On the other side of the lens is a concave mirror of focal length f_2 separated from the lens by a distance $2(f_1 + f_2)$. Light from the object passes rightward through the lens, reflects from the mirror, passes leftward through the lens and forms a final image of the object.



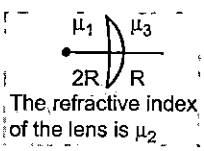
(a) The distance between the lens and the final image is equal to $2f_1$
(b) The distance between the lens and the final image is equal to $2(f_1 + f_2)$
(c) The final image is real, inverted and of same size as that of the object
(d) The final image is real, erect and of same size as that of the object

55. A ray of light enters into a thick glass slab from air as shown in figure. The refractive index of the glass slab varies with the depth x form the topmost surface of the slab as $\mu = \sqrt{3} - \frac{1}{\sqrt{3}}x$, where x is in meters.



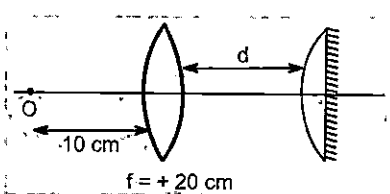
Slab is of sufficiently large thickness. Find the maximum depth reached by the ray inside the slab.
(a) 3 m (b) 1.5 m
(c) 1.0 m (d) 0.5 m

56. The diagram shows a concavo-convex lens. What is the condition on the refractive indices so that the lens is diverging ?



(a) $2\mu_3 < \mu_1 + \mu_2$ (b) $2\mu_3 > \mu_1 + \mu_2$
(c) $\mu_3 > 2(\mu_1 - \mu_2)$ (d) none of these

57. A convex lens of focal length 20 cm and another plano-convex lens of focal length 40 cm are placed co-axially (see figure). The plano-convex lens is silvered on plane surface. What should be the distance d (in cm) so that final image of the object 'O' is formed on O itself :



(a) 10 (b) 15
(c) 20 (d) 25

58. An object is placed at a distance of 12 cm from a converging lens on its principal axis and a virtual image of certain size is formed. Now the lens is moved 4 cm away from the object and a real image of the same size as that of the (previous) virtual image is formed. The focal length in cm of the lens is :
(a) 14 (b) 16
(c) 18 (d) 20

59. Two identical thin plano-convex lenses of refractive index n are silvered, one on the plane side and the other on the convex side. The ratio of their focal lengths is :
(a) $\frac{n}{n-1}$ (b) $\frac{n-1}{n}$
(c) $\frac{n+1}{n}$ (d) n

60. A vessel of depth d is half filled with a liquid of refractive index μ_1 and the other half is filled with a liquid of refractive index μ_2. The apparent depth of the vessel, when looked at normally from air is :
(a) $\frac{d}{2}\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right]$ (b) $d(\mu_1 + \mu_2)$
(c) $\frac{d}{2}(\mu_1 + \mu_2)$ (d) $d\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right]$

61. A ray incident at a point at an angle of incidence of 60° enters a glass sphere of $\mu = \sqrt{3}$ and is reflected and refracted at the farther surface of the sphere. The angle between the reflected and refracted rays at this surface is :
(a) 50° (b) 90°
(c) 60° (d) 40°

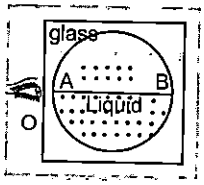
62. An equilateral prism deviates a ray through 45° for two angles of incidence differing by 20°. The angles of incidence are :
(a) 60° and 40° (b) 50° and 70°
(c) 62° 30′ and 42° 30′ (d) 60° 30′ and 40° 30′

63. Three right angled prisms of refractive indices are fitted together so that the faces of the middle prisms are in contact with one of the outside prisms. If the ray passes through the composite block and emerges without any deviation then :
(a) $\mu_1^2 + \mu_3^2 + \mu_2^2 = 1$ (b) $\mu_1^2 + \mu_3^2 - \mu_2^2 = 1$
(c) $\mu_1^2 - \mu_3^2 + \mu_2^2 = 1$ (d) $\mu_1 + \mu_2 - \mu_3 = 1$

64. Solar rays are incident at 45° on the surface of water $\mu = \frac{4}{3}$. What is the length of the shadow of a pole of length 1.2 m erected at the bottom of the pond if the pole is vertical assuming that 0.2 m of the pole is above the water surface ?
(a) 1 m (b) 0.75 m
(c) 0.825 m (d) 1.2 m

65. The image produced by a concave mirror is one quarter the size of object. If the object is moved 5 cm closer to the mirror, the image will only be half the size of the object. The focal length of mirror is :
(a) $f = 5.0$ cm (b) $f = 2.5$ cm
(c) $f = 7.5$ cm (d) $f = 10$ cm

66. The observer 'O' sees the distance AB as infinitely large. If refractive index of liquid is μ_1 and that of glass is μ_2, then μ_1/μ_2 is :

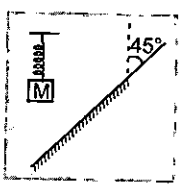


(a) 2 (b) 1/2
(c) 4 (d) none of these

67. A man is standing on a straight road in a summer noon. The refractive index of the air changes with height as $\mu = \mu_0\left(1+\frac{y}{3(\text{metre})}\right)$ due to variation of the vertical temperature where μ_0 is the refractive index of the air at road surface and y height. Eyes of the man are at height of 1.5 m above the road. What is the apparently visible length of the road.

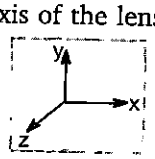
(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}$
(c) $\frac{4}{\sqrt{5}}$ (d) $\frac{6}{\sqrt{5}}$

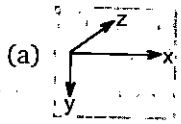
68. An insect of negligible mass is sitting on a block of mass M, tied with a spring of force constant K. The block performs simple harmonic motion with amplitude A infront of a plane mirror placed as shown. The maximum speed of insect relative to its image will be :

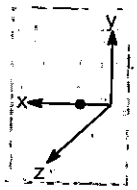


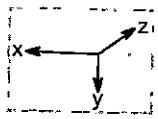
(a) $A\sqrt{\frac{K}{M}}$ (b) $\frac{A}{\sqrt{2}}\sqrt{\frac{K}{M}}$
(c) $A\sqrt{2}\sqrt{\frac{K}{M}}$ (d) $A\sqrt{\frac{M}{K}}$

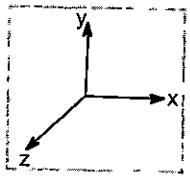
69. A coordinate axis as shown below is kept in front of a converging lens at a distance $2f$ from it. Where f is the focal length of the lens. What is the approximate shape of the image as seen by an observer on the other side of the lens at a distance $3f$ from it. Assume that x-axis is the principal axis of the lens :



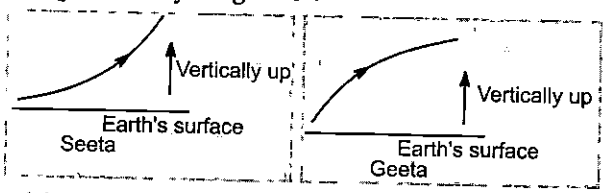
(a) 

(b) 

(c) 

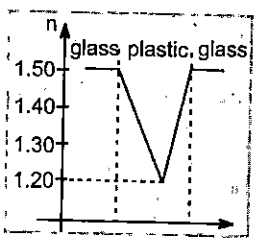
(d) 

70. Seeta and Geeta were asked to draw the trajectory of a light ray near, earth's surface when refractive index is decreasing with increase in height. Figure shows the respective ray diagrams :



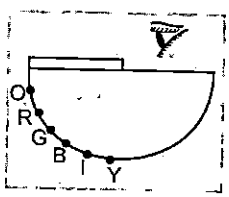
(a) Seeta is correct
(b) Geeta is correct
(c) Both are incorrect
(d) No conclusion can be drawn

71. A thin plate of transparent plastic is embedded in a thick slab of glass. The index of refraction of the glass is $n = 1.50$: the index of refraction of the plate changes as shown in the diagram. A beam of light passes through glass and strikes the surface of the plastic plate. What maximum angle of incidence enables the beam to pass through the plate ?



(a) 63° (b) 43°
(c) 53° (d) 73°

72. A person is looking at the flat surface of transparent hemi sphere of refractive index $\mu = \sqrt{2}$. Half of the flat surface is coloured black and half of curved surface is marked at 6 points equally spaced as shown in the figure. If the person sees only through the flat surface, he can see :



(a) all the points (b) Y and I only
(c) Y, I and B (d) O, Y and G

73. A plane mirror is placed in y-z plane facing towards negative x-axis. The mirror is moving parallel to y-axis with a speed of 2 cm/s. A point object P is moving infront of the mirror with a velocity $(1\,\text{cm/s})\hat{\mathbf{i}} + (1\,\text{cm/s})\hat{\mathbf{j}}$. Then which of the following statements is incorrect ?

(a) The velocity of image is $(-\hat{\mathbf{i}} + \hat{\mathbf{j}})$ cm/s

(b) The velocity of image with respect to mirror is $-(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ cm/s

(c) The velocity of image with respect to object is $(-2\hat{\mathbf{i}})$ cm/s

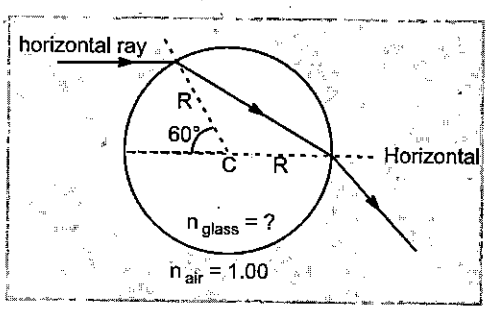
(d) The unit vector is the direction of reflected ray is $(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

74. A beam of the light incident vertically on a glass hemisphere of radius R and refractive index $\sqrt{2}$ lying with its plane side on a table. The axis of beam coincides with the vertical axis passing through the centre of base of the hemisphere and the cross sectional radius is $\frac{R}{\sqrt{2}}$. The luminous spot formed on the table is :

(a) R (b) $\frac{R}{\sqrt{2}}$

(c) $\frac{\sqrt{2}R}{(\sqrt{3}+1)}$ (d) $(\sqrt{3}+1)R$

75. A ray of light, initially horizontal, passes through a cylinder of glass of radius R, as shown. Assume that $n_{air} = 1$. Based on the geometry of the ray's path as shown, find index of refraction of the glass, n_{glass}



(a) 2.73 (b) 1.73

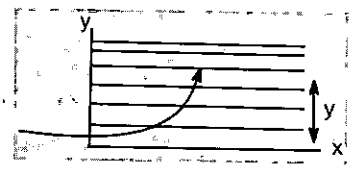
(c) 2.95 (d) 3.73

76. Surfaces of a thin equi-convex glass lens ($\mu = 1.5$) have radius of curvature R. Paraxial rays are incident on it. If the final image is formed at a distance $\frac{R}{13}$ from pole of the lens after n internal reflection. Then n is :

(a) 2 (b) 3

(c) 4 (d) infinity

77. The refractive index of the medium within a certain region, $x > 0, y > 0$, changes with y. A thin light ray traveling in the x-direction strikes the medium at right angles and moves through the medium along a circular arc. The maximum angular size of the arc is (The material having greatest refractive index is diamond with refractive index $n = 2.5$)



(a) 90° (b) 30°

(c) $\sin^{-1}\left(\frac{1}{2.5}\right)$ (d) $\cos^{-1}\left(\frac{1}{2.5}\right)$

78. A concave mirror has the form of a hemisphere of radius $R = 20$ cm. A thin layer of an unknown transparent liquid is poured into this mirror and it was found that the given optical system with the source in a certain position produces two real images, one of which (formed by direct reflection) coincides with the source and the other is at a distance = 8 cm from it. The refractive index μ of the liquid is :

(a) $\frac{7}{6}$ (b) $\frac{6}{7}$

(c) $\frac{4}{3}$ (d) $\frac{3}{2}$

79. The lateral magnifications of the lens with an object located at two different positions U_1 and U_2 are m_1 and m_2 respectively. Then the focal length of the lens is :

(a) $f = \sqrt{m_1 m_2}\,(u_2 - u_1)$ (b) $\frac{u_2 - u_1}{m_2 - m_1}$

(c) $\frac{(u_2 - u_1)}{\sqrt{m_1 m_2}}$ (d) $\frac{(u_2 - u_1)}{(m_2)^{-1} - (m_1)^{-1}}$

80. In a Young's double slit experiment $\lambda = 500$ nm, $d = 1$ mm and $D = 1$ m. The minimum distance from the central maximum for which the intensity is half of the maximum intensity is :

(a) 2×10^{-4} m (b) 1.25×10^{-4} m

(c) 4×10^{-4} m (d) 2.5×10^{-4} m

81. A bird is flying 3 m above the surface of water. To a fish underwater, the height of the bird from the water surface appears to be [Take $\mu_w = 4/3$]

(a) 2 m (b) 3.33 m

(c) 4 m (d) 4/3 m

82. In the above question 81, if the bird is diving vertically down with speed = 6 m/s, his apparent velocity as seen by a stationary fish underwater is :

(a) 8 m/s (b) 6 m/s

(c) 12 m/s (d) 4 m/s

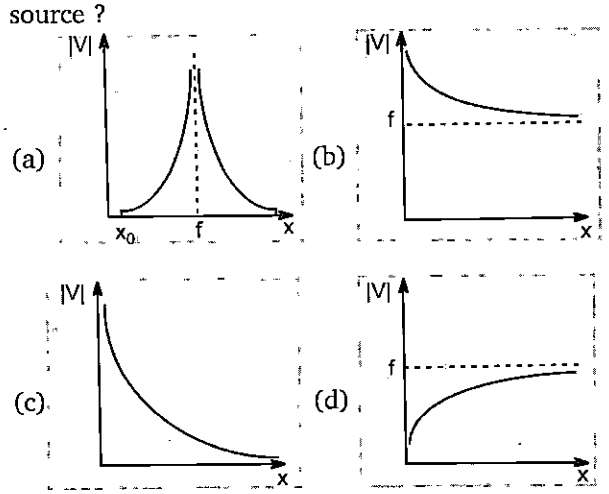
83. A plane mirror approaches a stationary person with some acceleration 'a'. The acceleration of his image, as seen by the person, will be :
(a) a (b) $2a$
(c) $a/2$ (d) none of these

84. A ray of light is passing from air into glass. If the angle of incidence, with respect to the normal to the interface, is increased :
(a) Total internal reflection will occur when the angle of incidence equals the critical angle.
(b) Total internal reflection will occur when the angle of incidence is less than the critical angle.
(c) Total internal reflection will occur when the angle of incidence is greater than the critical angle.
(d) Angle of refraction angle will increase but there will be no total internal reflection.

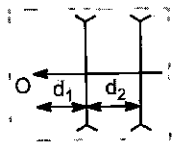
85. A convex mirror has a radius of curvature of 20 cm. An object is placed at such a distance from the mirror that the size of the image is exactly half that of the object. The object distance must be :
(a) 20 cm (b) 30 cm
(c) 10 cm (d) 40 cm

86. An air bubble is inside water. The refractive index of water is 4/3. At what distance from the air bubble should a point be placed so as form a real image at the same distance from the bubble ?
(a) $2R$
(b) $3R$
(c) $4R$
(d) the air bubble cannot form a real image of real object

87. A point source is situated at a distance $x_0 < f$ from the pole of convex mirror of focal length f. At time $t = 0$, the point source starts moving away from the mirror with constant velocity. Which of the graph below represents variation of image distance $|V|$ with the distance x between the pole of the mirror and the source ?



88. Two diverging lenses are kept as shown in figure. The final image formed will be :



(a) virtual for any value of d_1 and d_2
(b) real for any value of d_1 and d_2
(c) virtual or real depends on d_1 and d_2 only
(d) virtual or real depends on d_1 and d_2 and also on the focal lengths of the lens

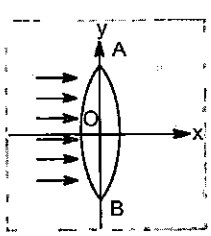
89. A diverging lens of focal length f_1 is placed infront of and coaxially with a concave mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement :
(a) The beam diameters of the incident and reflected beams will not be same.
(b) $d = 2|f_2| - |f_1|$
(c) If the entire arrangement is immersed in water, the conditions will remain unaltered.
(d) None of the above.

90. Sunbeam makes an angle $\theta = 40°$ with the surface of the Earth. At what angle to the horizontal should we place a flat mirror so that a ray of sunlight after reflection falls on the bottom of a deep well ?
(a) 40° (b) 50°
(c) 80° (d) 65°

91. An object of height $h = 5$ cm is located at a distance $a = 12$ cm from a concave mirror with focal length 10 cm. Find the height of the image.
(a) 10 cm (b) 15 cm
(c) 20 cm (d) 25 cm

92. A diver immersed in water sees the sun at an angle $\theta = 53°$ to the surface of water. What is the real angular position of the sun above the horizontal ? Take refractive index of water as $\frac{4}{3}$.
(a) 37° (b) 45°
(c) $\sin^{-1}\left(\frac{24}{25}\right)$ (d) $\sin^{-1}\left(\frac{9}{16}\right)$

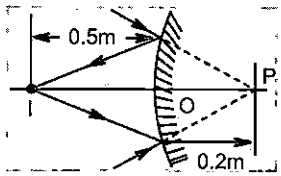
93. Monochromatic light rays parallel to x-axis strike a convex lens AB. If the lens oscillates such that AB tilts upto a small angle θ (in radian) on either side of y-axis, then amplitude of oscillation of image will (f = focal length of the lens) :



(a) $f\sec\theta$ (b) $f\sec^2\theta$

(c) $\frac{f\theta^2}{2}$ (d) $\frac{f\theta^2}{4}$

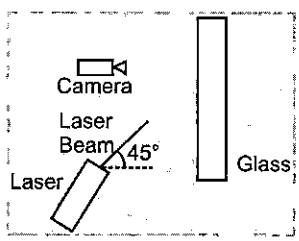
94. A beam of convergent light converges to a point 0.5 m in front of the mirror after reflection at a convex mirror but in the absence of the mirror the beam converges to a point 0.2 m behind the mirror. The radius of curvature of the mirror is :



(a) 20 cm (b) 50 cm

(c) 66.67 cm (d) 28.57 cm

95. A quality control engineer working at the Davinci Glass Company has invented a new method to measure the thickness of glass panes. By directing a laser beam at an incident angle of 45° w.r.t. the glass, he notices that not one but two parallel beams are reflected. He measures the distance between the reflected beams with an automated camera system, as shown. (The camera is pointed directly at the reflected beams). Assuming that the index of refraction of glass is $\sqrt{2}$, and the distance between the reflected beams is measured to be 1 mm, how thick is the glass ?



(a) $\frac{1}{\sqrt{6}}$ mm (b) $\sqrt{\frac{2}{3}}$ mm

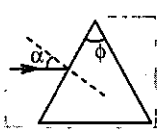
(c) $\sqrt{\frac{3}{2}}$ mm (d) None of these

96. The man, whose height is $l = 1.75$ m is standing at a distance $I = 6$ m from a vertical pillar of height $h = 7$ m. At what distance from the man should we put a small flat mirror on the ground horizontally to see in it the image of the top of the pillar ?

(a) 4.8 m (b) 1.2 m

(c) 5.2 m (d) 4.2 m

97. Refracting angle is the prism ϕ (see figure) is 30°. The angle of incidence of the beam on the face of the prism $\alpha = 37°$. Determine the angle of deviation δ (in degree) of ray the from the original direction after passing through the prism. The refractive index of the material of the prism $n = 1.2$. Round off the answer to the nearest integer :



(a) 30° (b) 37°

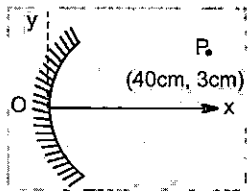
(c) 7° (d) 0

98. A ray of light is incident on a plane mirror, along the direction given by, $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{j}$. Find the unit vector along the reflected ray. Take normal to mirror along the direction of $\vec{B} = 3\hat{i} - 6\hat{j} + 2\hat{k}$.

(a) $\frac{-94\hat{i} + 237\hat{j} + 68\hat{k}}{49\sqrt{29}}$ (b) $\frac{-94\hat{i} + 68\hat{j} - 237\hat{k}}{49\sqrt{29}}$

(c) $\frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$ (d) None of these

99. The co-ordinates of the image of point object P formed by a concave mirror of radius of curvature 20 cm (consider paraxial rays only) as shown in the figure is:



(a) (13.33 cm, – 1 cm) (b) (13.33 cm, + 1 cm)

(c) (–13.33 cm, + 1 cm) (d) (–13.33 cm, – 1 cm)

100. A spherical mirror is polished on both sides. When the convex side is used as a mirror the image is erect with magnification 1/4. What is the magnification when the concave side is used as a mirror, the object remaining the same distance from the mirror ?

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$

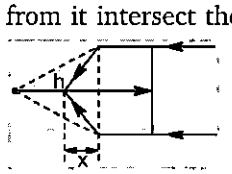
(c) $-\frac{1}{3}$ (d) $+\frac{1}{4}$

101. Rear window of a car has a size of $120 \times 45\,cm^2$. The driver sits at a distance $L = 2$ m from the rear window. What should be the minimum size of flat mirror, hanging at a distance of 0.5 m in front of the driver so that he has best view of road situation behind the car?

(a) (24 cm × 9 cm) (b) (30 cm × 11.25 cm)

(c) (40 cm × 15 cm) (d) (20 cm × 7.5 cm)

102. From the base of a hollow cone of height h with a small angle at the top, a small ring was cut off and placed in front of a parallel beam of light (the figure is exaggerated for clarity). At what distance x will rays of light reflected from it intersect the axis of the cone.



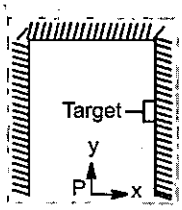
(a) h (b) $\frac{h}{2}$

(c) $\frac{h}{\sqrt{2}}$ (d) $\sqrt{\frac{2}{3}}h$

103. A large convex spherical mirror in an amusement park is facing a plane mirror 10 m away. A body of height 1 m standing midway between the two sees himself twice as tall as in plane mirror as in spherical one. In other words, the angles subtended at the observer by the image in plane mirror is twice the angle subtended by the image in the spherical mirror. The focal length of the convex mirror is :

(a) 1.5 m (b) 3.5 m
(c) 2.5 m (d) 2.0 m

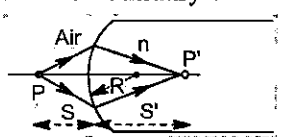
104. The drawing shows a top view of a square room. One wall is missing and the other three are each mirrors. From point P in the center of the open side, a laser is fired, with the intent of hitting a small target located at the center of one wall. Identify vector in whose direction the laser can be fired and score a hit, assuming that the light does not strike any mirror more than one:



(a) $\frac{\hat{i}}{3}+\hat{j}$ (b) $\hat{i}+\hat{j}$

(c) $-\hat{i}+\hat{j}$ (d) $-\hat{i}+\frac{\hat{j}}{3}$

105. Which of these actions will move the real image point P' farther from the boundary ?



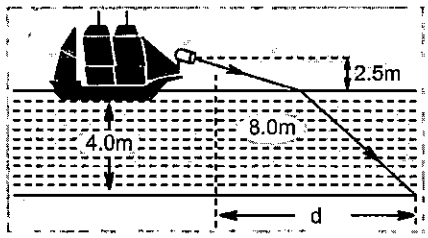
(1) Decrease the index of refraction n
(2) Increase the distance S
(3) Decrease the radius of curvature R

(a) 1, 2, 3 (b) 1 only
(c) 2 and 3 only (d) 2 only

106. You are required to design a little dentist's mirror to be fixed at the end a shaft for use in someone's mouth. The image should be erect and when held at 1.5 cm from a tooth, it should produce an image twice the size of the tooth.

(a) The mirror is convex $|f| = 3$ cm
(b) The mirror is concave with $|f| = 3$ cm
(c) The image of this tooth is 3 cm behind the mirror
(d) The image of this to this 3 cm in front of the mirror

107. A spotlight on a boat is 2.5 m above the water, and the light strikes the water at a point that is 8.0 m horizontally displaced from the spotlight (see the figure). The depth of the water is 4.0 m.The distance d (in m), which locates the point where the light strikes the bottom is . Take n for water $= \frac{4}{3}$, Round off the answer to nearest integer,



(a) 12.1 (b) 10.1
(c) 11.5 (d) 12.9

108. A bright point S is on the principal optical axis of a concave mirror of radius $R = 40$ cm at $d = 30$ cm from its pole. At what distance (in cm) in front of the concave mirror should a plane mirror be placed so that after two reflections, the rays converge back at point S.

(a) 30 (b) 35
(c) 40 (d) 45

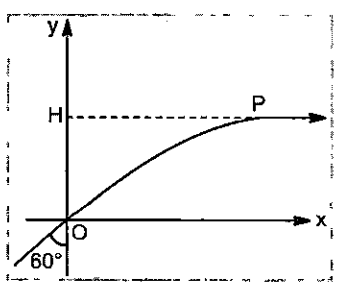
109. A point object is moving velocity $6\hat{i}+\hat{j}+2\hat{k}$ (m/s) in front of a plane mirror whose reflecting side is xz-plane. If the mirror also moves with a velocity $\hat{j}-\hat{j}$ (m/s), what is the speed of the image in m/s

(a) 7 (b) 5
(c) 9 (d) 6

110. A beam of light of width t is incident on an air-water boundary at an angle of incidence 45°. The width of the beam in water is (Refractive index of water = μ) :

(a) $(\mu-1)t$ (b) μt
(c) $\frac{\sqrt{2\mu^2-1}}{\mu}\cdot t$ (d) $\frac{\sqrt{\mu^2-t}}{\mu}\cdot t$

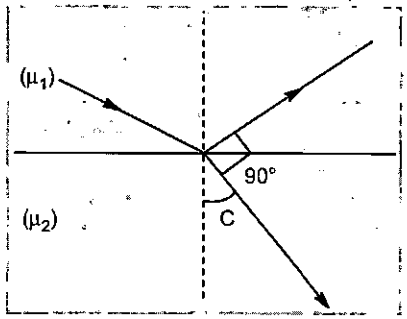
111. A system of coordinates is drawn in a medium whose refractive index varies as $\mu = \frac{\sqrt{2\mu^2-1}}{\mu}$, where



$0 \le y \le 1$. A ray of light is incident at origin at an angle 60° with y-axis as shown in the figure. At point P, the ray becomes parallel to x-axis. The value of H is :

(a) $\left\{\left(\frac{2}{\sqrt{3}}\right)-1\right\}^{1/2}$ (b) $\left\{\frac{2}{\sqrt{3}}\right\}^{1/2}$
(c) $\{(\sqrt{3})-1\}^{1/2}$ (d) $\{(\sqrt{3})+1\}^{1/2}$

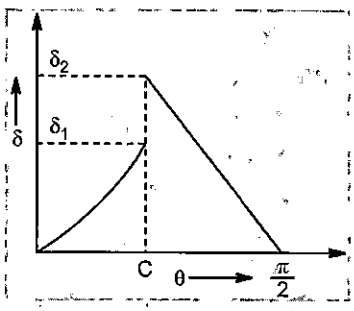
112. Light incident on a surface separating two media is partly reflected and partly refracted as shown in figure. μ_1, and μ_2 are the respective indices of the two media and C is the critical angle for the interface. Which of the equation is correct ?



(a) $\sin i = \frac{\mu_2}{(\mu_1^2+\mu_2^2)^{1/2}}$ (b) $\tan i = \frac{\mu_1}{\mu_2}$

(c) $\sin C = \tan i$ (d) $\sin C = \sin i$

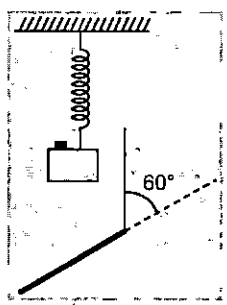
113. A ray of light travels from a medium of refractive index μ to air. Its angle of incidence in the medium is θ, measured from the normal to the boundary and its angle of deviation is δ with the angle of refraction ϕ. A graph is plotted between δ and θ which is shown in the figure. Choose the correct relation from the following :



(a) $\frac{\delta_2}{\delta_1} = \mu$ (b) $\frac{\delta_2}{\delta_1} = 2$

(c) $\sin\phi = \frac{1}{2}$ (d) $f = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\pi}\right)$

114. An insect of negligible mass is sitting on a block of mass M, tied with a spring of force constant k. The block performs simple harmonic motion with amplitude A in front of a plane mirror placed as shown. The maximum speed of insect relative to its image will be :



(a) $A\sqrt{\frac{k}{M}}$ (b) $\frac{A\sqrt{3}}{2}\sqrt{\frac{k}{M}}$

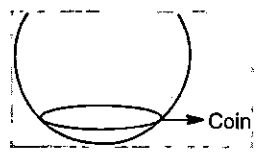
(c) $2A\sqrt{\frac{k}{M}}$ (d) $\frac{A}{2}\frac{k}{\sqrt{M}}$

115. A medium bounded by the planes $x = 0, x = a, y = b$, $y = -b, z = c$ and $z = 0$ has refractive index $\mu(x,z) = Az^3 - Bx + 2$. A ray of light travelling along positive z-axis enters the medium B at point $P\left(\frac{1}{B}, 0\right)$. Then the optical path introduced by the medium (when the light ray emerges out) will be :

(a) $\frac{Ac^4}{4}$ (b) $4Ac^4$

(c) $2c$ (d) $\frac{Ba^2}{2}$

116. A coin is placed at the bottom of an empty hemispherical bowl of radius R and is just not visible to an eye looking over the edge. When the basin is filled with water the whole of the coin is just visible to an eye in the same position. The diameter of the coin, if μ is refractive index of water is :



(a) $\frac{5}{63}$ (b) $\frac{10}{14}$

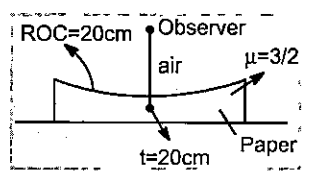
(c) $\frac{20}{9}$ (d) $\frac{30}{14}$

117. A submarine is 300 m horizontally out from the shore and 100 m beneath the surface of the water. A laser beam is sent from the submarine, so that it strikes the surface of the water at a point 200 m from the shore. If the outgoing beam from the surface of the water just strikes the top of building standing vertically at shore, then the height of the building is [$\mu_w = 4/3$] :

(a) 300 m (b) 140 m

(c) 240 m (d) 170.7 m

118. A planoconvex lens is placed on a paper on which flower is drawn. How far above its actual position does the flower appear to an observer looking down from top ?



(a) 10 cm (b) 15 cm

(c) 50 cm (d) None of these

119. A parallel beam of monochromatic light is incident on a plane glass surface. In the glass, the beam :

(a) remains parallel but the beam width increases

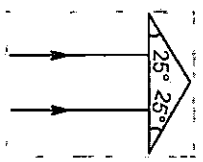
(b) remains parallel but the beam width decreases

(c) remains parallel and the beam width also remains the same
(d) becomes converging

120. A point object is moving with velocity $\vec{v}_O = 2\hat{i} - 3\hat{j} + 4\hat{k}$ in front of a moving plane mirror whose normal is along x-axis. The mirror is moving with velocity $\vec{v}_M = \hat{i} - 4\hat{j} + 3\hat{k}$. Find the velocity vector of the image :
(a) $5\hat{j}$ (b) $-3\hat{j} + 4\hat{k}$
(c) $-4\hat{j} + 2\hat{k}$ (d) $2\hat{i} - 3\hat{j} + 2\hat{k}$

121. A clock hung on a wall has marks instead of numbers on its dial. On the opposite wall there is mirror, and the image of the clock in the mirror is read, and it indicates the time as 8:20. What is the time in the clock ?
(a) 3:40 (b) 4:40
(c) 5:20 (d) 4:20

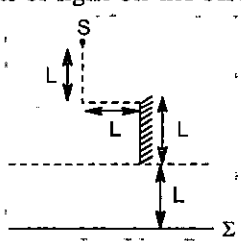
122. A beam is incident-parallel on the prism as shown in the figure below :
Find the angle between the emerging rays (Given $\mu = 1.66$) :



(a) 180° (b) 140°
(c) 40° (d) 320°

123. If a convergent beam of light passes through a diverging lens, the result :
(a) may be a conversing beam
(b) may be a diverging beam
(c) may be parallel beam
(d) all of the above

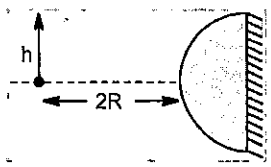
124. A point source of light is placed in front of a plane mirror as shown in the figure. Determine the length of reflected patch of light on the screen Σ :



(a) L (b) $2L$
(c) $\frac{3L}{2}$ (d) $\frac{L}{2}$

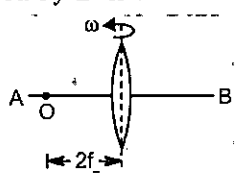
125. Ram is looking at his face in a mirror kept 10 cm away and he finds that his image is erect and magnified ($m = 1.8$). If he holds the mirror 50 cm away :
(a) He cannot see the image because reflected rays falling on his eyes are converging
(b) He sees a magnified and erect image
(c) He sees a diminished and inverted image
(d) He sees a magnified and inverted image

126. A glass hemisphere of radius R and material having refractive index 1.5 is silvered on its flat face as shown in the figure. A small object of height h is located at a distance $2R$ from surface of hemisphere. The final image will form :



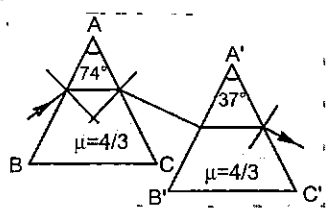
(a) at a distance of R from silvered surface, on right side
(b) on the hemispherical surface
(c) at the hemispherical surface
(d) at a distance of $2R$ from the silvered surface on left side

127. A point object O is placed at a distance $2f$ from a thin convex lens of focal length f as shown in figure. The lens then starts rotating about an axis perpendicular to the plane of paper, (object remains fixed and OA remains in the same plane), the speed of image when object is rotated by 2° is :



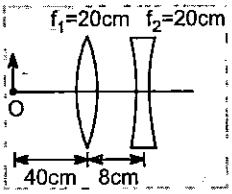
(a) $\frac{4f\pi\omega}{90}$ m/s along AB (b) $\frac{4f\pi\omega}{90}$ m/s along BA
(c) $\frac{2f\pi\omega}{90}$ m/s along AB (d) $\frac{2f\pi\omega}{90}$ m/s along BA

128. A light ray is incident ton face AB of prism ABC as shown in figure. The second prism is kept in such a manner that emergent ray from prism ABC is falling normally on face $A'B'$ of prism $A'B'C'$. The net deviation produced by the optical system of the two prisms is :



(a) 48° (b) 112°
(c) 32° (d) 18°

129. Consider a system of two thin lenses as shown in figure. An object of height 1 cm is placed at 40 cm from convex lens. Mark the correct option related to final image formed by the two-lens-system :



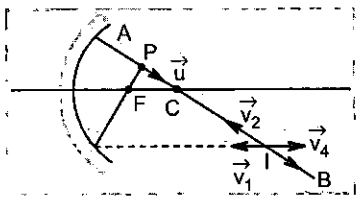
(a) Final image is formed at 32 cm on right of concave lens and is $\frac{1}{2.2}$ cm in size

(b) Final image is formed at 32 cm on left side of convex lens and is 1 cm is size

(c) Final image is formed at 14.5 cm on the left side of concave lens and is $\frac{1}{2.2}$ cm is size

(d) None of the above

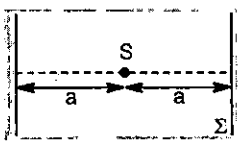
130. A point object P moves along line AB passing through centre of curvature of concave mirror as shown in figure. The object velocity is constant and is approaching the centre of curvature of mirror. For the instant shown in figure the image is formed at I. At this instant, the image is having velocity along :



(a) $\vec{v}_1$ (b) $\vec{v}_2$

(c) $\vec{v}_3$ (d) $\vec{v}_4$

131. A point source of light S is place in front of a perfectly reflecting mirror as shown in the figure. Σ is a screen. The intensity at the centre of screen is found to be I.

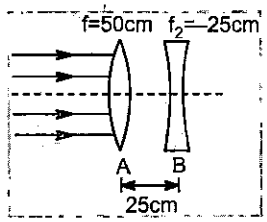


If the mirror is removed, then the intensity at the centre of screen would be :

(a) I (b) $\frac{10I}{9}$

(c) $\frac{9I}{10}$ (d) $2I$

132. The two lenses shown are illuminated by a beam of parallel light from the left. Lens B is then moved slowly toward lens A. The beam emerging from lens B is :



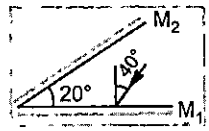
(a) always diverging

(b) initially parallel and then diverging

(c) always parallel

(d) initially converging and then parallel

133. Two plane mirrors are arranged as shown in figure. A ray has been incident on M_1 at an angle of 40°, find the deviation produced in it after three reflections :



(a) 40° clockwise

(b) 40° anti-clockwise

(c) 140° clockwise

(d) 140° anti-clockwise

134. A convex spherical refracting surface with radius R separates a medium having refractive index $\frac{5}{2}$ from air.

As an object is moved towards the surface from far away from the surface along the central axis, its image:

(a) changes from real to virtual when it is at a distance R from the surface

(b) changes from virtual to real when it is at a distance R from the surface

(c) changes from real to virtual when it is a distance $\frac{2R}{3}$ from the surface

(d) changes from virtual to real when it is at a distance $\frac{2R}{3}$ from the surface

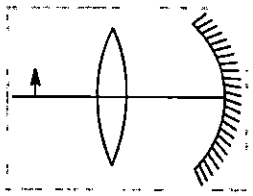
135. A real object is placed in front of a convex mirror (fixed). The object is moving towards the mirror, if v_O is the speed of the object, v_I be speed of image, then :

(a) $v_I = v_O$, always (b) $v_I > v_O$, always

(c) $v_I > v_O$, initially and then $v_O > v_I$

(d) $v_I < v_O$, initially and then $v_I > v_O$

136. Figure shows a thin converging lens for which the focal length is 5.00 cm. The lens is in front of a concave spherical mirror of radius R = 30 cm. If the lens and mirror are 20.0 cm apart and an object is placed 15 cm to the left of the lens, determine the approximate distance of the final image from the lens (in cm) :

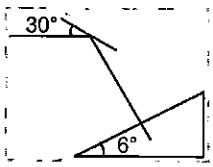


(a) 5.3 cm (b) 4.6 cm
(c) 6.1 cm (d) 12.7 cm

137. A ray enters a glass slab at an angle α from air and the refractive index varies along the slab as $\mu = \mu_0 - kt^2$ where t is the distance measured along the normal. μ_0 and k are positive constants. If the glass slab is sufficiently thick, how far along the normal will the ray go before it reflects back ?

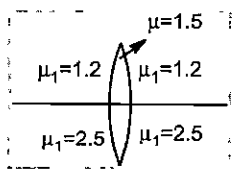
(a) $\frac{\mu_0 - \sin\theta}{k}$ (b) $\frac{\mu_0 + \sin\theta}{k}$
(c) $\sqrt{\frac{\mu_0 - \sin\theta}{k}}$ (d) $\sqrt{\frac{\mu_0 + \sin\theta}{k}}$

138. Find the net deviation produced in the incident ray for the optical instrument shown in figure below. (Take refractive index of the prism material as 2) :



(a) 66° clockwise (b) 66° anti-clockwise
(c) 54° anticlockwise (d) 54° clockwise

139. A thin lens of material having refractive index $\mu = 1.5$ and focal length of 20 cm when placed in air is taken and two mediums of different refractive indicies $\mu_1 = 1.2$ and $\mu_2 = 2.5$ are there covering upper and lower halves of the lens, respectively as shown in figure. If an object is placed on the principal axis, then its two images will form one after refraction from upper part and other after refraction from lower part. Consider the object to be at ∞, the separation between two images formed would be :

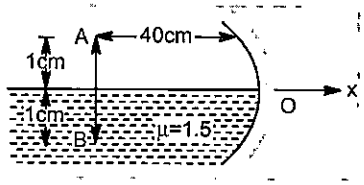


(a) 15 cm (b) 40 cm
(c) 25 cm (d) 65 cm

140. The object is at distance 45 cm from the screen. With lens we obtain a small image of the object on the screen. By moving the lens, we receive a different image on the screen, whose size is 4 time greater than the first. What is the focal length (in cm) of the lens?

(a) 5 (b) 15
(c) 10 (d) 12.5

141. An object AB of length 2 cm is kept in front of concave mirror of radius of curvature 20 cm as shown in figure. The upper half of mirror is exposed to air and lower half is exposed to a medium having refractive index 1.5. For this situation mark out the correct statement (s) about the image of AB formed by mirror :



(a) The separation between x-coordinates of image of ends A and B is zero
(b) The separation between x-coordinates of image of ends A and B is 2 cm
(c) The separation between x-coordinates of image of ends A and B is $\frac{2}{3}$ cm
(d) The image formed is perpendicular to the principal axis of mirror

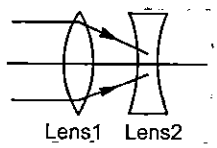
142. On the surface of a liquid reservoir of depth 3 m is a circular raft of radius 6 m. The surface is illuminated by light incident from all the direction. Determine the radius (in m) of the full shadow of the raft at the bottom of the reservoir. The refractive index of the liquid 1.25 :

(a) 1m (b) 2m
(c) 3m (d) 2.5m

143. A 2 cm diameter coin rests flat on the bottom of a bowl in which the water is 20 cm deep $\left(\mu_w = \frac{4}{3}\right)$. If the coin is viewed directly from above, what is its apparent diameter ?

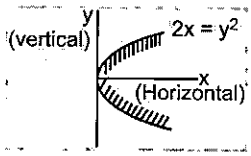
(a) 2 cm (b) 1.5 cm
(c) 2.67 cm (d) 1.69 cm

144. The second lens in this optical instrument can not :



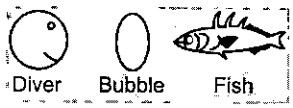
(a) Cause the light rays to focus closer than they would with the first lens acting along
(b) Cause the light rays to focus farther away than they would with the first lens acting alone
(c) Cause the light beam to diverge after refraction from it
(d) Make the beam parallel

145. The reflecting surface represented by the equation $2x = y^2$ as shown in figure. A ray travelling horizontal becomes vertical after reflection. The co-ordinates of the point of incidence can be :



(a) (1/2, 1) (b) (1, 1/2)
(c) (1/2, 1/2) (d) (1/2, ± 1)

146. A fish sees the smiling face of a scuba diver through a bubble of air between them, as shown,



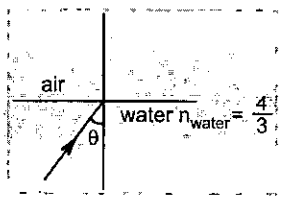
compared to the face of the diver, the image seen by the fish will be:
(a) Smaller and erect
(b) Smaller and inverted
(c) Larger and erect
(d) Can be either of above depending on the distance of the diver

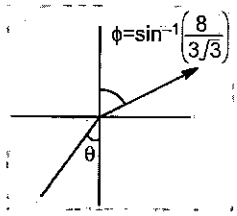
147. Ram sees his friend Shyam through a thick glass slab ($n = 1.5$). Shyam is at a distance of 100 cm from him, but he seems to be at a distance 98 cm from himself. What is the thickness of the slab ?
(a) 2 cm (b) 4 cm
(c) 6 cm (d) 5 cm

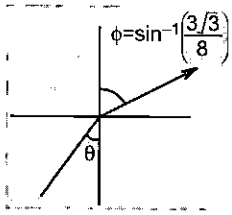
148. When an object is kept a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm. If the object is moved with a speed of 9 m/s, the speed with which image moves is :
(a) 10 cm/s (b) 1 m/s
(c) 9 m/s (d) 5 cm

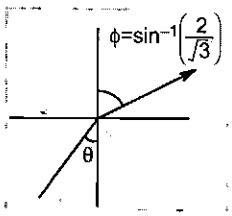
149. A luminous object is kept is at a distance D from a screen. In order to obtain the image of the same size as the object, the focal length of a convex lens to be inserted between them must be :
(a) D (b) $D/2$
(c) $2D$ (d) $D/4$

150. A light ray in water strikes the water surface at an angle of 60° to the normal. The path of the ray after striking is :



(a) 

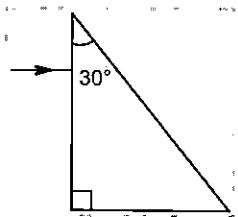
(b) 

(c) 

(d) None of these

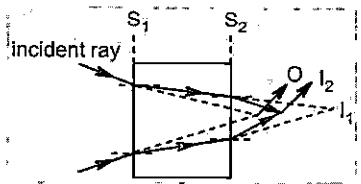
151. At what angle with the normal (in degree) should a light beam fall on the surface of material with refractive index $\mu - 1.732$, if the angle of refraction was half the angle of incidence ?
(a) 30° (b) 60°
(c) 45° (c) 90°

152. A light ray is normally incident on a right angled prism of refractive index 2.1 as shown. How many total internal reflections can the ray undergo inside the prism before coming out of the prism. Consider possible cases :



(a) 1 (b) 2
(c) 3 (d) 4

153. Figure shows converging rays on a glass slab placed in air. O is point of intersection of incident rays I_1 and I_2 are image formed after refraction at S_1 and S_2 surface respectively. Mark the correct option (s) :



(a) For refraction at S_2, I_1 is real object and I_2 is real image
(b) For refraction at S_1, O is real object and I_1 is its real image
(c) For refraction at surface S_1, O is virtual object and I_1 is real image
(d) For refraction at surface S_2, I_1 is virtual object and I_2 is real image

154. A particle is projected above a plane mirror. Taking up as positive and down as negative. Mark the correct options :
(a) Image is moving upwards during upward motion of object
(b) Image is moving downward during upward motion of object
(c) Sign of acceleration of object is positive and image negative
(d) Sign of acceleration of object is negative and image positive

155. Two objects are placed on the principal axis of a thin converging lens. One is 10 cm from the lens and the other is on the other side of the lens at a distance of 40 cm from the lens. The image of both objects are in the same plane. What is the focal length of the lens (in cm) ?
(a) 14 (b) 15
(c) 16 (d) 20

ANSWERS

Level-1: Only One Alternative is Correct

1.	(a)	**2.**	(a)	**3.**	(d)	**4.**	(b)	**5.**	(b)	**6.**	(b)	**7.**	(a)	**8.**	(c)
9.	(b)	**10.**	(c)	**11.**	(c)	**12.**	(b)	**13.**	(c)	**14.**	(d)	**15.**	(a)	**16.**	(c)
17.	(d)	**18.**	(b)	**19.**	(a)	**20.**	(c)	**21.**	(a)	**22.**	(c)	**23.**	(d)	**24**	(d)
25.	(d)	**26.**	(c)	**27.**	(d)	**28.**	(c)	**29.**	(d)	**30.**	(b)	**31.**	(b)	**32.**	(d)
33.	(c)	**34.**	(a)	**35.**	(d)	**36.**	(c)	**37.**	(a)	**38.**	(a)	**39.**	(a)	**40.**	(d)
41.	(a)	**42.**	(d)	**43.**	(b)	**44.**	(c)	**45.**	(a)	**46.**	(a)	**47.**	(c)	**48.**	(a)
49.	(b)	**50.**	(c)	**51.**	(a)	**52.**	(a)	**53.**	(d)	**54.**	(c)	**55.**	(b)	**56.**	(b)
57.	(c)	**58.**	(a)	**59.**	(a)	**60.**	(a)	**61.**	(b)	**62.**	(c)	**63.**	(b)	**64.**	(c)
65.	(b)	**66.**	(a)	**67.**	(a)	**68.**	(c)	**69.**	(a)	**70.**	(b)	**71.**	(c)	**72.**	(c)
73.	(d)	**74.**	(c)	**75.**	(b)	**76.**	(c)	**77.**	(d)	**78.**	(c)	**79.**	(d)	**80.**	(b)
81.	(c)	**82.**	(a)	**83.**	(b)	**84.**	(d)	**85.**	(c)	**86.**	(d)	**87.**	(d)	**88.**	(a)
89.	(b)	**90.**	(d)	**91.**	(d)	**92.**	(a)	**93.**	(d)	**94.**	(c)	**95.**	(c)	**96.**	(a)
97.	(c)	**98.**	(a)	**99.**	(d)	**100.**	(b)	**101.**	(d)	**102.**	(b)	**103.**	(c)	**104.**	(d)
105.	(c)	**106.**	(d)	**107.**	(a)	**108.**	(d)	**109.**	(a)	**110.**	(c)	**111.**	(a)	**112.**	(a)
113.	(b)	**114.**	(b)	**115.**	(a)	**116.**	(a)	**117.**	(d)	**118.**	(a)	**119.**	(a)	**120.**	(b)
121.	(b)	**122.**	(c)	**123.**	(d)	**124.**	(c)	**125.**	(c)	**126.**	(d)	**127.**	(b)	**128.**	(a)
129.	(c)	**130.**	(b)	**131.**	(c)	**132.**	(b)	**133.**	(c)	**134.**	(c)	**135.**	(a)	**136.**	(a)
137.	(c)	**138.**	(d)	**139.**	(d)	**140.**	(c)	**141.**	(c)	**142.**	(b)	**143.**	(a)	**144.**	(c)
145.	(d)	**146.**	(a)	**147.**	(c)	**148.**	(b)	**149.**	(d)	**150.**	(c)	**151.**	(b)	**152.**	(b)
153.	(d)	**154.**	(d)	**155.**	(c)										

Solutions

Level-1: Only One Alternative is Correct

1. (a) $\delta = e = A$

$\delta = i + e - A$

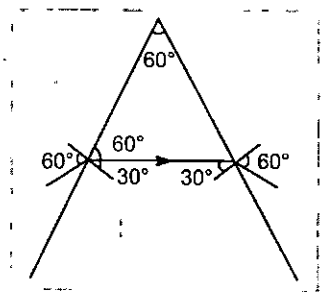
$\delta = i = 60°$

$e = 60°$

$A = 60°$

$1 \times \sin 60° = \mu \sin 30°$

$\mu = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$



3. (d) For dispersion without average deviation

$(n_y - 1)A = -(n_y - 1)A'$

for no dispersion

$A(\delta_V - \delta_R) = -(\delta'_V - \delta'_R)A'$

$\Rightarrow \quad (n_y - 1)A\omega = -(n'_y - 1)A'\omega'$

$\Rightarrow \quad \omega = \omega' [\because (n_y - 1)A\omega = -(n'_y - 1)A']$

because $\delta_V - \delta_R = \omega\delta$

$\Rightarrow \quad \omega = \frac{n_y - n_g}{n_y - 1} = \frac{1.51 - 1.49}{1.5 - 1} = \frac{0.02}{0.5} = 0.04$

4. (b) $50 \times 10^{-6} \times 3 \times 10^8 \times \frac{1}{10}$

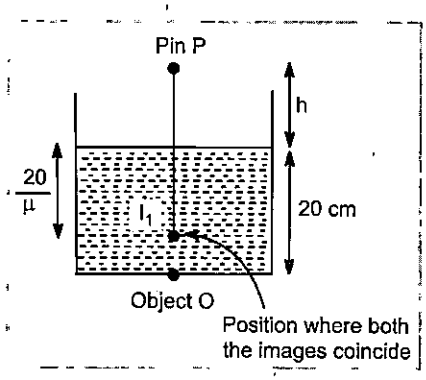
$= 150 \times 10^{-6} \times 10^7$

$= 1500\,\text{m}$

$= 1.5\,\text{km}$

6. (b) I_1 is image of pin which is formed at h below surface, also apparent depth of object $= \frac{20}{\mu}$

Thus $\quad h = \frac{20}{4/3} = 15\,\text{cm}$



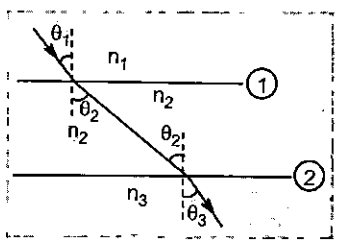
8. (c) From 1st interface $n_2 < n_1$

From 2nd interface $n_3 > n_2$

but since $\theta_3 < \theta_1$ hence $n_3 > n_1$

So $n_3 > n_1 > n_2$

$L_1 = \frac{t_1}{t_1} < 1 \quad [L_1 < L_2]$



9. (b) $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ ...(1)

Now, $\frac{1}{f_v} - \frac{1}{f_R} = (\mu_v - \mu_r)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$+ (\mu_{2v} - \mu_{2r})\left(\frac{1}{R_1^2} - \frac{1}{R_2^1}\right) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$

Thus must be zero for achromatic combination.

i.e., $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ ...(2)

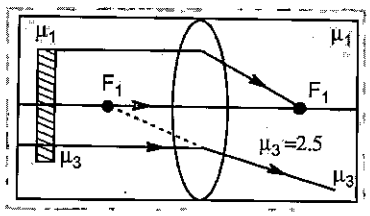
Solving eqn. (1) and (2)

$$f_1 = \left(1 - \frac{\omega_1}{\omega_2}\right) f \Rightarrow f_2 = \left(1 - \frac{\omega_2}{\omega_1}\right) f$$

12. (b) $\delta = i + e - A \Rightarrow A = 45°$

13. (c) For upper portion :

$$f_1 = \frac{\mu_2 - 1}{\frac{\mu_2}{\mu_1} - 1}(f_a) = \frac{1.5 - 1}{\frac{1.5}{1.2} - 1} \times 20 = 40\,\text{cm}$$



For lower portion, $f_1 = \frac{\mu_2 - 1}{\frac{\mu_2}{\mu_3} - 1}(f_a)$

$$= \frac{1.5 - 1}{\frac{1.5}{2.5} - 1} \times 20 = -25\,\text{cm}$$

$\therefore$ Object is at infinity, images will form at focal points.

Hence the distance between two images will $= 40 + |-25| = 65\,\text{cm}$

15. (a) Deviation produced by one is cancelled by the other.

Use $\delta = A(\mu - 1)$

17. (d) For lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Magnification, $m = -\frac{v}{u} = \frac{v}{f} - 1$

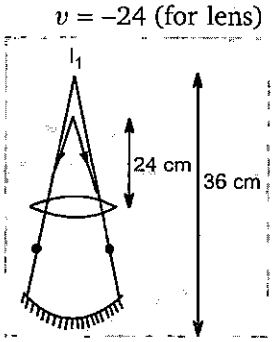
Graph between m and v will be a straight line with -1 intercept on m-axis and slope $\tan\theta = 1/f$.

Putting $m = 0$, it gives $v = f$

$\therefore$ (b), (c) and (a) are correct.

19. (a) The lens forms the image on the centre of curvature of mirror, so that the rays will retrace their path *i. e.*

$v = -24$ (for lens)



$$\frac{1}{-24} + \frac{1}{d} = \frac{1}{40} \Rightarrow d = 15\,\text{cm}$$

20. (c) $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 3$

$\therefore \quad 3 = (1.25 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(1)

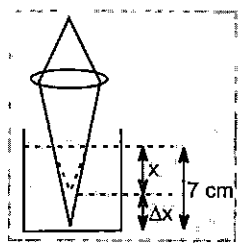
and $-2 = \left(\frac{1.25}{\mu} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(2)

$$-\frac{3}{2} = \frac{0.25\mu}{1.25 - \mu} \Rightarrow -0.5\mu = 3.75 - 3\mu$$

$$\Rightarrow \mu = 3.75/2.5 = 1.5$$

21. (a) Use the concept of pole for the paraxial rays.

23. (d) $\frac{7\,\text{cm}}{x} = 1.4$



$x = 5\,\text{cm}$

$\therefore \quad \Delta x = 2\,\text{cm}$

26. (c) $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

$$= (\mu_1 - 1)\left[\frac{1}{\infty} + \frac{1}{R}\right] + (\mu_3 - 1)\left[-\frac{1}{R} + \frac{1}{R}\right]$$

$$+(\mu_2 - 1)\left[-\frac{1}{R} - \frac{1}{\infty}\right]$$

$$= \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{\mu_1 - \mu_2}{R} \Rightarrow F = \frac{R}{\mu_1 - \mu_2}$$

29. (d) $d\sin\theta = \left(\frac{2n-1}{2}\right)\lambda \Rightarrow \left(\frac{2n-1}{2}\right)\frac{\lambda}{d} < 1$

$$\Rightarrow \quad 2n - 1 < \frac{2d}{\lambda}$$

$$\Rightarrow \quad n < \frac{d}{\lambda} + \frac{1}{2} = \frac{0.0756 \times 10^{-3}}{420 \times 10^{-3}} + \frac{1}{2} = 180.5$$

$$\frac{2n-1}{2m-1} = \frac{9}{7} = \frac{27}{21} = \ldots.$$

n	5	14	....
m	4	11	....

$5 + (n-1)9 < 180$

30. (b) $\mu_1 \sin i = \mu_2 \sin r$

$2\sqrt{3}\sin 30° = \sqrt{3}\sin r \Rightarrow r = 90°$

33. (c) $u = -24$

$$v = \frac{uf}{u + f}$$

$$v = \frac{-24\times 15}{-24+15} = \frac{124\times 15}{8} = 40$$

$\Rightarrow \quad D = 200 - 24 - 40 = 136\,\text{cm}$

$$\frac{R_1}{-0.03} = \frac{v}{u} = \frac{40}{-24}$$

$$R_1 = \frac{40}{24}\times 0.03 = 0.05\,\text{mm}$$

$\Rightarrow \quad d = 0.05\times 2 + 0.06 = 0.16\times 10^{-3}\,\text{m}$

$$\beta = \frac{\lambda D}{d} = \frac{6000\times 10^{-10}\times 136\times 10^{-2}}{0.16\times 10^{-3}}$$

$$\beta = 51\times 10^{-4}\,\text{m} = 5.1\,\text{mm}$$

35. (d) $\dfrac{dv}{dt} = \dfrac{v^2}{u^2}\dfrac{du}{dt}$

Since direction of object velocity and image velocity is always same, thus image should lie in object side *i.e.* $u < f$

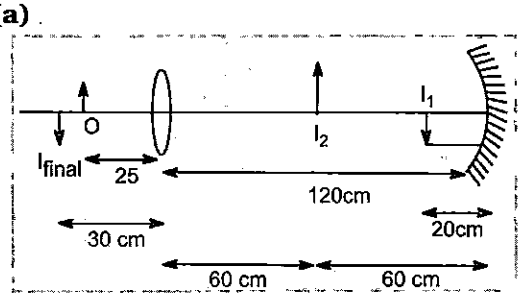
36. (c) $\mu = \dfrac{\sin\left(\dfrac{A+D}{2}\right)}{\sin\dfrac{A}{2}}$ gives

$2\cos(A/2) = \mu$ or $\cos(A/2) = 1.732/2 = \sqrt{3}/2$

or $A = 60°$

37. (a) $\beta = \dfrac{\lambda}{\theta}$ when θ is small

38. (a)



I_1 is first image of lens and object for mirror and I_2 is image of mirror and object for second refraction at lens.

For refraction at lens

$$\frac{1}{V} - \frac{1}{(-25)} = \frac{1}{20} \quad \Rightarrow \frac{1}{V} = \frac{1}{20} - \frac{1}{25} = \frac{1}{100}$$

$$V = 100\,\text{cm}$$

For mirror

$$\frac{1}{V} + \frac{1}{-20} = \frac{-1}{15}$$

$$\frac{1}{V} = -\frac{1}{60} \quad \Rightarrow \quad V = -60\,\text{cm}$$

For final refraction at lens (ray moving leftward)

$$\frac{1}{V} + \frac{1}{60} = \frac{1}{20}$$

$$\frac{1}{V} = \frac{3-1}{60}$$

$$V = 30\,\text{cm}$$

Final magnification $= (-4)\times(-3)\times(-1/2) = -6$

Final image is real, inverted and enlarged.

39. (a) μ varies with λ which is different for different waves.

Velocity in a medium $V = \dfrac{C}{\mu}$

which is different for different media

Frequency of a wave remain same in different media.

41. (a) For the near end of the rod,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \Rightarrow \quad v = \frac{uf}{u-f} = \frac{Lf}{L-f}$$

For far end of the rod,

$$\frac{1}{f} = \frac{1}{u+L} + \frac{1}{v_1} \Rightarrow \quad \frac{1}{v_1} = \frac{1}{f} - \frac{1}{2L} = \frac{2L-f}{(f)(2L)}$$

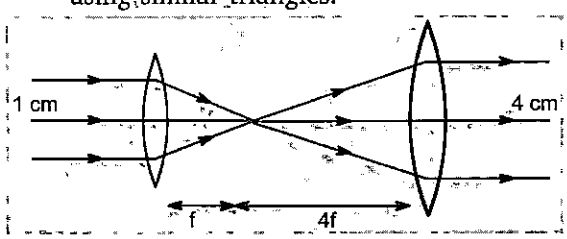
$$\therefore \quad v_1 = \frac{(f)(2L)}{2L-f}$$

$$\text{Length of image} = v - v_1 = \frac{Lf}{L-f} - \frac{(2L)(f)}{(2L-f)}$$

$$= \frac{Lf(2L-f) - 2Lf(L-f)}{(L-f)(2L-f)}$$

$$= \frac{2L^2f - Lf^2 - 2L^2f + 2Lf^2}{(L-f)(2L-f)} = \frac{Lf^2}{(L-f)(2L-f)}$$

42. (d) This is solved by sketching a ray diagram and using similar triangles.



43. (b) δ = angle of minimum deviation $= (\mu - 1)A$

$$\therefore \quad \delta_{air} = ({}_a\mu_g - 1)A \quad \text{and} \quad \delta_L = ({}_L\mu_g - 1)A$$

$$\frac{\delta_{air}}{\delta_L} = \frac{({}_a\mu_g - 1)}{({}_L\mu_g - 1)} = \frac{[(3/2)-1]}{\left[\dfrac{(3/2)}{(9/7)} - 1\right]}$$

$$= \frac{(1/2)}{[(21/18)-1]} = \frac{1}{2}\times\frac{18}{3} = 3$$

44. (c) $i = 40$

$A = 60°$

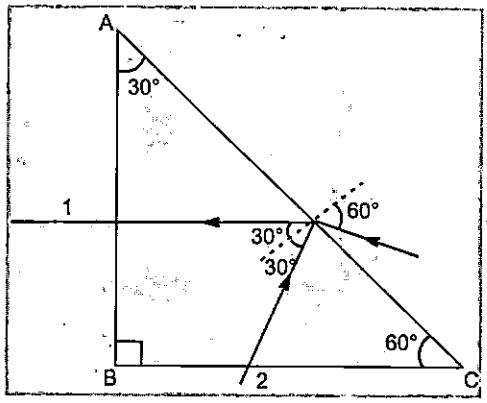
for $\delta_{min} = i = e$

$\delta_{min} = (i + e) - A$

$= 80 - 60°$

$= 20°$

45. (a) For the surface AC,



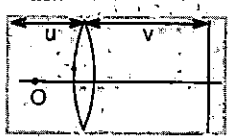
$\sqrt{3}\sin 30° = \sin r$

$\Rightarrow \quad r = 60°$

clearly angle between ray 2 and 3 is 90°

47. (c) $v/u = 2$

$m_{new} = u/v = 1/2$

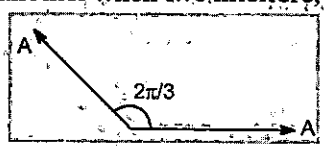


Alternative

According to principle of reversibility of light image size will become half.

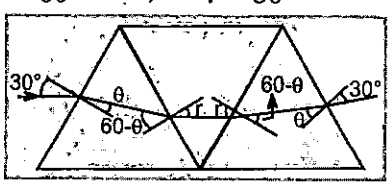
49. (b) Mirror is convex in nature and produces diminished image at closer distance but due to small size, it appears distant.

50. (c) They must be at phase difference of $2\pi/3$ from one another when two interfere, phasor will be :



Thus result will be still I_1.

51. (a) $2r = 60° \quad \Rightarrow \quad r = 30°$



$\sin 30° = 1.5 \sin\theta$

$1.5\sin(60-\theta) = \mu \sin 30°$

$1.5\sin 60°\cos\theta - 1.5\cos 60°\sin\theta = \mu \sin 30°$

52. (a) $\delta = i + e - A$

$d_{min} = 60°$ when $i = e$

$\therefore \quad 60° = 2i - A = 2(60°) - A$

$\therefore \quad A = 60°$

$$\therefore \quad \mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60+60}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \sqrt{3}$$

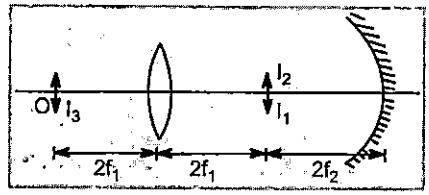
53. (d) When angle of incidence is i_1, $e = 40°$ (from reversibility of ray)

also $\delta = 70°$

$\therefore \quad 70° = i_1 + 40° - A$

$\therefore \quad i_1 = 90°$

54. (c)



55. (b) (1) $\sin 60° = \left(\sqrt{3} - \frac{1}{\sqrt{3}}x\right)\sin 90°$

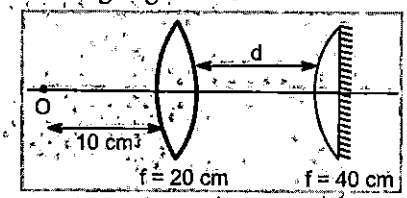
$$\frac{\sqrt{3}}{2} = \sqrt{3} - \frac{1}{\sqrt{3}}x$$

56. (b) $\frac{1}{f} = \frac{\mu_2 - \mu_1}{-2R} + \frac{\mu_3 - \mu_2}{-R}$

$$= -\frac{1}{R}\left(\frac{\mu_2 - \mu_1}{2} + \mu_3 - \mu_2\right) < 0$$

$$\Rightarrow \quad \frac{2\mu_3 - (\mu_1 + \mu_2)}{2} > 0$$

57. (c) The silvered lens can be replaced by a mirror of focal length given as

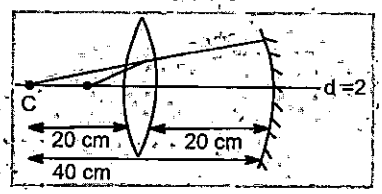


$$\frac{1}{F_M} = \frac{1}{f_m} - \frac{2}{f_l}$$

$$\frac{1}{F_M} = 0 - \frac{2}{40} \quad F_M = -20$$

For lens $v = \frac{uf}{u+f}$

$$v = \frac{-10\times 20}{-10+20} = -20$$



So this position has to be centre of curvature of mirror in order for the ray to retrace its path

so $d = 40 - 20 = 20\,\text{cm}$.

58. (a) $\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$

$$\frac{u}{V} = 1 + \frac{u}{f}$$

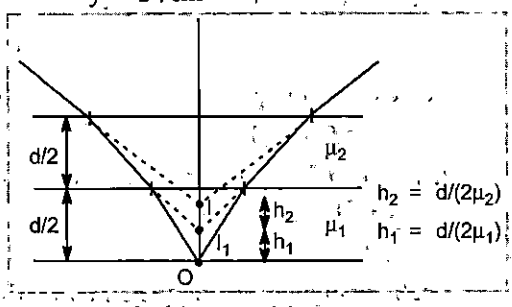
$$m = \frac{v}{u} = \frac{f}{u+f}$$

$$\frac{f}{-12+f} = -\left[\frac{f}{-16+f}\right]$$

m is same in both the cases

$f = 14\,\text{cm}$

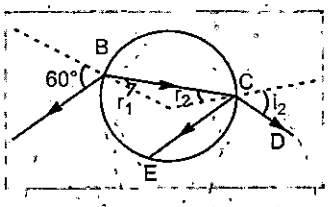
60. (a)



O— Initial object position

I— Final image position

61. (b) The path of ray AB after refraction at opposite face and reflection at opposite face as shown in figure. We have to find $\angle DCE$



By snell's law $(\sin 60°/\sin r_1) = \sqrt{3}$

or $\sin r_1 = \sin 60°/\sqrt{3} = 1/2.$

$\therefore$ $r_1 = 30°$ and hence $r_2 = 30°$

Considering refraction at C $(\sin r_2/\sin i_2) = 1/\sqrt{3}$

or $\sin i_2 = \sqrt{3}/2$

$\therefore$ $i_2 = 60°$

$\therefore$ $\angle ECD = 180° - (i_2 + r_2)$

$= 180° - 90° = 90°$

62. (c) $\delta = i_1 + i_2 - A$

$45° = i_1 + i_2 - 60°$

$i_1 + i_2 = 105°$

$i_1 - i_2 = 20°$

or $2i_1 = 125°$

$i_1 = 62°\,30'$, $i_2 = 42°\,30'$

63. (b) The figure displays the given problem.

$r_1 + r_2 = 90°$ ...(1)

$r_3 + r_4 = 90°$ ...(2)

$r_5 + r_6 = 90°$ ...(3)

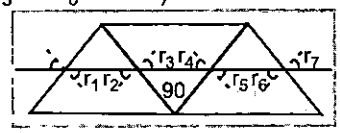
Since the emergent ray is parallel to incident ray,

$r_7 = 90° - i$ ...(4)

$\sin i = \mu_1 \sin r_1$ $\mu_1 \sin r_2 = \mu_2 \sin r_3$

$\mu_2 \sin r_4 = \mu_3 \sin r_5$

$\mu_3 \sin r_6 = \sin r_7$



Using equations (1) to (4) in the above equations

$\sin i = \mu_1 \sin r_1$ ...(5)

$\mu_1 \cos r_1 = \mu_2 \sin r_3$ ...(6)

$\mu_2 \cos r_3 = \mu_3 \sin r_5$ ...(7)

$\mu_3 \cos r_4 = \cos i$ ...(8)

Carrying out the manipulation

(equation 5)^2 – (equation 6)^2 + (equation 7)^2 – (equation 8)^2

we get $1 + \mu_2^2 = \mu_1^2 + \mu_3^2$

$\mu_1^2 + \mu_3^2 - \mu_2^2 = 1$

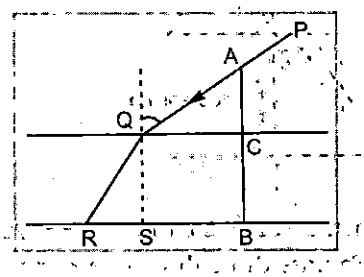
64. (c) In the figure AB is pole. The shadow formed of the pole is indicated as $BR = BS + SR$

$BS = QC = AC$

$\frac{\sin 45°}{\sin r} = \frac{4}{3}$ or $\sin r = \frac{3}{4\sqrt{2}}$

$SR = QS \tan r = BC \tan r = 1 \tan r$

$\tan r = \frac{3}{\sqrt{23}} = \frac{3}{4.8} = \frac{1}{1.6} = 0.625$



$SB = 0.2$

$SR = 0.625$

$RB = 0.825$ metre

65. (b) $\frac{1}{4} = \frac{f}{x_0 - f}$

$\frac{1}{2} = \frac{f}{x_0 - 5 - f}$

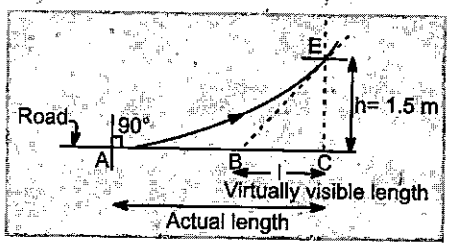
Solving $f = 2.5$

66. (a) B must be appearing at infinity

$\frac{h_2}{\infty} - \frac{\mu_1}{-2R} = \frac{\mu_2 - \mu_1}{-R}$

Solving $\frac{\mu_1}{\mu_2} = 2$

67. (a)



$$\mu_0 \sin 90° = \mu_0\left(1+\frac{1.5}{3}\right)\sin\theta$$

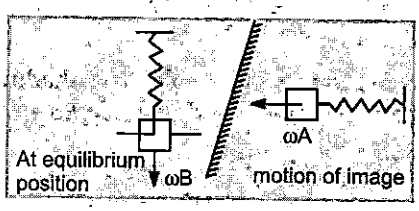
$$\sin\theta = \frac{2}{3}$$

$$\cot\theta = \frac{\sqrt{5}}{2}$$

$$\cot\theta = \frac{h}{l}$$

$$l = \frac{h}{\cot\theta} = 1.5 \times \frac{2}{\sqrt{5}} = \frac{3}{\sqrt{5}}\text{ m}$$

68. (c)

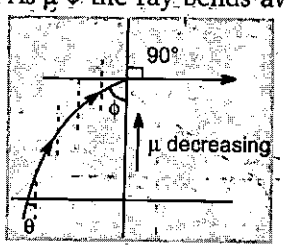


$$V_{max} = V\sqrt{2} = \sqrt{2}WA$$

69. (a) Since image is read and inverted.

70. (b) As $\mu \downarrow$ the ray bends away from normal.

71. (c)



When μ varies continuosly $\theta = 90°$

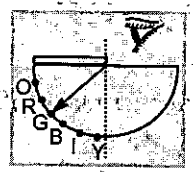
$\therefore$ To pass through plate $\phi < 90°$

$$\frac{3}{2}\sin\theta = 1.2\sin\phi$$

for θ_{max} $\phi = 90°$

$$\frac{3}{2}\sin\theta = 1.2 \Rightarrow \sin\theta = \frac{2.4}{3} \Rightarrow \theta \approx 53°$$

72. (c) By Snell's law



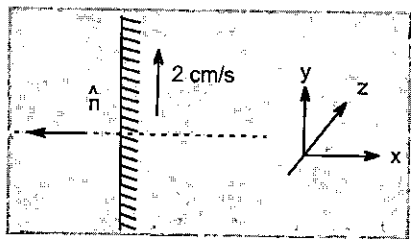
$$1 \cdot \sin 90° = \sqrt{2}\sin r$$

$$r = 45°$$

73. (d) The velocity of image,

$$v' = \vec{v} = 2(\vec{v}\cdot\vec{n})\hat{n}$$

$$= (\hat{i}+\hat{j}) - 2\left[(\hat{i}+\hat{j})-\hat{i}\right](-\hat{i}) = (-\hat{i}+\hat{j})\text{ cm/s}$$



Velocity of image with respect to mirror,

$$v'' = \vec{v'} - \vec{v}_m = (-\hat{i}+\hat{j}) - 2\hat{j} = (-\hat{i}-\hat{j})\text{ cm/s}$$

velocity of image with respect to object

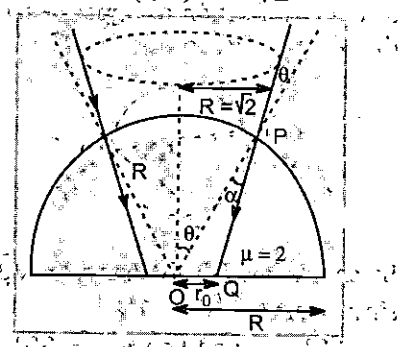
$$v' - v_0 = (-\hat{i}+\hat{j}) - (\hat{i}+\hat{j}) = -2\hat{i}\text{ cm/s}$$

Unit vector in the direction of reflected ray

$$= \frac{(-\hat{i}+\hat{j})}{\sqrt{2}}$$

74. (c) Let θ be angle of incidence and a be the angle of refraction for the extreme rays

$$\sin\theta = \left(\frac{R}{\sqrt{2}}\right)\frac{1}{R} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$



$$\mu = \frac{\sin\theta}{\sin\alpha} = \frac{1}{\sqrt{2}\sin\alpha} = \sqrt{2}$$

In ΔOPQ, $\angle OQP = 90°+\theta-\alpha$

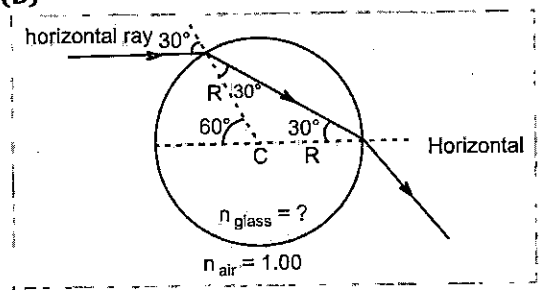
Applying sine rule

$$\frac{R}{\sin(90°+\theta-\alpha)} = \frac{r}{\sin\alpha}$$

$$r_0 = \frac{r}{\sin(90°+45°-30°)}\sin 30° = \frac{R\sin 30°}{\cos 45°}$$

$$= \frac{R}{2\left[\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}\right]} = \frac{\sqrt{2}R}{(\sqrt{3}+1)}$$

75. (b)



$1 \times \sin 60° = \mu \sin 30°$

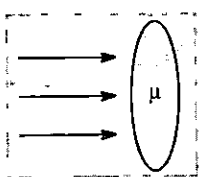
$\mu = \sqrt{3} = 1.73$

76. (c) The rays will first get refracted, then n-times reflected and finally again refracted.

using $\frac{\mu_2}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$ for first refraction, we have

$$\frac{\mu}{v_i} - \frac{1}{\infty} = \frac{(\mu - 1)}{R}$$

$$v_i = \left(\frac{\mu}{\mu - 1}\right) R$$



For first reflection, $\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$

$$\frac{1}{v_1} + \frac{(\mu - 1)}{\mu R} = \frac{-2}{R} \Rightarrow \frac{1}{v_1} = \frac{-(3\mu - 1)}{\mu R}$$

For second reflection,

$$\frac{1}{v_2} + \frac{(3\mu - 1)}{\mu R} = \frac{-2}{R} \Rightarrow \frac{1}{v_2} = \frac{-(5\mu - 1)}{\mu R}$$

Similarly after n^{th} reflections,

$$\frac{1}{v_n} = -\left[\frac{(2n+1)\mu - 1}{\mu R}\right]$$

Finally using $\frac{\mu_2}{v} - \frac{\mu_1}{v} = \left(\frac{\mu_2 - \mu_1}{R}\right)$

$\mu_1 = \mu, \mu_2 = 1, \mu = -v_n$

$$\frac{1}{v_1} - \frac{[(2n+1)\mu - 1]}{R} = -\frac{1-\mu}{R}$$

$$\Rightarrow \quad v_f = \frac{R}{2(\mu n + \mu + 1)}$$

Given $v_f = \frac{R}{13}; \mu = \frac{3}{2}$

$$\frac{R}{2\left(\frac{3}{2}n + \frac{3}{2} - 1\right)} = \frac{R}{13} \Rightarrow (3n+1) = 13 \Rightarrow n = 4.$$

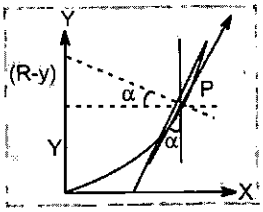
77. (d) By Snell's law $n\sin\alpha$ = constant

At $y = 0, \alpha = 90°, n = n_0$ At $n = n(y)$

$n(y)\sin\alpha = n_0 \sin 90°$

$$n_0 = n(y)(\sin\alpha) = n(y)\frac{(R-y)}{R}$$

$n(y) > n_0$

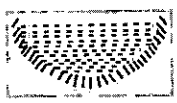


The material with the greatest known refractive index is diamond but even the refractive index of this material does not reach the value $n_{max} = 2.5$. To find the maximum angular size of arc, the refractive index changes from $n_0 = 1$ to $n_{max} = 2.5$.

$$\sin\alpha = \frac{1}{2.5}(90° - \alpha) = \cos^{-1}\left(\frac{1}{2.5}\right)$$

Maximum angular size of arc is $\cos^{-1}\left(\frac{1}{2.5}\right)$.

78. (c) Formed by direct reflection, real image coincides with object, then $u = R = 20$ cm,



II image is at a distance $v = R \pm l$

Let F be the focal length of system formed

$$\frac{1}{F} = \frac{2}{f_{lens}} + \frac{1}{f_{mirror}}$$

$$= \frac{2(\mu - 1)}{R} + \frac{2}{R} = \frac{2\mu}{R} = \frac{1}{v} + \frac{1}{u}$$

$$\mu = \left(\frac{1}{(R \pm l)} + \frac{1}{R}\right)\frac{R}{2}$$

for

$$v = R + l, \mu = \frac{(2R + l)}{2(R + l)} = \frac{(40 + 8)}{2 \times 28} = \frac{48}{56} = \frac{6}{7} \geq 1$$

This is not possible

For $v = R - l, \mu = \frac{(2R - l)}{2(R - l)} = \frac{32}{2 \times 12} = \frac{4}{3}$

79. (d) For lens

$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\frac{u_1}{f} = \frac{u_1}{v_1} - 1 \quad \Rightarrow m_1 = \frac{u_1}{v_1} = \frac{f}{u_1 + f}$$

and $m_2 = \frac{f}{u_2 + f}$

$$\frac{1}{m_1} - \frac{1}{m_2} = \frac{(u_2 - u_1)}{f} \Rightarrow \frac{(u_2 - u_1)}{(m_2)^{-1} - (m_1)^{-1}}$$

80. (b) Intensity of central maxima $I_u = (2A_0)^2 = 4A_0^2 = 4I_0$

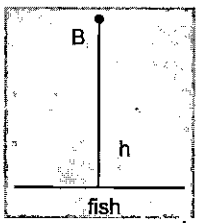
Intensity at distance x from the central maxima is half of the maximum intensity if

$$I_\phi = 4I_0 \cos^2\frac{\phi}{2} = \frac{4I_0}{2}$$

if $\cos^2\frac{\phi}{2} = \frac{1}{2} \Rightarrow \frac{\phi}{2} = \frac{\pi}{4} \Rightarrow \phi = \frac{\pi}{2}$

$$\phi = \frac{2\pi d}{\lambda D}x = \frac{\pi}{2} \Rightarrow x = \frac{\lambda D}{4d} = \frac{500\times10^{-9}\times1}{4\times10^{-3}}$$
$$= 1.25\times10^{4}\,\text{m}$$

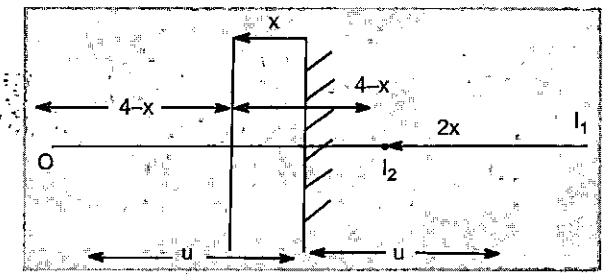
81. (c)



$$sh_{(ap)} = \mu h = \frac{4}{3}\times3 = 4\,\text{m}$$

82. (a) $V_{ap} = \mu V_{ac} = \frac{4}{3}\times6 = 8\,\text{m/s}.$

83. (b)



Since in the same time in which mirror moves by x, the image moves by $2x$. So same relation will carry in velocity as well as acceleration.

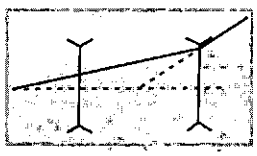
84. (d) Since light is going from rarer to denser medium hence no *TIR* will occur.

Hence option (d) is correct.

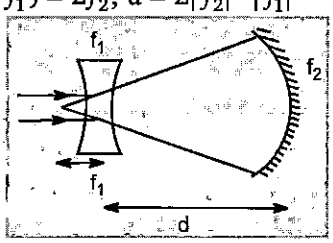
85. (c) $f = 10\,\text{cm}$ $\quad m = \frac{f}{f-u}$

$$m = \frac{1}{2} \qquad \frac{1}{2} = \frac{10}{10-u}$$
$$10-u = 20 \Rightarrow u = -10$$

88. (a)



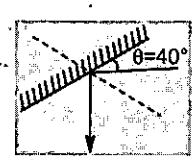
89. (b) $(d+f_1) = 2f_2;\ d = 2|f_2| - |f_1|$



90. (d) $i = 65 \Rightarrow \theta = 25°$

$\Rightarrow$ Angle of mirror

$\Rightarrow$ 40°+25° = 65°

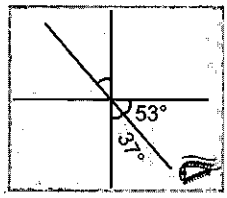


91. (d) $v = \frac{uf}{u-f} = -60\,\text{cm}$

$$\frac{h_1}{5} = \frac{-v}{u} \Rightarrow h_1 = -25\,\text{cm}$$

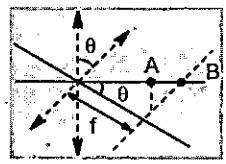
92. (a) $1\sin\theta = \frac{4}{3}\sin37°$

$$\sin\theta = \frac{4}{3}\times\frac{3}{5} = \frac{4}{5}$$
$$\sin\theta = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$$



93. (d) Along principal axis

$$\text{shift} = \frac{f}{\cos\theta} - f$$
$$= f\left[\frac{1-\cos\theta}{\cos\theta}\right] = \frac{f\theta^2}{2}$$

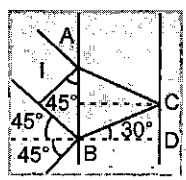


$\therefore$ amplitude $= \frac{f\theta^2}{4}$

94. (c) $\frac{1}{-0.5} + \frac{1}{0.2} = \frac{2}{r}$

$$\Rightarrow \frac{1}{r} = \frac{3}{2};\ r = 0.667\,\text{m}$$

95. (c)

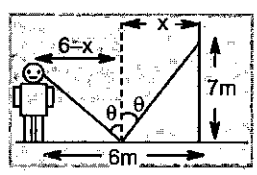


$$\frac{\sin i}{\sin r} = \sqrt{2} \Rightarrow r = 30°$$

$AB = \sqrt{2},$

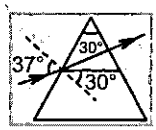
so $CD = \frac{\sqrt{2}}{2};\ BD = \frac{1}{\sqrt{2}\tan30°} = \sqrt{\frac{3}{2}}\,\text{mm}$

96. (a) $\frac{6-x}{x} = \frac{1.75}{7} \Rightarrow x = 4.8\,\text{m}$



97. (c) $1\sin37° = \frac{6}{5}\sin\theta$

$$\sin r_1 = \frac{1}{2},\ r_1 = 30°\text{h}$$



$\Rightarrow \quad r_2 = 0°$

$\Rightarrow \quad e = 0°$

$\delta = i + e - A = 37° + 0 - 30° = 7°$

98. (a) $\frac{2\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{29}} - 2\left(\left(\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}\right)\left(\frac{2\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{29}}\right)\right)\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$

$\frac{2\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{29}} - 2\left[\frac{6+18+8}{7\times\sqrt{29}\times 7}\right](3\hat{i}-6\hat{j}+2\hat{k})$

$= \frac{98\hat{i}-147\hat{j}+196\hat{k}-(192\hat{i}-384\hat{j}+128\hat{k})}{\sqrt{29}\times 49}$

$= \frac{-94\hat{i}+237\hat{j}+68\hat{k}}{49\sqrt{29}}$

99. (d) $\frac{1}{v}-\frac{1}{40}=\frac{1}{-10}$

$\frac{1}{v}=\frac{-1}{10}+\frac{1}{40}$

$\frac{1}{v}=\frac{-4+1}{40}=\frac{-3}{40}$

$v=\frac{-40}{3}$

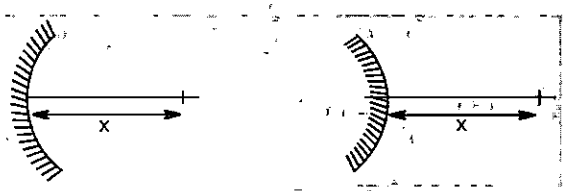
$u=-40; f=-10$

$v=\frac{uf}{u-f}=\frac{-40\times -10}{-40+10}=\frac{-40}{3}$ cm

$\frac{h_1}{3}=\frac{-\left(\frac{-40}{3}\right)}{-40}=\frac{-1}{3}$

$\Rightarrow \quad h_1=-1$ cm

100. (b)

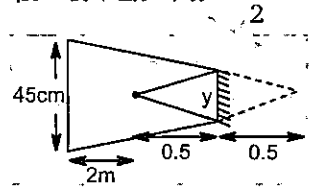


$m=\frac{f}{f-u}=\frac{\frac{-R}{2}}{\frac{-R}{2}+\frac{3R}{2}}=-\frac{1}{2}$

$m=\frac{f}{f-u}=\frac{\frac{R}{2}}{\frac{R}{2}+x}=\frac{1}{4}$

$\Rightarrow \quad 4R=R+2x \Rightarrow x=\frac{3R}{2}$

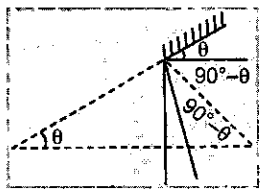
101. (d)



$\frac{0.5}{3}=\frac{y}{0.45} \qquad \frac{0.5}{3}=\frac{x}{1.2}$

$y=7.5$ cm $\quad \Rightarrow x=20$ cm

102. (b)



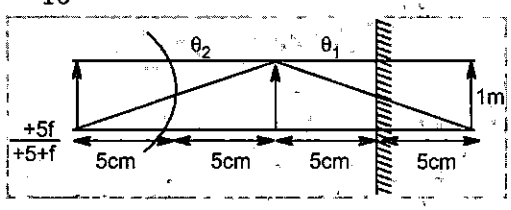
$\theta-(90°-\theta)=2\theta-90°$

$\tan(2\theta-90°)=\frac{x}{r}$

$-\cot 2\theta=\frac{x}{r} \Rightarrow \frac{2\tan\theta}{1-\tan^2\theta}=\frac{-r}{x}$

$\frac{2\times\frac{r}{h}}{1}=\frac{r}{x} \Rightarrow x=\frac{h}{2}$

103. (c) $\frac{1}{10}=\theta_1=0$



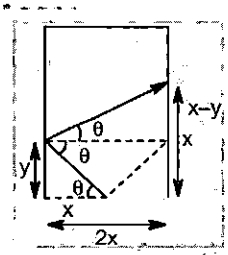
$\frac{h_1}{1}=\frac{5f}{5+f}$

$\theta_2=\frac{\frac{f}{5+f}}{5+\frac{5f}{5+f}}=\frac{f}{25+10f}=\frac{0.1}{2}$

$2f=2.5+f$

$f=2.5$ m

104. (d) $\tan\theta=\frac{y}{x}=\frac{x-y}{2x}$



$2xy=x^2-yx$

$3xy=x^2$

$y=\frac{x}{3}$

$-x\hat{i}+\frac{x}{3}\hat{j}$

105. (c) $\frac{n}{v}-\frac{1}{-S}=\frac{n-1}{R}$

$\frac{n}{v}=\frac{n-1}{R}-\frac{1}{S}$

$v>0 \Rightarrow \frac{n-1}{R}>\frac{1}{S} \Rightarrow S>\frac{R}{n-1}$

$$v\uparrow \Rightarrow \frac{1}{v}\downarrow$$

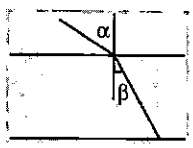
$$\frac{1}{v}=\frac{1}{R}-\frac{1}{nR}-\frac{1}{nS}=\frac{1}{R}\left(1-\frac{1}{n}\right)-\frac{1}{nS}$$

106. (d) $$m=2=\frac{f}{f-u}=\frac{f}{f-(-1.5)}$$

$$2f+3=f$$

$$v=\frac{uf}{u-f}=\frac{-1.5\times -3}{-1.5+3}=\frac{1.5\times 3}{1.5}=3\,\text{cm}$$

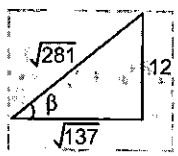
107. (a) $$\tan\alpha=\frac{8}{2.5}=\frac{16}{5}=3.2$$



$$\tan\beta=\frac{d-8}{4}$$

$$1\sin\alpha=1\times\frac{16}{\sqrt{281}}=\frac{4}{3}\sin\beta$$

$$\sin\beta=\frac{12}{\sqrt{281}}$$



$$\frac{12}{\sqrt{137}}=\frac{d}{4}-2$$

$$\Rightarrow \quad d=8+\frac{48}{\sqrt{137}}=12.1\,\text{m}$$

108. (d) For n mirror 1

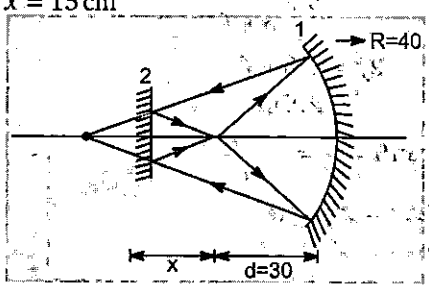
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Big|\begin{matrix}u=-30\\ f=-20\end{matrix}$$

$$\frac{1}{v}-\frac{1}{30}=-\frac{1}{20}$$

$$\frac{1}{v}=\frac{1}{30}-\frac{1}{20}$$

$$v=-60\,\text{cm}$$

So $x=15\,\text{cm}$



So distance between two mirrors = 45 cm

109. (a) $$v_{11}=6\hat{\mathbf{i}}+2\hat{\mathbf{k}}$$

$$v_{O\perp}=v_{M\perp}=-(v_{O\perp}-v_{M\perp})$$

$$v_{1\perp}=v_{M\perp}=-v_{O\perp}=-2\hat{\mathbf{i}}-\hat{\mathbf{j}}=3\hat{\mathbf{j}}$$

$$\vec{v}_I=6\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$$

$$v_I=\sqrt{36+9+4}=7\,\text{m/s}$$

110. (c) AB is the width of the beam in air and CD is the width of the beam in water of refractive index μ.

By law of refractive $\mu=\dfrac{\sin 45^\circ}{\sin r}=\dfrac{1}{\sqrt{2}\sin r}$

or $$\sin r=\frac{1}{\sqrt{2}\mu}$$

From the figure

$$\sin(90^\circ-r)=\frac{CD}{AC}$$

$$\cos r=\frac{CD}{AC}$$

$$\frac{AB}{AC}=\sin 45^\circ$$

$$AB=t \qquad \therefore \quad AC=\frac{t}{\sin 45^\circ}$$

$$CD=AC\cos r$$

$$=\frac{t}{\sin 45^\circ}\cos r=t\sqrt{2}\sqrt{1-\frac{1}{2\mu^2}}$$

$$=\frac{t\sqrt{2}\sqrt{2\mu^2-1}}{\sqrt{2}\mu}=\frac{t\sqrt{2\mu^2-1}}{\mu}$$

111. (a) Apply Snell's law between the points O and P, we have

$$2\times\sin 60^\circ=(\sin 90^\circ)\times\frac{2}{(1+H^2)}$$

$$2\times\frac{\sqrt{3}}{2}=1\times\frac{2}{(1+H^2)}$$

$$(1+H^2)=\frac{2}{\sqrt{3}}, \quad H=\sqrt{\left(\frac{2}{\sqrt{3}}-1\right)}$$

112. (a) $$\frac{\mu_2}{\mu_1}=\tan i$$

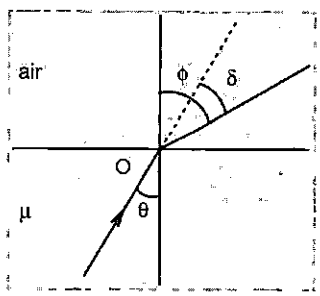
$$\frac{\mu_2}{\mu_1}=\frac{1}{\sin C}$$

$$\therefore \quad \tan i=\frac{\mu_2}{\mu_1}$$

$$\therefore \quad \sin i=\frac{\mu_2}{\sqrt{\mu_1^2+\mu_2^2}}$$

113. (b)

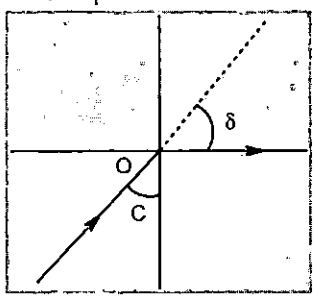
(i)



$\theta < C$

$\delta = \phi - \theta$

(ii)

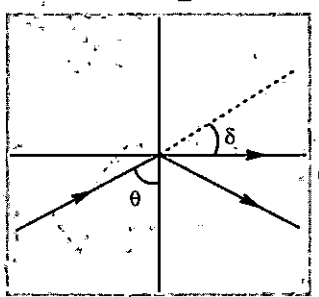


$\theta = C$

$\delta = \frac{\pi}{2} - C$

$\delta = \frac{\pi}{2}$

(iii)



$\theta > C$

$\delta = \pi - 2\theta$

$\mu = \frac{\sin\phi}{\sin\theta}, \phi = \sin^{-1}(\mu \sin\theta)$

(1) $\theta < C : \delta = \sin^{-1}(\mu \sin\theta) - \theta \longrightarrow$ nonlinear in the lim. $\theta \to C$ (from left)

$\lim_{\theta \to C^-} \delta_1 = \frac{\pi}{2} - C$

(2) $\theta > C : \delta = \pi - 2\theta \sin^{-1} \longrightarrow$ linearly δ decreasing as θ increasing

$\lim_{\theta \to C^-} \delta_2 = \pi - 2C$

$\therefore \quad \delta_2 = 2\delta_1$

[Discontinuity at $\theta = C$ LCL $\neq$ RHL]

114. (b) The maximum velocity of the insect is $A\sqrt{\frac{k}{M}}$

Its component perpendicular to the mirror is $A\sqrt{\frac{k}{M}} \sin 60°$.

Thus, maximum relative speed $= \frac{\sqrt{3}}{2} A\sqrt{\frac{k}{M}}$.

115. (a) The ray will always be normal, its x and y coordinates remains same

$\Delta S_{extra} = \int_0^C (\mu - 1)dz = \frac{Ac^4}{4}$

116. (a)

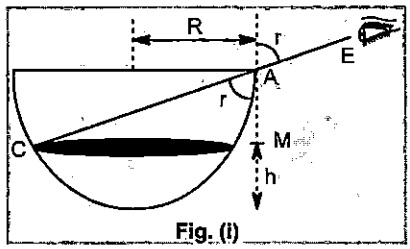


Fig. (i)

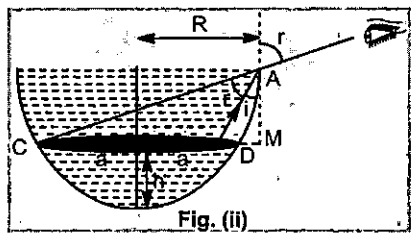


Fig. (ii)

As show in fig. (i) when the hemisphere was empty the ray from the extreme end C of the coin CA passes just above the eye E.

When the hemisphere is full of liquid of refractive index μ the ray DA from the near end of the coin after refraction just reaches the eye and hence the whole coin in visible.

From fig. (ii), $\frac{1}{\mu} = \frac{\sin i}{\sin r} = \frac{DM/DA}{CM/CA}$

$$= \frac{(R-a)}{\frac{\sqrt{(R-h)^2 + (R-a)^2}}{(R+a)}} \Big/ \sqrt{(R-h)^2 + (R+a)^2}$$

$$\frac{1}{\mu} = \frac{(R-a)\sqrt{(R-h)^2 + (R-a)^2}}{(R+a)\sqrt{(R-h)^2 + (R-a)^2}}$$

From fig. (iii), $a^2 = h(2R - h)$

$\therefore \quad a^2 + h^2 = 2hR$

$\therefore \quad \frac{1}{\mu} = \frac{(R-a)\sqrt{2R(R+a)}}{(R+a)\sqrt{2R(R-a)}} = \frac{\sqrt{R-a}}{\sqrt{R+a}}$

$\frac{1}{\mu^2} = \frac{R-a}{R+a}$ or $\mu^2 = \frac{R+a}{R-a}$

$\mu^2 R - \mu^2 a = R + a$

$a(\mu^2 + 1) = R(\mu^2 - 1)$

$\therefore \quad a = \frac{R(\mu^2 - 1)}{(\mu^2 + 1)}$

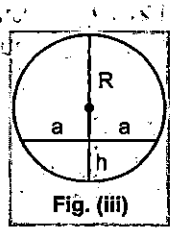
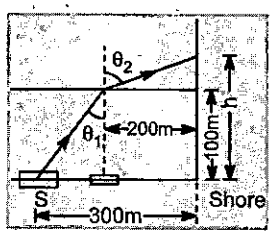


Fig. (iii)

Diameter of coin $= 2a = \frac{2R(\mu^2 - 1)}{(\mu^2 + 1)}$

117. (d) Situation is shown clearly in the figure



From Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{1}{4/3}$

$\Rightarrow \sin\theta_2 = \frac{4}{3\sqrt{2}}$

$\tan\theta_2 = \frac{200}{(h-100)}$

$\Rightarrow h = \frac{200}{\tan\theta_2} + 100$

$= \left(\frac{200\sqrt{2}}{4} + 100\right)\text{m} = 170.7\text{ m}$

118. (a) For refraction at the spherical surface,

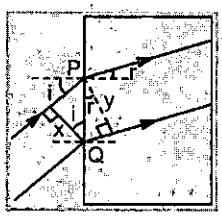
$\mu_1 = \frac{3}{2}, \mu_2 = 1, u = -20\text{ cm}, v = ?, R = 20\text{ cm}$

$\Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{1}{v} - \frac{3}{2\times(-20)} = \frac{1-1.5}{20}$

$\Rightarrow v = -10\text{ cm}$

119. (a) The situation is shown clearly in figure



We have, $i > r$

$x = PQ\cos i, \quad y = PQ\cos r$

So, $y > x$

i.e., inside the glass, the beam remains parallel but increases in width.

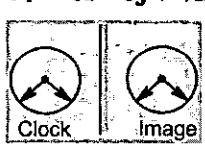
120. (b) Using $\vec{v}_I + \vec{v}_O = 2\vec{v}_M$ [Be careful that this equation is valid only along the normal *i.e.*, in present case along *x*-axis]

$\Rightarrow v_I = 2\times1 - 2 = 0$

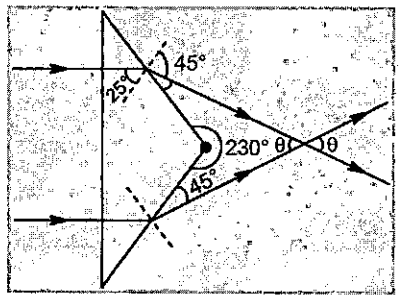
So, the velocity vector of image is

$\vec{v}_I = 0\hat{i} - 3\hat{j} + 4\hat{k}$

121. (b)

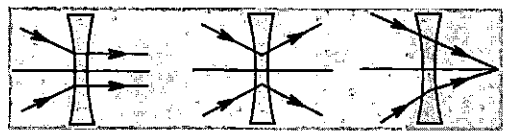


122. (c) $\frac{\sin 25°}{\sin r} = \frac{1}{1.66}$

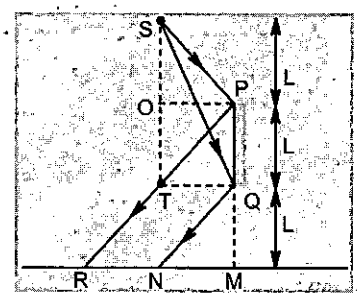


$r \simeq 45°$ So, $\theta = 40°$

123. (d)



124. (c) Ray diagram is as shown in the figure below



Here, $\frac{SO}{OP} = \frac{PM}{RM}$

$\Rightarrow RM = 2L$

Also $\frac{ST}{TQ} = \frac{QM}{SN}$

$\Rightarrow SN = \frac{L}{2}$

$\therefore \quad RS = RM - SN = \frac{3L}{2}$

125. (c) $1.8 = \frac{f}{f+10}$

$1.8f + 18 = f$

$18 = -0.8f$

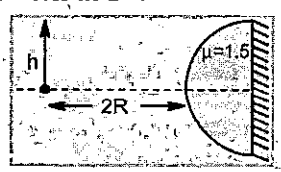
$f = -22.5\text{ cm}$

In second case, $u = -50$

$\Rightarrow$ object is beyond *C*.

$\Rightarrow$ image is inverted and diminished

126. (d) Here, three optical phenomena take place-first refraction, then reflection, and finally refraction. For refraction at 1st.

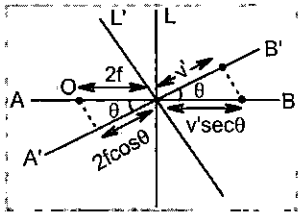


$\frac{1.5}{v} - \frac{1}{-2R} = \frac{1.5-1}{R} \Rightarrow \frac{1.5}{v} = 0$

$\Rightarrow \quad v = \infty$

i.e., rays after refraction get parallel to axis and strike the mirror normally, get retraced back and the final image is formed at the same place where the object is and of the same size. Image would be real.

127. (b) As the lens rotates by θ, the *OA* also rotates by θ as shown in figure. Primed letters in figure are corresponding to rotated position of lens.



The object distance along the *OA* in rotated position of lens is $u' = -2f\cos\theta$.

From lens formula,

$$\frac{1}{v'} - \frac{1}{-2f\cos\theta} = \frac{1}{f}$$

$\Rightarrow \quad v' = \dfrac{2f\cos\theta}{2\cos\theta - 1}$ as θ is small so v' comes out to be + ve.

So, the image distance along initial *OA* is,

$$v = v'\sec\theta = \frac{2f}{2\cos\theta - 1}$$

Velocity of image is,

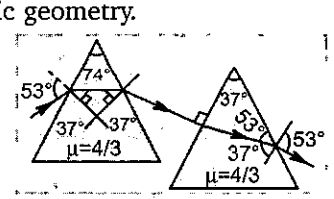
$$\frac{dv}{dt} = \frac{2f}{(2\cos\theta - 1)^2} \times (-2\omega \sin\theta)$$

where $\theta = \omega t$

For $\theta° = 1° = \dfrac{\pi}{90}$ rad, $\sin\theta \simeq \dfrac{\pi}{90}$ and $\cos\theta \simeq 1$

$$v_{\text{image}} = -\frac{4f\omega \times \frac{\pi}{90}}{(2-1)^2} = \left|-\frac{4f\pi\omega}{90}\right| \text{ m/s along } BA.$$

128. (a) Here we have drawn the ray diagram with all angles. The values of all angles can be found by using concepts of prism theory, refraction and basic geometry.



Deviation produced by 1st prism is,

$\delta_1 = 53° + 53° - 74° = 32°$ clockwise

Deviation produced by 2nd prism is,

$\delta_2 = 53° - 37° = 16°$ – clockwise

So, $\delta = 32° + 16° = 48°$ – clockwise

129. (c) Let image formed by 1st lens be at distance v_1 from convex lens.

Now, $\dfrac{1}{v_1} - \dfrac{1}{-40} = \dfrac{1}{20}$

$\Rightarrow \quad v_1 = 40\,\text{cm}$

Size of intermediate image is, $\dfrac{h_I}{h_O} = \dfrac{v_1}{u} = \dfrac{40}{-40}$

$\Rightarrow \quad h_I = -1\,\text{cm}$

– ve sign tells that image formed is inverted.

We can find this image distance directly also, as object distance is $2f$.

This image formed by 1st lens would be treated as object for 2nd lens, let final image forms at a distance v from concave lens.

$$\frac{1}{v} - \frac{1}{32} = \frac{1}{-10} \Rightarrow v = -14.5\,\text{cm}$$

– ve sign tells that image is formed on left side of concave lens.

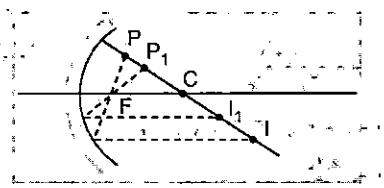
The magnification produced by 2nd lens is,

$$m_2 = \frac{v}{m} = -\frac{14.5}{32}$$

So, size of final image is $h = \dfrac{14.5}{32} \times 1 = \dfrac{1}{2.2}$ cm

So, final image is virtual, erect and diminished.

130. (b) As the object approaches *C*, image is also approaching *C*, so velocity of image is along $\vec{\mathbf{v}}_2$.



Aliter : Let us draw the ray diagram at two very close instants, among one of which is the given instant. It si clear from the ray diagram that as object moves from P to P_1, the image moves from I to I_1 and hence velocity of image, when object is at P is along $\vec{\mathbf{v}}_2$.

131. (c) Let the power of light source be *P*, then intensity at any point on the screen is due to light rays directly received from source and that due to after reflection from the mirror.

$\Rightarrow \quad I = \dfrac{P}{4\pi a^2} + \dfrac{P}{4\pi \times (3a)^2}$

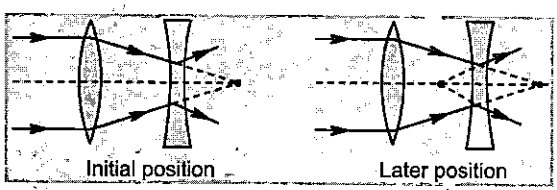
$$= \frac{P}{4\pi a^2}\left(1 + \frac{1}{9}\right)$$

$$= \frac{P}{4\pi a^2}\left(\frac{10}{9}\right)$$

When mirror is taken away,

$$I_1 = \frac{P}{4\pi a^2} = \frac{9I}{10}$$

132. (b)

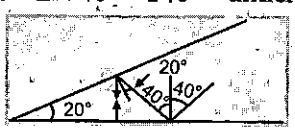


Initially, the object for lens B (the image formed by lens A) is at its focus, so rays are parallel.

As the separation between A and B decreases, the object for B is lying always from focus and hence, the rays get diverged.

133. (c) $\delta_1 = \pi - 2\times 40° = 100°$– clockwise

$\delta_2 = \pi - 2\times 40° = 140°$ – anticlockwise

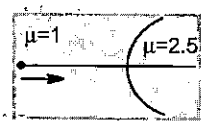


$\delta_3 = \pi - 2\times 20° = 140°$ – clockwise

$\Rightarrow$ Deviation (Net) = 140° – clockwise

134. (c) $\frac{2.5}{v} - \frac{1}{-u} = \frac{2.5-1}{R}$

$$\Rightarrow \frac{2.5}{v} = \frac{1.5}{R} - \frac{1}{u}$$



$$\Rightarrow v = \frac{2.5uR}{1.5u - R}$$

$v = \infty$ at $1.5u - R = 0$

$$\Rightarrow u = \frac{2R}{3}$$

For $u < \frac{2R}{3}$, $v < 0$ virtual image

$u > \frac{2R}{3}$, $v > 0$, real image

So, image is changing from real to virtual at $u = \frac{2R}{3}$

135. (a) As object moves from infinity to pole of convex mirror, image moves from focus to pole, so, $v_I < v_O$, always.

136. (a) $\frac{1}{v} - \frac{1}{-15} = \frac{1}{5}$

$$\frac{1}{v} = \frac{1}{5} - \frac{1}{15}$$

$v = 7.5$ cm

$v' = -12.5$ cm

$$\frac{1}{v'} = \frac{2}{12.5} = \frac{1}{-15}$$

$$\frac{1}{v'} = \frac{2}{25} - \frac{1}{15}$$

$$\frac{1}{v'} = \frac{12-10}{150}$$

$v' = 75$ cm

$v'' = -95$ cm

$$\frac{1}{v''} + \frac{1}{95} = \frac{1}{5}$$

$v'' = 5.3$ cm

137. (c) As t is increasing, μ is decreasing, so refraction is taking place from denser to rarer medium as light is proceeding in the medium.

Using continuous varying refractive index, let the ray reflect back after travelling $t = t_0$ along the normal, then at this location, angle of refraction is $\frac{\pi}{2}$. So

$$1\times \sin\theta = (\mu_0 - kt_0^2)\times \sin\frac{\pi}{2}$$

$$\Rightarrow \quad t_0 = \sqrt{\frac{\mu_0 - \sin\theta}{k}}$$

138. (d) Deviation produced by mirror is

$\delta_1 = \pi - 2\times 60° = 60°$; clockwise

Deviation produced by prism is,

$\delta_2 = (2-1)\times 6° = 6°$; anticlockwise

So, net deviation,

$\delta = \delta_1 + \delta_2$

$= (60°-6°)$; clockwise $= 54°$; clockwise

139. (d) Focal length for upper half is

$$f_{1_1} = \left(\frac{\mu - 1}{\mu/\mu_1 - 1}\right) f_{air} = \left(\frac{1.5-1}{\frac{1.5}{1.2}-1}\right) 20 = 40\text{ cm}$$

Focal length for lower half is,

$$f_2 = \left(\frac{\mu - 1}{\mu/\mu_2 - 1}\right) f_{air} = \left[\frac{1.5-1}{\frac{1.5}{2.5}-1}\right] \times 20 = -25\text{ cm}$$

If the object is at infinity, two images will form at corresponding focuses.

So, the required separation is

$x = |f_1| + |f_2| = 40 + 25 = 65$ cm

140. (c) $h_{I_1} \times h_{I_2} = h_0^2$

$$4h_{I_1}^2 = h_0^2 \Rightarrow \frac{h_1}{h_0} = \frac{-1}{2}$$

$$\frac{-1}{2}=\frac{(45-x)}{-x}$$

$\Rightarrow \quad x=90-2x$

$x=30\,\text{cm}$

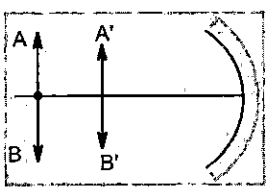
$$\frac{1}{30}+\frac{1}{15}=\frac{1}{f}$$

$\Rightarrow \quad f=10\ \text{cm}$

141. (c) As focal length of mirror is independent of surrounding medium, so the image of ends A and B formed by concave mirror are having the same x-coordinates and image is perpendicular to PA.

For y-coordinates,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$



$$\Rightarrow \quad \frac{1}{v}-\frac{1}{40}=\frac{1}{-10}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{40}-\frac{1}{10}$$

$$\Rightarrow \quad v=\frac{-40}{3}\,\text{cm}$$

From magnification formula

$$A'B'=\frac{2}{3}\,\text{cm}$$

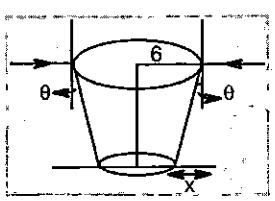
From magnification formula

$$A'B'=\frac{2}{3}\,\text{cm}$$

142. (b) $1\sin 90°=1.25\sin\theta$

$\theta=53°$

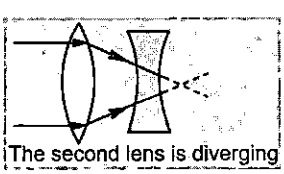
$$\frac{x}{3}=\tan\theta \Rightarrow x=4$$



$r=6-4=2\,\text{m}$

143. (a) Dimension parallel to surface does not change

144. (c)



The second lens is diverging

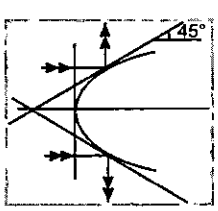
145. (d) At the point of incidence slope should be ±1

$$y^2=2x$$

$$y=\sqrt{2x}$$

$$\frac{dy}{dx}=\sqrt{2}.\frac{1}{2\sqrt{x}}=\frac{1}{\sqrt{2x}}=1$$

$\Rightarrow \quad \sqrt{2x}=1$



$\Rightarrow \quad 2x=1$

$$\Rightarrow \quad x=\frac{1}{2}\Rightarrow y=\pm 1$$

146. (a) The bubble acts as a diverging lens.

$\Rightarrow$ Image is virtual, erect and diminished

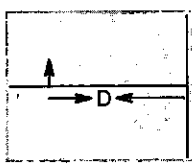
147. (c) $\text{NS}=100-98=2\,\text{cm}=t\left(1-\frac{1}{n}\right)$

$$2=t\left(\frac{0.5}{1.5}\right)\Rightarrow t=6\,\text{cm}$$

148. (b) $v_I=-m^2v_O$

$$=-\left(\frac{10}{30}\right)^2\times 9=1\,\text{m/s}$$

149. (d)

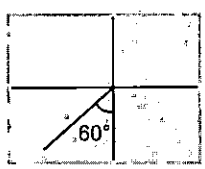


$|h_1|=h_0 \quad \Rightarrow m=\pm 1$

$\Rightarrow$ object is at $2f$ and image is $2f$ from lens

$D=4f \Rightarrow f=\frac{D}{4}$

150. (c)



$$\frac{4}{3}\sin 60°=\frac{4}{3}\frac{\sqrt{3}}{2}=\frac{2}{\sqrt{3}}>1\approx\sin 90°$$

$\Rightarrow$ Ray will undergo TIR

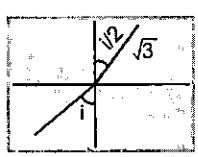
151. (b) $1\sin i=\sqrt{3}\sin\frac{i}{2}$

$$2\cos i=\sqrt{3}$$

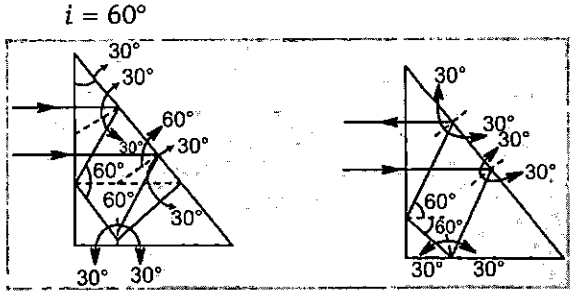
$$\cos\frac{i}{2}=\frac{\sqrt{3}}{2}$$

$$\frac{i}{2}=30°$$

$$i=60°$$



152. (b)



154. (d) v_O is decreasing $\Rightarrow a_O$ is $-$ve

$\Rightarrow a_I$ is in opposite direction $\Rightarrow a_I$ is $+$ve

155. (c) In one case image is virtual ($u = -10\,\text{cm}$)

In another case image is real ($u = -40\,\text{cm}$)

$$v_1 = \frac{uf}{u+f} = \frac{-10f}{-10+f}$$

$$v_2 = \frac{-40f}{-40+f}$$

In both situations, sign convention is opposite

$$\Rightarrow \quad v_2 = -v_1$$

$$\Rightarrow \quad v_2 = \frac{-10f}{-10+f} = \frac{40f}{-40+f}$$

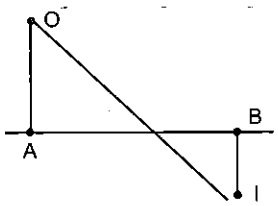
$$f = 16\,\text{cm}$$

Problems

Level 2 More than One Alternative is/are Correct

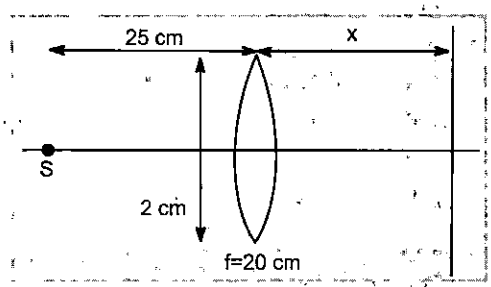
1. In displacement method, the distance between object and screen is 96 cm. The ratio of length of two images formed by a convex lens placed between them is 4.84.
(a) Ratio of the length of object to the length of shorter image is 11/5
(b) Distance between the two positions of the lens is 36 cm
(c) Focal length of the lens is 22.5 cm
(d) Distance of the lens from the shorter image is 30 cm

2. A luminous point object is placed at O, whose image is formed at I as shown in figure. Line AB is the optical axis. Which of the following statement is/are correct?



(a) If a lens is used to obtain the image, then it must be a diverging lens and its optical centre will be the intersection point of line AB and OI.
(b) If a lens is used to obtain the image, then it must be a converging lens and its optical centre will be the intersection point of line AB and OI.
(c) If a mirror is used to obtain the image then the mirror must be concave and object and image subtend equal angles at the pole of the mirror.
(d) I is a real image.

3. A point source of light S is placed on the axis of a lens of focal length 20 cm as shown. A screen is placed normal to the axis of lens at a distance x from it. Treat all rays as paraxial :

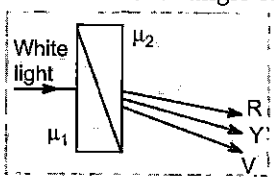


(a) As x is increased from zero intensity continuously decreases
(b) As x is increased from zero intensity first increases then decreases
(c) Intensity at centre of screen for $x = 90$ cm and $x = 110$ cm is same
(d) Radius of bright circle obtained on screen is equal to 1 cm for $x = 200$ cm

4. A ray of light from a denser medium strikes the plane boundary of a rarer medium at an angle of incidence I. The angle of refraction is r. If the reflected and the refracted rays are mutually perpendicular the critical angle of the medium is :
(a) $\sin^{-1}(\cot I)$ (b) $\sin^{-1}(\tan I)$
(c) $\tan^{-1}(\sin I)$ (d) $\sin^{-1}(\tan r)$

5. A plano-convex lens ($\mu = 1.5$) of focal length 20 cm has its plane side silvered :
(a) The radius of curvature of its curved surface is half that of a surface of equi-convex lens of focal length 20 cm made of same material.
(b) An object placed at 15 cm on the axis on the convex side gives rise to an image at a distance of 30 cm from it.
(c) An object placed at a distance of 20 cm on the axis on the convex side gives rise to an image at 40 cm from it.
(d) It acts as a convex mirror.

6. A beam of white light is incident on a combinations of thin prism with equal angles. In the out going beam the rainbow colours are observed as shown. μ stands for refractive index and θ for angle of dispersion. Then:



(a) $\mu_1 < \mu_2$ (b) $\mu_2 = \mu_1$
(c) $\theta_1 > \theta_2$ (d) $\mu_1 > \mu_2$

7. A lens made from a material of refractive index 1.5 behaves as a converging lens in air. When placed in liquid of refractive index 8/5, it will :
(a) still behave as a converging lens
(b) behave as a diverging lens
(c) have its focal length increased
(d) have its focal length decreased

8. The slit 1 of a Young's double slit experiment is winder than slit 2, so that the light from slits are given as $A_1 = A_0 \sin\omega t$ and $A_2 = 3A_0 \sin\left(\omega t + \frac{\pi}{3}\right)$. Then resultant amplitude and intensity at a point where the path difference them is zero, is A and t respectively, then :
(a) $A = \sqrt{13}A_0$ (b) $A = 4A_0$
(c) $I \propto 16A_0^2$ (d) $I \propto 13A_0^2$

9. Two transparent plane parallel plates are stacked face - to - face. The plate whose thickness $t_1 = 4.5\,\text{cm}$ has a refractive index $n_1 = \frac{3}{2}$ while the other plate whose thickness $t_2 = 2\,\text{cm}$ has a refractive index $n_2 = \frac{4}{3}$. A narrow light beam is incident on the first plate at an angle $i = 37°$ to the normal. The lateral shifts of the beam emerging from the second plate into air space due to I plate only, due to II plate only, and due to both the plates are d_1, d_2 and d respectively, then :
(a) $d_1 \simeq 0.5\,\text{cm}$ (b) $d_1 \simeq 1.129\,\text{cm}$
(c) $d \simeq 1.5\,\text{cm}$ (d) $d_2 \simeq 2\,\text{cm}$

10. A plano-convex lens of focal length 20 cm has its place side silvered :
(a) The radius of curvature of its curved surface of plano-convex lens is equal to half of radius of curvature of a surface of equi-convex lens of focal length 20 cm
(b) A object placed at 15 cm on the axis on the convex side of silvered plano-convex lens gives rise to an image at a distance of 30 cm from it
(c) An object placed at a distance of 20 cm on the axis on the convex side of silvered plano-convex lens gives rise to an image at 40 cm from it
(d) Silvered plano-convex lens acts as a concave mirror of focal length 10 cm

ANSWERS

1.	(a, b, d)	**2.**	(b, c, d)	**3.**	(b, c, d)	**4.**	(a, d)	**5.**	(a, b)	**6.**	(c, d)
7.	(b, c)	**8.**	(a, d)	**9.**	(b, c)	**10.**	(a, c, d)				

Solutions

Level-2: More than One Alternative is/are Correct

1. (a, b, d)

$D = 96$

Given $\frac{I_2}{I_1} = 4.84$

Let $I_2 = 4.84\ a$ and $I_1 = a$

$O = \sqrt{I_1 I_2} = 2.2a$

(a) Required ratio $= \frac{2.2a}{a} = \frac{11}{5}$

(b) $\frac{v}{u} = \frac{11}{5}$ ...(1)

$v + u = 96$ ...(2)

$\therefore \quad v + \frac{5v}{11} = 96$

$\Rightarrow \quad \frac{16v}{11} = 96$

$\Rightarrow \quad v = 66$ and $u = 30$

Distance between two position

$= v - u = 66 - 30 = 36\,\text{cm}$

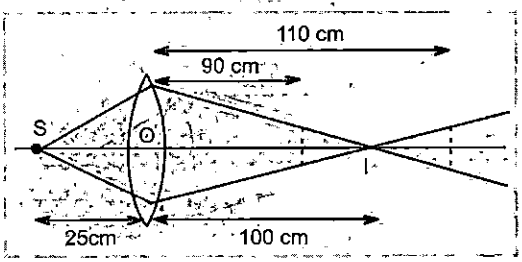
(c) $\frac{1}{f} = \frac{1}{66} + \frac{1}{30} = \frac{30 + 66}{30 \times 66}$

$\Rightarrow \quad f = \frac{30 \times 66}{96} = \frac{330}{16} = 20.625\,\text{cm}$

(d) $u = 30$

3. (b, c, d)

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-25)} = \frac{1}{20}$

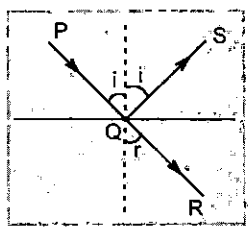


$\Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{25} = \frac{1}{100} \Rightarrow V = 100\,\text{cm}$

from O to I intensity increases and then decreases at $x = 90\,\text{cm}$ and $110\,\text{cm}$ intensity is same (d) Radius at $x = 200\,\text{cm}$ is equal to radius of lens.

4. (a, d)

PQ = Incident ray QS = Reflected ray



QR = Refracted ray

$\therefore \quad \angle SQR = 180 - (r + r)$

$= 90°$

$\therefore \quad i + r = 90° \qquad r = 90° - i$

$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90 - i)} = \tan i$

If C is the critical angle $\mu = \frac{1}{\sin C}$

$\tan i = \frac{1}{\sin C}$

$\sin C = \frac{1}{\tan i} = \cot i$

$C = \sin^{-1}(\cot i)$

5. (a, b)

$\frac{1}{F} = \frac{(\mu - 1)}{R} = \frac{1}{20} \Rightarrow 20 = \frac{R}{(\mu - 1)}$ ...(1)

If equiconvex lens of $f = 20$

then $\frac{1}{20} = (\mu - 1)\frac{2}{R}$

From eq. (1) and (2) we conclude

$\therefore \quad R = 40(\mu - 1)$ ...(2)

that option (a) is correct.

After silvering :

$F_{eq} = -10\,\text{cm}$ (combination will behave like a concave mirror)

(b) $u = -15$

$\frac{1}{v} + \frac{1}{\mu} = \frac{1}{F_{eq.}} \Rightarrow \frac{1}{v} - \frac{1}{15} = -\frac{1}{10}$

$$\frac{1}{v}=\frac{1}{15}-\frac{1}{10} \Rightarrow v=-30\,\text{cm}$$

So (b) is correct.

(c) $u=-20$

$v=-20$

So (c) is wrong

Alternative:

For planoconvex

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$R_1=\infty\ R_2=R$

$r=20(\mu-1)$

For equiconvex $\frac{1}{20}=(\mu-1)\left(\frac{1}{R}+\frac{1}{R}\right)$

$R=40(\mu-1)$

6. (c, d)

$\delta_1+\delta_2>0$

$\Rightarrow \quad \mu_1>\mu_2$

$\theta>0$

$\theta_1-\theta_2>0$

7. (b, c)

$$\frac{1}{f_a}=({}_a\mu_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

[f_a = focal length of lens in air]

$$\frac{1}{f_L}=({}_Lm_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\frac{({}_L\mu_g-1)}{({}_a\mu_g-1)}\frac{1}{f_a}$$

$$=\frac{\left(\frac{3/2}{8/5}-1\right)}{\left(\frac{3}{2}-1\right)}\frac{1}{f_a}=\frac{\left(\frac{15}{16}-1\right)}{\frac{1}{2}}\frac{1}{f_a}=-\frac{1}{8}\cdot\frac{1}{f_a}$$

$\therefore f_L=-8f_a$; negative sign indicate that lens is diverging.

8. (a, d)

If the amplitude due to two individual sources at point P is A_0 and $3A_0$ then the resultant amplitude at P, will be :

$$A=\sqrt{A_0^2+(3A_0)^2+2(A_0)(3A_0)\cos\frac{\pi}{3}}=\sqrt{13}A_0$$

Resultant intensity, $I\propto 13A_0^2$

9. (b, c)

Shift $$d=\left[1-\frac{\cos i}{\sqrt{n_1^2-\sin^2 i}}\right]t_1\sin i$$

$$+\left[1-\frac{\cos i}{\sqrt{n_2^2-\sin^2 i}}\right]t_2\sin i$$

$i=37^\circ\ n_1=3/2\ t_1=4.5\,\text{cm}$

$n_2=4/3,\ t_2=2\,\text{cm}$

$$d=\left[1-\frac{0.8}{\sqrt{(3/2)^2-0.6^2}}\right]\frac{9}{2}\times 0.6$$

$$+\left[1-\frac{0.8}{\sqrt{(4/3)^2-0.6^2}}\right]2\times 0.6$$

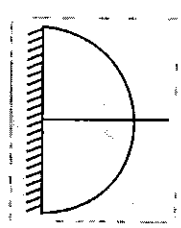
$=1.129+0.39=1.5\,\text{cm}\ (d=d_1+d_2)$

10. (a, c, d)

For plano-convex lens $\frac{1}{f}=\frac{(\mu-1)}{R}$

For equi-convex lens $\frac{1}{f}=(\mu-1)\frac{2}{R'}$

$R=\frac{R'}{2}$ (choice (a) is correct)



Let F be the focal length of silvered plano-convex lens

$$\frac{1}{F}=\frac{2(\mu-1)}{R}+\frac{1}{\infty}=\frac{2(\mu-1)}{R}$$

Given $\frac{1}{f}=\frac{1}{20}=\frac{(\mu-1)}{R}$

$F=10\,\text{cm}$ (it acts as concave mirror) choice (d) is correct

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{F}$$

If $u=15\,\text{cm}$, $\frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{3-2}{30}=\frac{1}{30}$

$\Rightarrow \quad v=30$

If $u=20\,\text{cm}$, $\frac{1}{v}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}\Rightarrow v=20$

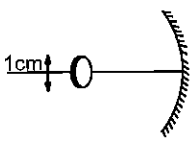
(choice c is correct)

Problems

Level 3 Comprehension Based Problems

Passage 1

A concave mirror of radius of curvature 20 cm is shown in the figure. A circular disc of diameter 1 cm is placed on the principle axis of mirror with its plane perpendicular to the principal axis at a distance 15 cm from the pole of the mirror. The radius of disc increasing according to the law $r = (0.5 + 0.1t)$ cm/sec.



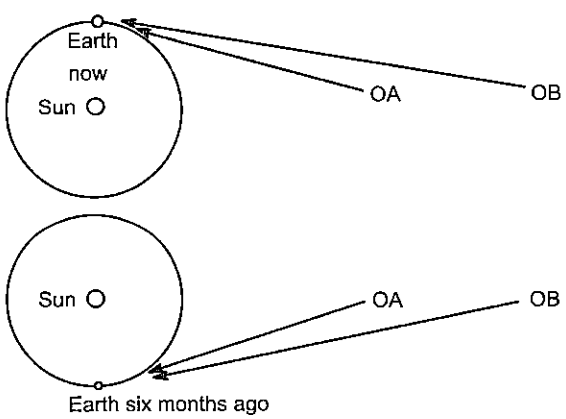
1. The image formed by the mirror will be in the shape of a :
- (a) circular disc
- (b) elliptical disc with major axis horizontal
- (c) elliptical disc with major axis vertical
- (d) distorted disc

2. In the above question, the area of image of the disc at $t = 1$ second is :
- (a) 1.2π cm^2
- (b) 1.44π cm^2
- (c) 1.52π cm^2
- (d) none of these

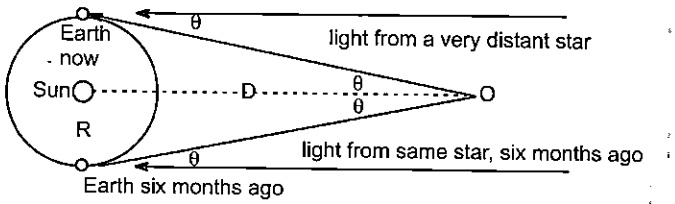
3. What will be the rate at which the horizontal radius of image will be changing :
- (a) 0.2 cm/sec increasing
- (b) 0.2 cm/sec decreasing
- (c) 0.4 cm/sec increasing
- (d) 0.4 cm/sec decreasing

Passage 2

Parallax refers to the different views that you see from two different positions. Try this experiment. Hold the index finger of your left hand vertical, 20 cm infront of you. Hold the index finger of your right hand vertical, 40 cm infront of you. Now close your left eye and, using just your right eye, move the two finger sideways until they line up. Now close your right eye and open the left. The closer finger has 'jumped' to the right of the further finger. Repeat a few times. Compared to a distant background, both fingers have both jumped to the right, but the closer one jumps farther. If you measure the angles through which they jump and the distance between your eyes, you can work out how far away the fingers are. For distant objects, the distance between our viewing positions must be greater than the distance between your eyes. Fortunately for astronomers, the Earth shifts our telescopes round the sum, so we can get a separation equal to the diameter of the orbit of the Earth (16 light minutes) if we wait six months, as shown in this diagram.



In this sketch, which is not to scale, imagine an observer looking at objects A and B, standing at the pole of the Earth with his head towards us. Now he sees object A to be to the right of B. Six months ago, he saw it to be to the left of B. Now most stars are so far away from us that we cannot observe any relative motion in this way. However, for close stars it is possible. The next sketch shows the path of light from a close object and from a very distant star.



From trigonometry,
$D = R/\tan\theta = R/\theta$
where we have used the small angle approximation for θ measured in radians. A parsec is defined as the distance to an object that 'moves' (w.r.t. to the distant stars) by an angle of 1 second (1/3600 of a degree) when the Earth moves by the mean radius of its orbit. In terms of this sketch, if θ = one second, D = 1 parsec. Now all stars except the sun are more than one parsec distant, so to measure their distance by parallax, we need to be able to resolve angles of about 1 second or better.

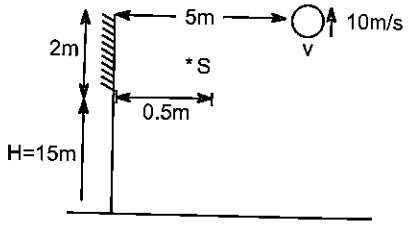
1. In the first experiment if we have two point objects A and B in place of your left and right fingers respectively then :
(a) your two eyes and A and B will lie on a circle
(b) your two eyes and A and B will always lie in a plane
(c) your two eye and A will form a triangle area of which is be equal to the area of the triangle formed by A, B and your left eye
(d) none of the above

2. In **above** if we have three objects A, B and C such that you see only A and B from your left eye and A and C from your right eye. Angle formed by AB on your left eye equals that formed by AC on your right eye. Then AB will :
(a) always equal to AC
(b) be equal to AC if your two eye and A form an isosceles triangle
(c) be equal to AC if your two eye and A form a right angled triangle
(d) none of the above

3. Parallex method is useful only if the star whose distance is to be measure should be closer to the earth and the reference star should be :
(a) very close (b) very far away
(c) any other star (d) only pole star

Passage 3

A balloon is moving upwards with a speed of 10 m/s at a distance 5m infront of a plane mirror of length 2m as shown in figure. A source S is placed symmetrically before plane mirror. The plane mirror is placed at a height $H = 15$ m above the surface of ground.



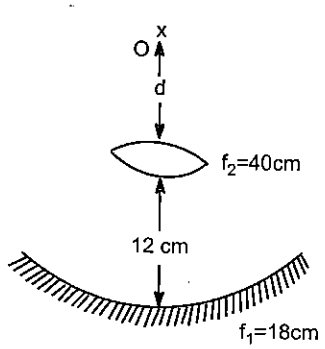
1. The maximum time during which a man inside the balloon can see the image of the source in the mirror is:
(a) 2.2 sec (b) 1.7 sec
(c) 4.6 sec (d) 0.5 sec

2. When the balloon is at a height of 17 m from ground, the man drops himself from the balloon, the greatest distance over which he can see the image of source in plane mirror during free fall and corresponding time duration are ($g = 10\,m/s^2$) respectively :
(a) 22 m, 2.2 s (b) 17m, 2.84 s
(c) 12 m, 1.7 s (d) 2m, 2s

3. If the balloon is moving upwards with a speed of 20 m/s, the man drops himself from the balloon at a height of 17 m from ground, the time duration for which he will see the image of source in mirror is :
(a) 2.2 m (b) 1.7 m
(c) 1 s (d) 2.84 s

Passage 4

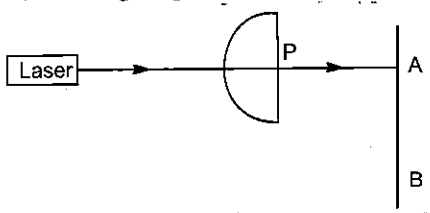
A convex lens sof focal length 40 cm is held at a distance 12 cm coaxially above a concave mirror of focal length 18 cm.



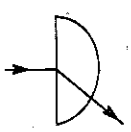
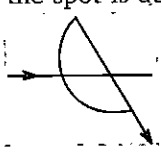
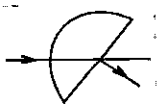
1. A luminous point object placed d cm above the lens on its axis gives rise to an image coincident with itself, then the value of d is equal to :
   (a) 15 cm (b) 30 cm
   (c) 24 cm (d) 18 cm
2. In the above problem, if the convex lens is replaced by a concave lens of focal length 60 cm, it gives rise to an image coincident with itself. Then the value of d is :
   (a) 20 cm (b) 30 cm
   (c) 40 cm (d) 60 cm
3. In the above problem, if the convex lens is replaced by a glass plate of thickness 6 cm, refractive index $\mu = \frac{3}{2}$ and gives rise to an image coincident with itself, then the value of d is equal to :
   (a) 20 cm (b) 30 cm
   (c) 40 cm (d) 60 cm

Passage 5

Light from a laser is directed at a semi-circular glass block. The light passes undeviated through the block and on to a screen, forming a spot as shown.

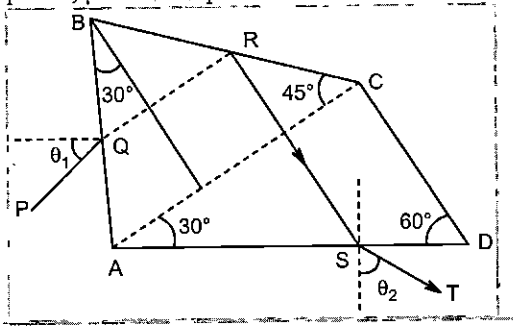


The semi-circular block is rotated about the point P. The spot of light on the screen is seen to move downwards. When the spot reaches point B, it disappears.

1. Which of the following diagram shows the position of the semi-circular block when the spot is at point B.
   (a) 
   (b) 
   (c) 
   (d) none of these
2. In a particular experiment, the distance PA is 120 cm and distance AB is 90 cm. Calculate the refractive index of the glass of the block :
   (a) $\frac{5}{3}$ (b) $\frac{5}{4}$
   (c) $\frac{4}{3}$ (d) none of these

Passage 6

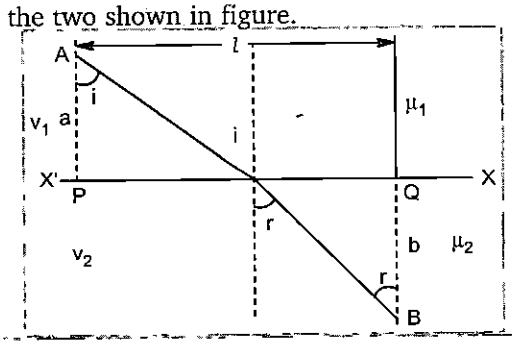
The figure shows a constant deviation prism $ABCD$. The incident ray is PQ and the emergent ray is ST. Although it is made-up of one piece of glass it is equivalent to two 30°–60°–90° prisms and one 45°–45°–90° prism. The angle θ_1 is the angle of incidence on face AB. The path of the ray inside the prism is indicated in the figure.
For this prism, $\mu = 2\sin\theta_1$.



1. The ratio of $\frac{\theta_1}{\theta_2}$ is :
   (a) 1 (b) $\frac{1}{2}$
   (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$
2. The total deviation of the incident ray when it emerges out of the prism is :
   (a) 90° (b) 60°
   (c) 30° (d) 45°
3. The angle of refraction on the face AB is :
   (a) 30° (b) 45°
   (c) 15° (d) 40°

Passage 7

A ray of light goes from point A in a medium where the speed of light is v_1 to a point B in a medium where the speed of light is v_2 as shown in the figure. The path of the rays in the two shown in figure.



1. The time taken for the light to go from the point A to the point B in the figure is :

(a) $\frac{a\sin i}{v_1}$ (b) $\frac{b\sin r}{v_2}$

(c) $\frac{v_2 a\sin i}{v_1 b\sin r}$ (d) $\frac{a\sec i}{v_1}+\frac{b\sec r}{v_2}$

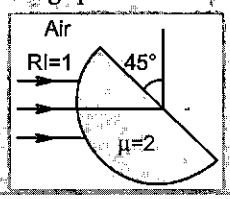
2. The slope of $(i-r)$ curve is :

(a) $\frac{-b\cos^2 i}{a\cos^2 r}$ (b) $\frac{a\cos r}{b\cos i}$

(c) $\frac{a\sin^2 r}{b\sin^2 i}$ (d) $\frac{-a\tan i}{b\tan r}$

Passage 8

A parallel beam of light is incident on a transparent hemisphere of RI 2 as shown in figure. O is the centre of hemisphere. The radius of the hemisphere is R. Give the answer of the following questions :



1. For the situation described above, mark out the correct statement (s) :

(a) The central ray of the beam suffers, total internal reflection at the plane surface of hemisphere.

(b) As we move up or down from central ray of beam, chances of TIR at plane surface is more

(c) As we move down from central ray of beam, chances of TIR at plane surface is more

(d) Both (a) and (b) are correct

2. Considering beam to be narrow, determine the location of image formed after refraction from curved surface of hemisphere :

(a) At $2R$ from P, real

(b) At $2R$ from P, virtual

(c) At R from P, real

(d) At R from P, virtual

3. Considering beam to be narrow, determine the location of the image formed after TIR from plane surface of hemisphere :

(a) Vertically below O, on the surface of hemisphere, virtual

(b) Vertically below O, at a distances of $2R$, virtual

(c) Vertically below O, on the surface of hemisphere real

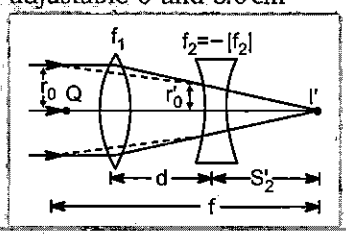
(d) Vertically below O, at a distance of $2R$ real

Passage 9

Figure shows a simple version of a zoom lens. The converging lens has focal length f_1, and the diverging lens has focal length $f_2 = -|f_2|$. The two lenses are separated by a variable distance d that is always less than f_1. Also, the magnitude of the focal length of the diverging lens satisfies the inequality $|f_2| > (f_1 - d)$. To determine the effective focal length of the combination lens, consider a bundle of parallel rays of radius r_0 entering the converging lens. It can be shown that the radius of the ray bundle decreases to $r_0' = \frac{r_0(f_1-d)}{f_1}$ at the point that it enters the diverging lens.

Final image I' is formed a distance $s_2' = \frac{|f_2|(f_1-d)}{(|f_2|-f_1+d)}$ to the right of the diverging lens. If the rays that emerge from the diverging lens and reach the final image point are extended backward to the left of the diverging lens, they will eventually expand to the original radius r_0 at same point Q. The distance from the final image I' to the point Q is the effective focal length f of the lens combination; if the combination were replaced by a single lens of focal length f placed at Q, parallel rays would still be brought to a focus at I'. The effective focal length is given by $f = \frac{f_1|f_2|}{(|f_2|-f_1+d)}$.

Assume that $f_1 = 12.0\,\text{cm}$, $f_2 = -18.0\,\text{cm}$, and the separation d is adjustable 0 and 3.0 cm



1. If the incident beam is parallel the image for the converging lens will act as an object for the diverging lens. This point will be :

(a) Real image for 1st lens and virtual object for 2nd lens

(b) Virtual image for 1st lens and virtual object for 2nd lens

(c) Real image for 1st lens and real object for 2nd lens

(d) Virtual image for 1st lens and real object for 2nd lens

2. Which of the following can be a value of the effective focal length of the zoom lens ?

(a) 18 cm (b) 12 cm

(c) 28 cm (d) 38 cm

3. The final image distance s_2' will be such that :

(a) $0 \le s_2' \le |f_2|$ (b) $|f_2| \le s_2' \le 2|f_2|$

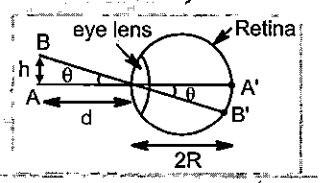
(c) $2|f_2| \le s_2' \le \infty$

(d) Any of the above three options

Passage 10

An object AB is seen by the eye lens. The length $A'B'$ of the image formed on the retina by the eye lens is dependent on 'θ' because radius of the eye is constant for a person during observation of an object $R \approx 1.25$ cm. An object of fixed height will appear to be of different heights as seen from different distances because of different values of 'θ'. Stars are of very large size but they appear to be very small because they are at very-very large distances 'θ' is called angle of vision or visual angle. For larger 'θ', the object will appear larger, θ depends on both 'h' and 'd'. If 'θ' is very small, then the object is not visible to us. To increase 'θ' the object should be kept nearest to the eye but then the object cannot be seen clearly because in that case the eye lens cannot adjust its focal length to make the image on the retina. The objects infront of the eye can have a minimum distance 'D' from the eye for their image to be formed on retina 'D' is called LEAST DISTANCE OF CLEAR VISION. $D \approx 25$ cm for average grown-up person. If an object is kept from distance 'D' to '∞' from a defect free eye, then its image can be formed on retina and it will be clearly visible. Sometimes an object kept in this range forms a very small angle and hence cannot be seen like dust particles, bacteria etc. To increase their 'θ' microscope (for small object near the eye) or telescope (for large objects but far away) is used. A simple microscope is a converging lens (for example lens used by watch repairer). The image formed by this lens acts as the object for the eye. Their performance is represented by a parameter called' angular magnification' or'magnifying power'. It is represented by 'm' $m = \dfrac{\theta}{\theta_{max}}$

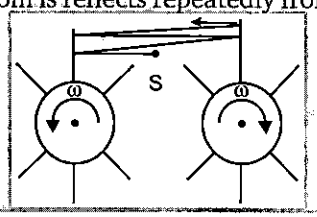
here θ = angle of vision of the image formed by the optical instrument (like microscope or telescope) θ_{max} = maximum angle of vision that an object can subtend on the eye when no optical instrument is used (for example if the object is kept at 'D' then 'θ' is maximum).



1. The minimum focal length of the eye lens is :
   (a) 1.2 cm (b) 2.5 cm
   (c) ∞ (d) 25/11 cm
2. If the eye is kept very close to a converging lens (focal length = 10 cm) and at the optical centre of the lens and an object is kept at distance 'd' then the maximum distance 'd' of the object from the lens so that its image can be seen clearly by the defect free eye is :
   (a) 10 cm (b) 25 cm
   (c) 50/3 cm (d) 50/7 cm
3. The value of angular magnification of this simple microscope (mentioned in the previous question) is :
   (a) 2.5 (b) 1.25
   (c) 5 (d) 3.5

Passage 11

The plane mirrors are placed symmetrically on disks revolving in different directions as shown. 'S' is the source and the beam from is reflects repeatedly from the mirrors.



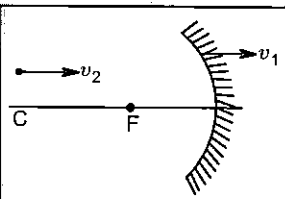
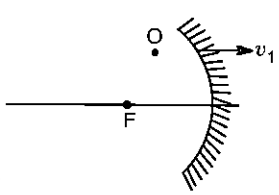
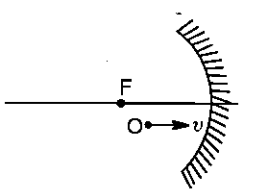
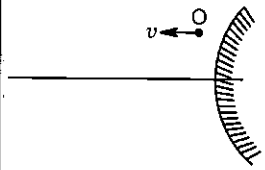
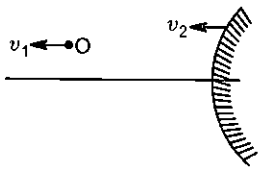
1. If the angular velocity of rotation of the disks is ω, then the angular velocity Ω of rotation of a beam that is consecutively reflected n times from the mirrors is :
   (a) $2n\omega$ (b) $(2n-1)\omega$
   (c) $(2n-1)2\omega$ (d) none of these
2. If the disks rotate in the same direction with angular velocity of rotation ω then the angular velocity Ω of rotation of a beam that is consecutively reflected n times will be :
   (a) $2n\omega$
   (b) $2n\omega$ if n is odd and if n is even
   (c) $(2n-1)2\omega$
   (d) 2ω if n is odd and 0 if n is even

MATCHING TYPE PROBLEMS

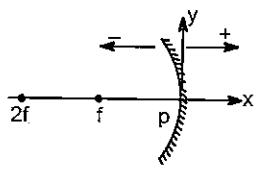
1. Column shows O_1-object and O_2-image. The optical system responsible can be a spherical mirror, plane mirror or a thin lens. In the case of the lens and spherical mirror, straight line shows the principal axis. Match column-I and column-II :

	Column-I		Column-II
(a)	O_1 O_2	(p)	Concave mirror between O_1 and O_2
(b)	O_2 O_1	(q)	Diverging lens between O_1 and O_2
(c)	O_2 O_1	(r)	Convex mirror between O_1 and O_2
(d)	O_2 O_1	(s)	converging lens between O_1 and O_2
		(t)	Inclined plane mirror somewhere between

2. In column-I possible instantaneous velocity vector of the image with respect to ground are shown. These are corresponding to the situations shown in column-II. Match column-I with the situations shown in column-II.

	Column-I		Column-II
(a)		(p)	
(b)		(q)	
(c)		(r)	
(d)		(s)	
		(t)	

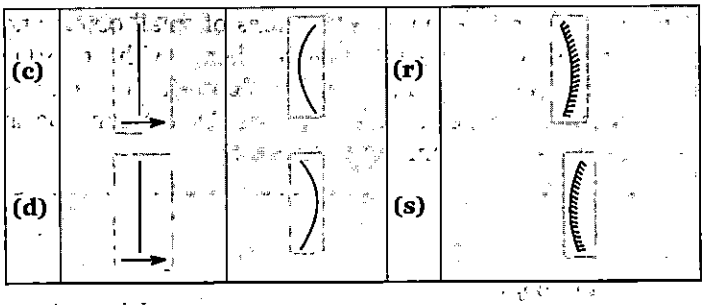
3. A small extended object can move infront of a fixed concave mirror considering pole as origin and sign convention, match column-I and column-II. Column-I shows position of object and its velocity v column-II shows characteristics of image.



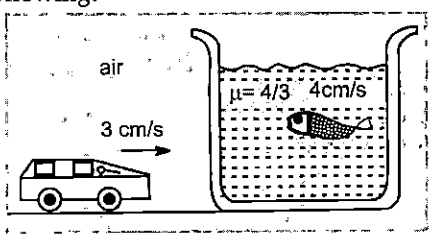
	Column-I		Column-II
(a)	Object between f and c approaching towards mirror ($v > 0$)	(p)	Velocity of image $v > 0$
(b)	Object between f and p and receding from mirror ($v < 0$)	(q)	Velocity of image $v < 0$
(c)	Object between c and ∞ and receding from mirror ($v < 0$)	(r)	Size of image increasing
(d)	Virtual object and approaching mirror ($v < 0$)	(s)	Size of image decreasing

4. Column-I shows incident wavefront and corresponding refracted or reflected wavefront. Arrow below incident wavefront shows direction. Refracted wavefront can be propagating right or left.

	Column-I Incident wave front	Refracted or reflected wave front		Column-II Optical Components
(a)			(p)	
(b)			(q)	



5. A child in a toy car is moving towards a cubical container with a velocity of 3 cm/s, whereas fish is moving towards child with a velocity of 4 cm/s as shown in the figure. The wall behind the fish is silvered (Refractive index of water is $\mu = 4/3$). Match the following.



	Column-I		Column-II
(a)	Velocity of child as observed by fish by one refraction only	(p)	Zero
(b)	Velocity of fish as observed by child by one refraction only	(q)	6 cm/s
(c)	Velocity of image of fish by reflection from mirror as observed by child	(r)	8 cm/s
(d)	Velocity of image of child by reflection from, mirror as observed by fish.	(s)	7 cm/s

6. Match the following

An object O is kept perpendicular to the principal axis of a spherical mirror. Each situation (a, b, c and d) gives object coordinate u in centimeters with sign, the type of mirror, and then the distance (centimeters, without sign) between the focal point and the pole of the mirror. On the right side information regarding the image is given. Correctly match the situations on the left side with the images described on the right side.

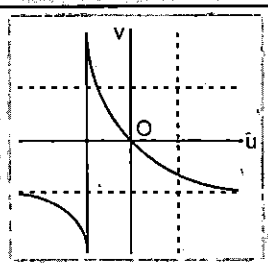
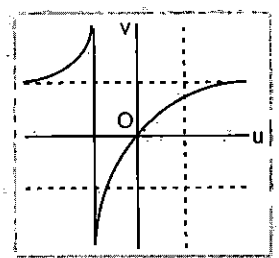
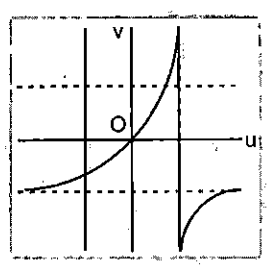
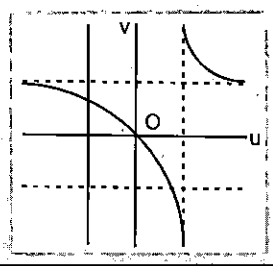
Column-I

Situation	u	Mirror
a	–18	Concave, 12
b	–12	Concave, 18
c	–8	Convex, 10
d	–10	Convex, 8

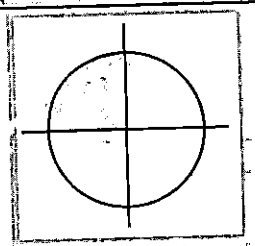
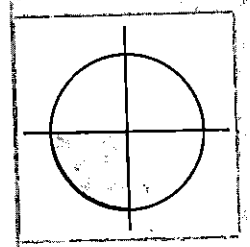
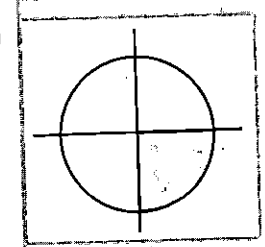
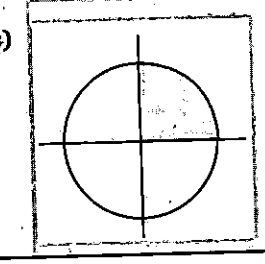
Column-II (Image)

(p) Real, Errect, Enlarged
(q) Virtual, Errect, Diminished
(r) Real Inverted, Enlarged
(s) Virtual, Erect, Enlarged

7. Match the following graphs (in column-II) between position of object (u) and position of image (v). Take positive sign in the direction of ray.

	Column-I		Column-II
(a)	Concave mirror	(p)	
(b)	Convex mirror	(q)	
(c)	Convex lens	(r)	
(d)	Concave lens	(s)	

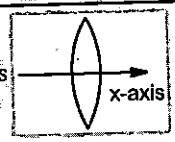
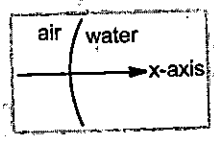
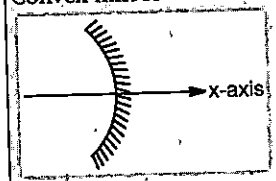
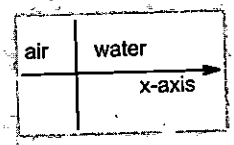
8. A very small circular object is kept infront of an optical device as shown in figure. The plane of object is parallel to the optical device. Match the images as seen by the observer (ignoring magnification).

	Column-I		Column-II
(a)	Plane mirror (The observer is at *A*)	(p)	
(b)	Concave mirror (The observer is at *A*)	(q)	
(c)	Convex mirror (The observer is at *A*)	(r)	
(d)	Convex lens (The observer is at *B*)	(s)	

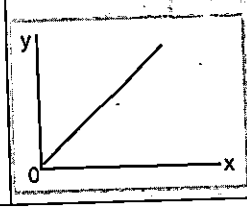
9. A metal plate containing an illuminator circular aperture is placed at one end of an optical bench and a screen, at the other end. By means of a convex lens the image of the aperture is formed on the screen, the diameter of the image being 4 cm. If the lens is displaced 17.5 cm along the optic bench, an image of the aperture again appears on the screen, the diameter of the image being 2.25 cm. Then match the following:

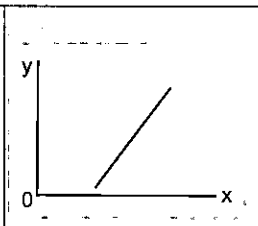
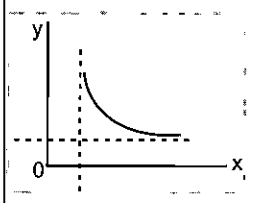
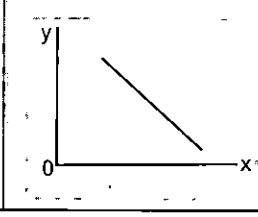
	Column-I		Column-II
(a)	Focal length of the lens in cm	(p)	122.5
(b)	Diameter of the aperture in cm	(q)	3.3
(c)	Distance between the aperture and the screen in cm	(r)	3
(d)	Power of lens in Dioptre	(s)	30

10. In column-I certain combinations of small object and image are marked. Match them with possible refracting/reflecting body is column-II. In all cases, object is perpendicular to *x*-axis. Object can't be at zero distance from the interface :

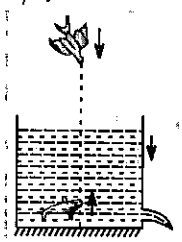
	Column-I		Column-II
(a)	Virtual image of virtual object	(p)	Converging lens 
(b)	Real image of real object	(q)	 Light is incident from air on the spherical surface
(c)	Erect real image of same size as object	(r)	Convex mirror 
(d)	Erect and virtual image size than the object	(s)	 Light is incident from right to left
		(t)	Diverging lens

11. The graphs given apply to a convex lens of focal length *f*, producing a real image at a distance *v* from the optical centre when self luminous object is at distance *u* from the optical centre. The magnitude of magnification is *m*. Identify the following graphs with the first named quantity being plotted along *y*-axis.

	Column-I		Column-II
(a)	*v* against *u*	(p)	

(b)	$\frac{1}{v}$ against $\frac{1}{u}$	(q)	
(c)	m against v	(r)	
(d)	$(m+1)$ against $\frac{v}{f}$	(s)	

12. A bird in air is diving vertically over a tank with speed 6 cm/s. Base of tank is silvered. A fish in the tank is rising upward along the same line with speed 4 cm/s. Water level is falling at rate of 2 cm/s.
(take: μ water = 4/3)



	Column-I		Column-II
(a)	Speed of the image of fish as seen by the bird directly	(p)	12
(b)	Speed of the image of fish formed after reflection from the mirror as seen by the bird	(q)	4
(c)	Speed of image of bird relative to the fish looking upwards	(r)	9
(d)	Speed of image of bird relative to the fish looking downwards in the mirror	(s)	3

ASSERTION AND REASON

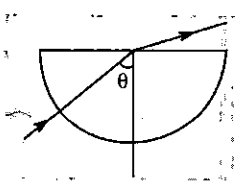
1. Direction: Each question contains statement **1 (assertion) and statement 2 (reason)**.
Each question has 4 choice (a),(b),(c) and (d) out of which only one is correct.
(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.
(c) Statement-1 is true, statement-2 is false.
(d) Statement-1 is false, statement-2 is true.

1. Statement-1: The brilliant colours that often appears on the throat and belly of a humping bird will vary with the viewing angle.
Statement-2: The indescence of that makes the brilliant colour is not due to pigment but is due to an interference effect caused by structures in the feathers.

2. Statement-1: A parallel beam of light travelling in air can be displaced laterally by a parallel transparent slab by distance more than the thickness of the plate.
Statement-2: The lateral displacement of light travelling in air increases with rise in value of refractive index of slab.

3. Statement-1: A solid glass sphere is placed in air. A light ray entres into the sphere from outside. The ray cannot undergo total internal reflection inside the sphere.
Statement-2: The angle of incidence at 1st surface cannot be greater than the critical angle for air-glass system.

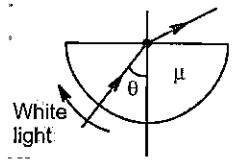
4. Statement-1: A beam of white light enters the curved surface of semicircular piece of glass along the normal. The incoming beams is moved clockwise (so that the angle θ increases), The color of the refracted beam is red before it steps energing from flat surface.



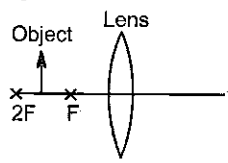
Statement-2: The index of refraction for light at the red end of the visible spectrum is lesser than at the violet end.

5. Statement-1: A convex lens suffers from chromatic aberration.
Statement-2: All parallel rays of monochromatic light passing through a convex lens do not come to a focus at the same point.

6. **Statement-1:** In a glass medium red light travels faster than blue light.
   Statement-2: Red light has a wavelength longer than blue.
7. **Statement-1:** The focal length of a lens depends on the material of the lens but the focal length of a mirror is independent of the material from which the mirror is made.
   Statement-2: Law of reflection is independent of material and surrounding medium where as law of refraction depends on the mediums.
8. **Statement-1:** Deviation of a ray of light produced by reflections from two inclined mirrors is independent of the angle of incidence.
   Statement-2: Deviation of a ray of light produced by reflection from one mirror is dependent of the angle of incidence.
9. **Statement-1:** Real images form on the side of a refracting surface that is opposite to the object and virtual images form on the same side as the object.
   Statement-2: Virtual images cannot be taken on screen.
10. **Statement-1:** Convex mirrors are used as rear mirrors in cars or scooters
    Statement-2: A convex mirror gives a wider field of view than a plane mirror.
11. **Statement 1:** The sun seems to rise before it actually rises and it appears to set after it actually sets.
    Statement 2: Near the horizon total internal reflection takes place.
12. **Statement 1:** The coating of optical fibers of a light pipe is of a material having refractive index more than that of the fiber, to optically insulate from each other.
    Statement 2: The optical fiber works on the principal of total internal reflection.
13. **Statement-1:** A bright metal ball is first blackened with soot in a candle flame and then immersed in water, it will again appear bright.
    Statement-2: At water-air surface total internal reflection can take plane.
14. **Statement-1:** An observer on the west facing beach of a large lake is observing the beginning of sunset. The water is very smooth except for some areas with small ripples. The observer notices that some area of the water are blue and the remaining is pink.
    Statement-2: Laws of reflection are valid only for smooth, polished surfaces.
15. **Statement-1:** A beam of white light is incident on a transparent glass hemisphere as shown in figure. The beam is rotated clockwise so that angle θ increases, as the refracted beam approaches a direction parallel to the horizontal, it appears red.



    Statement-2: Critical angle for a pair of medium aepends on RIS of the medium and is given by $i_C = \sin^{-1}\left(\frac{1}{R\mu_D}\right)$ and RI in turn depends upon wave-length of light.
16. **Statement-1:** The colour of green object seen through red glass appears to be dark.
    Statement-2: Red glass absorbs all other light except the red colored light.
17. **Statement-1:** Dry sand appears bright.
    Statement-2: Dry sand is smooth.
18. **Statement-1:** If there were no atmosphere, the duration of the day on the earth would decrease.
    Statement-2: Due to refraction in the atmosphere, the sun appears to rise earlier and sets later.
19. **Statement-1:** We can hear around corners, but we cannot see around corners.
    Statement-2: Wavelength of sound in much greater than wavelength of light.
20. **Statement-1:** In the shown situation if middle portion of the lens is painted black, then complete image of the object will form but of low intensity.



    Statement-2: More the reflected or refracted ray intersect, more would be the intensity of then corresponding image.

ANSWERS

Level-3: Comprehension Based Problems

Passage-1:	**1.** (a)	**2.** (b)	**3.** (a)	**Passage-2:**	**1.** (b)	**2.** (b)	**3.** (b)
Passage-3:	**1.** (a)	**2.** (b)	**3.** (b)	**Passage-4:**	**1.** (a)	**2.** (c)	**3.** (a)
Passage-5:	**1.** (b)	**2.** (b)		**Passage-6:**	**1.** (a)	**2.** (a)	**3.** (a)
Passage-7:	**1.** (d)	**2.** (a)		**Passage-8:**	**1.** (a)	**2.** (b)	**3.** (c)
Passage-9:	**1.** (b)	**2.** (a)	**3.** (d)	**Passage-10:**	**1.** (d)	**2.** (a)	**3.** (a)
Passage-11:	**1.** (c)	**2.** (b)					

Matching Type Problems

1. (a)-r, t; (b)-r, t; (c)-s, t; (d)-s, t
2. (a)-p, q; (b)-r, t; (c)-p, t; (d)-p, s, t
3. (a)-q, r; (b)-p, r; (c)-p, s; (d)-p, r
4. (a)-p, s; (b)-q, r; (c)-q, s; (d)-p ,r
5. (a)-r; (b)-q; (c)-p; (d)-p
6. (a)-r; (b)-s; (c)-q; (d)-q
7. (a)-p; (b)-s; (c)-q; (d)-r
8. (a)-p; (b)-p, q; (c)-p; (d)-q, s
9. (a)-s; (b)-r; (c) -p; (d) -q
10. (a)-r, t; (b)-p, q; (c)-s; (d)-p, q, t
11. (a)-r; (b)-s; (c)-q; (d)-p
12. (a)-r; (b)-s; (c)-p; (d)-q

Assertion and Reason

1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(a)	**5.**	(b)	**6.**	(a)	**7.**	(a)	**8.**	(b)
9.	(b)	**10.**	(a)	**11.**	(c)	**12.**	(d)	**13.**	(b)	**14.**	(b)	**15.**	(b)	**16.**	(b)
17.	(a)	**18.**	(a)	**19.**	(a)	**20.**	(a)								

Solutions

Level-3: Comprehension Based Problems

Passage-1

2. (b)

$r_0 = 0.6$

$r_i = 0.6 \times 2 \quad (m = 2)$

$= 1.2$

$A = \pi \times (1.2)^2 = 1.44\pi \text{ cm}^2$

3. (a)

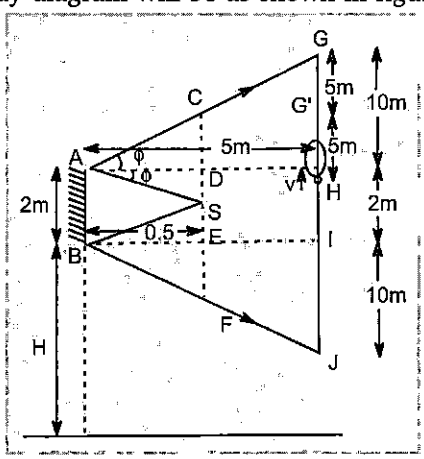
$r_i = 2(0.5 + 0.1t) = 1 + 0.2t$

$$\frac{dr_i}{dt} = 0.2$$

Passage-3

1. (a)

The ray diagram will be as shown in figure.



$HI = 2\text{m} = AB$

$DS = CD = 1\text{m}$

$AH = 5\text{m} = 10AD$

$GH = 10CD = 10 \times 1 = 10\text{m}$

$GJ = GH + HI + IJ = 10\text{m} + 2\text{m} + 10\text{m} = 22\text{m}.$

The greatest distance over which the man in the balloon can see the image of light source in the mirror S is 22m

Required time $= \frac{S}{v} = \frac{22}{10} = 2.2\text{s}$

2. (b)

Initial velocity given to man at the time of drop is 10 m/s upwards

The height through which he can rise,

$$h_1 = \frac{v^2}{2g} = \frac{10^2}{2 \times 10} = 5\text{m}$$

$\therefore$ The greatest distance over which he can see the image of source is $G'J = (5\text{m} + 12\text{m}) = 17\text{m}$

$$S = ut + \frac{1}{2}gt^2$$

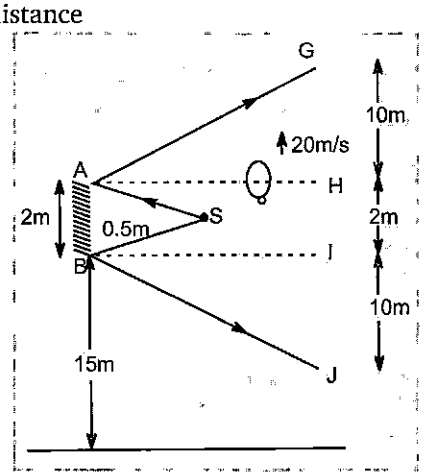
$$-12 = 10t - 5t^2$$

$$5t^2 - 10t - 12 = 0$$

$$t = \frac{+10 \pm \sqrt{100 + 240}}{10} = 2.84\text{s}$$

3. (b)

If the velocity of balloon is 20 m/s, the man can rise upto a distance



$$S_2 = \frac{u^2}{2g} = \frac{20^2}{2 \times 10} = 20\text{m}$$

But he can see the image only upto G during upward motion. Time taken from H to $G = t_1$

$$10 = 20t_1 - \frac{1}{2}gt_1^2$$

$t_1^2 - 4t_1 + 2 = 0$

$$t_1 = \frac{4 \pm \sqrt{16-8}}{2} = 0.586\,s$$

Then he can see the image from G to J during downward motion

Time taken from G to $H = t_2 = t_1 = 0.586\,s$

Then taken from H to $J = t_3$

$$12 = 20\,t_3 + \frac{1}{2} g t_3^2$$

$$5t_3^2 + 20t_3 - 12 = 0 \Rightarrow t_3 = \frac{-20 \pm \sqrt{400+240}}{10} = 0.53\,s$$

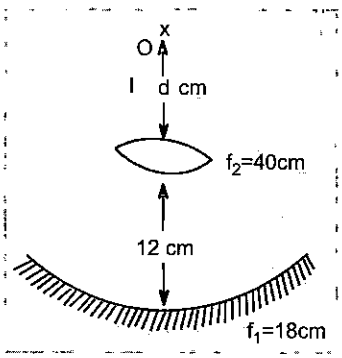
Total time taken

$$= t_1 + t_2 + t_3 = (0.586)\,2 + 0.53 = 1.7\,s$$

Passage-4

1. (a)

Since final image will coincide with the object if the ray retraces its path after reflection from the concave mirror *i.e.* the ray strike the mirror normally. The normal rays at the mirror after being extended must pass through the centre of curvature of the mirror for the refraction at the convex lens



$$u = (2f - 12) = 24\,cm$$

$$v = d$$

$$\frac{1}{2} = \frac{1}{24} = \frac{1}{40}$$

$$\frac{1}{d} = \frac{1}{40} = \frac{1}{24} = \frac{3+5}{120}$$

$$d = 15\,cm$$

Aliter

Let the object be placed at a distance d above the lens

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{d} = \frac{d-40}{40d}$$

$$v = \frac{40d}{d-40}$$

A real image is formed due to first refraction at the lens this image is an object for reflection from the convex mirror

Object distance $v_1 = \left(12 - \frac{40d}{(d-40)}\right) = \frac{(-28d-480)}{(d-40)}$

From mirror equation

$$\frac{1}{v_1} + \frac{(d-40)}{(28d+480)} = \frac{-1}{18}$$

$$\frac{1}{v_1} = \frac{-1}{18} - \frac{(d-40)}{28d+480} = \frac{(240-46d)}{18(28+480)}$$

$$v_1 = \frac{18(28d+480)}{(46d-240)}$$

The image is formed at a distance

$$= 12 - \frac{18(28d+480)}{(-240+46d)}$$

For the second refraction at the convex lens

$$= \frac{1}{d} - \frac{1}{-\left[12 - \frac{18(28d+480)}{(46d-240)}\right]} = \frac{1}{40}$$

Solving $d = 15\,cm$

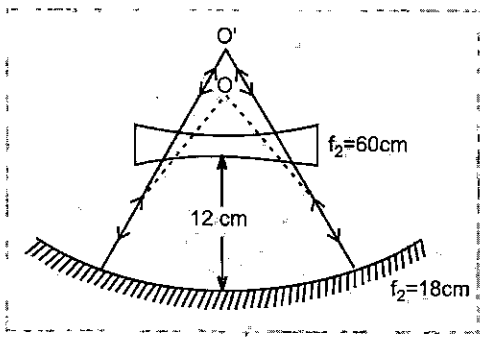
[**Note:** *First method is simple for calculation*]

2. (c)

The final image will coincide with the object if the ray retraces its path after reflection from the concave mirror.

$$u = -(36-12)\,cm + 24\,cm$$

$$v = d$$



$$f_2 = -60\,cm$$

$$\frac{1}{d} - \frac{1}{24} = -\frac{1}{60}$$

$$\frac{1}{d} = \frac{-1}{60} + \frac{1}{24}$$

$$= \frac{-2+5}{120} = \frac{3}{120}$$

$$d = 40\,cm$$

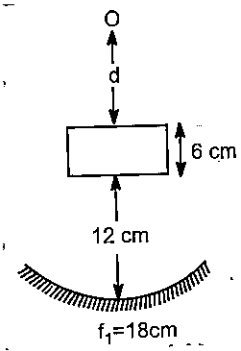
3. (a)

Shift produced by the glass plate

$$\Delta x = \left(1 - \frac{1}{\mu}\right) t = \left(1 - \frac{2}{3}\right) 6 = 2\,cm$$

for the mirror, the object is placed at a distance

$$[12 + 6 + d - \Delta x] = (16 + d)$$

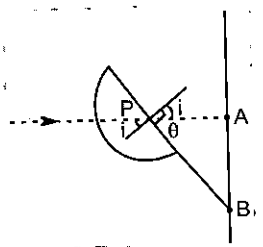


The image is coincident with the object itself if

$(16+d) = 36\,\text{cm} \Rightarrow d = 20\,\text{cm}$

Passage-5

2. (b)



$$PB = \sqrt{PA^2 + AB^2} = 150\,\text{cm}$$

$$\theta = 90° - i \angle 90°$$

independent of radius.

disapearing relates to angle.

$$r = i + \theta = 90°$$

$$\mu = \frac{\sin 90°}{\sin i} = \frac{1}{\cos\theta}$$

$$\mu = \frac{PB}{PA} = \frac{150}{120} = \frac{5}{4}$$

Passage-6

1. (a); 2. (a); 3. (a)

Let r be the angle of refraction at Q.

$$m = \frac{\sin\theta_1}{\sin r}, \sin\theta_1 = \frac{\mu}{2}$$

$$\sin r = \frac{1}{2}, r = 30°$$

Considering ABC as a triangular prism

$r + r' = A$

$30° + r' = 75° \Rightarrow r' = 45°$

Thus, the ray is incident on BCat 45°. It is reflected at the same angle on to AD.

From the figure $RSD = 360° - (45° + 135° + 60°) = 120°$ (consider the quadrilateral)

Angle of incidence at $AD = 120° - 90° = 30°$

$\theta_1 = \theta_2$

Total deviation of the ray $= (\theta_1 - 30°) + (180° - 90°) - (\theta_2 - 30°) = 90°$

(last one subtracted because it is opposite)

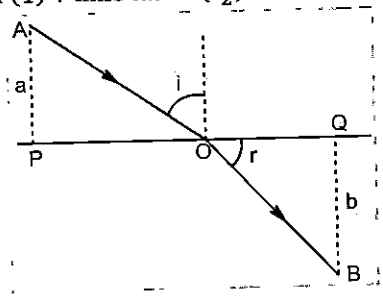
The deviation being 90° the incident ray and emergent ray are at right angles.

Passage-7

1. (d)

Let $X'X$ be the boundary separating the two media.

Total time taken = time taken (t_1) from A to O in medium (1) + time taken (t_2) from O to B in medium.



$$t_1 = \frac{AO}{v_1} = \frac{a\sec i}{v_1}$$

$$t_2 = \frac{BO}{v_2} = \frac{b\sec r}{v_2}$$

$$t = t_1 + t_2 = \frac{a\sec i}{v_1} + \frac{b\sec r}{v_2}$$

2. (a) To optimize time, $\frac{dt}{dr} = 0$

$$\Rightarrow \frac{d}{dr}\left(\frac{a\sec i}{v_1} + \frac{b\sec r}{v_2}\right) = 0$$

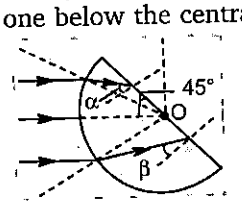
$$\Rightarrow \frac{a\sec i\tan i}{v_1}\frac{di}{dr} + \frac{b\sec r\tan r}{v_2} = 0$$

$$\Rightarrow \frac{di}{dr} = -\frac{b}{a}\frac{\cos^2 i}{\cos^2 r}$$

Passage-8

1. (a)

The ray diagram shows the refraction at curved surface for three rays, one for central ray, one above central ray at one below the central ray.



For glass-air pair, critical angle is, $i_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$$\Rightarrow i_C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

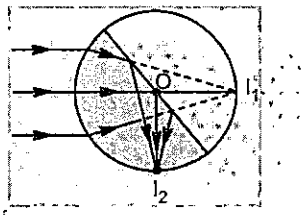
For central ray, angle of incidence is 45°, so TIR takes place. For ray above central ray, $\angle i > 45°$, so TIR takes place. For ray below central ray, $\angle i$ is decreasing *i.e.*, $< 45°$, and at one place may become less than critical

angle for glass-air pair and rays may refract at plane surface.

2. (b)

Here, we are draw the ray diagram for narrow parallel beam.

For refraction at 1st surface, using refraction at spherical surface :



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\Rightarrow \quad \frac{2}{v}-\frac{1}{\infty}=\frac{2-1}{R}$$

$\Rightarrow v = 2R$ *i.e.*, image will form at a distance of $2R$ from P on a straight line joining P and O-directly opposite to P. And the image I, would be virtual.

3. (c)

As the beam is narrow, all the rays get TIR from plane surface and form a real image at I_2 *i.e.*, on the surface of hemisphere and vertically below O.

Passage-9

2. (c)

$$f=\frac{f_1|f_2|}{|f_2|-f_1+d}=\frac{12\times 18}{18-12+d}=\frac{216}{6+d}$$

$24 \le f \le 36\,\text{cm}$

3. (d)

$$s_2'=\frac{|f_2|}{\left(\frac{|f_2|}{f_1-d}-1\right)}$$

Passage-10

1. (d)

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$v = 2.5\,\text{cm}$ in all cases

$u = -x$

$$\frac{1}{2.5}+\frac{1}{x}=\frac{1}{f}$$

$f_{max} \Rightarrow x$ is max. $\Rightarrow x = \infty$

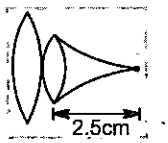
$f_{max} = 2.5\,\text{cm}$

$f_{min} \Rightarrow x$ is min

$$\frac{1}{2.5}+\frac{1}{25}=\frac{1}{f}$$

$$f_{min}=\frac{25}{11}\,\text{cm}$$

2. (a) $\Rightarrow \frac{1}{f_{eq}}=\frac{1}{v}-\frac{1}{u}=\frac{1}{2.5}-\frac{1}{-x}$



$x_{max} \Rightarrow f_{eq} = \text{max.} \Rightarrow$ eye is focussed as ∞

$$\frac{1}{f_e}+\frac{1}{f_{lens}}=\frac{1}{2.5}+\frac{1}{x}$$

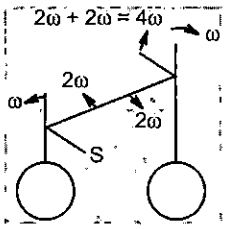
$$f_{lens} = 10\,\text{cm} \Rightarrow x = 10\,\text{cm}$$

3. (a)

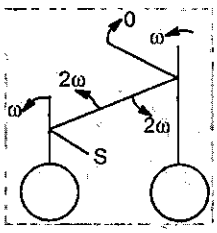
$$m=\frac{\theta}{\theta_{max}}=\frac{h/F}{h/D}=\frac{D}{F}=\frac{D}{10}=2.5\,\text{cm}$$

Passage-11

1. (c)



2. (b)



Matching Type Problems

2. $(v_{I/M})_x = -\left(\frac{v}{u}\right)^2 (v_{O/M})_x$

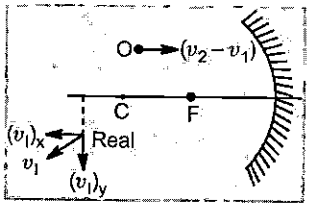
$$(v_I)_x = -\left(\frac{v}{u}\right)^2 ((v_O)_x - v_M) + v_M$$

So, if $(v_O)_x >> v_M$, then,
$(v_I)_x$ will be in backward direction
and if $(v_O)_x << v_M$ then,
$(v_I)_x$ will be in forward direction

In case of $(v_I)_y = \left(\frac{dm}{dt}\right) y_0 = \left[\frac{d}{dt}\left(\frac{-v}{u}\right)\right] y_0$

If object is approaching focal point (F) then, 'm' increases and $|y_I|$ will decrease.

(p) $v_1 < v_2$: In the frame of mirror

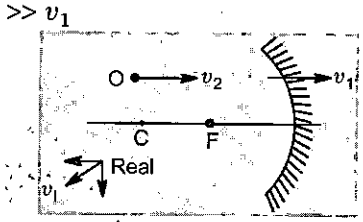


$$(v_I)_x = -\left(\frac{v}{u}\right)^2 (v_O)_x$$

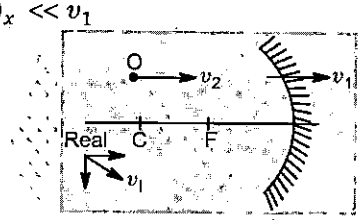
$$(v_I)_y = \left(\frac{dm}{dt}\right)(y_O)$$

In the frame of ground :

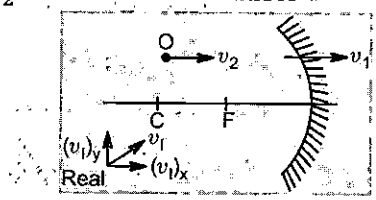
If $(v_I)_x >> v_1$



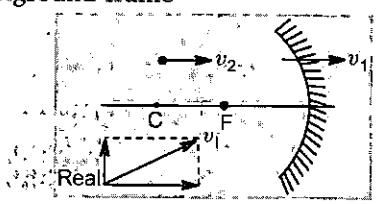
If $(v_I)_x << v_1$



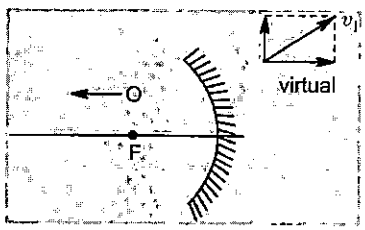
$v_I > v_2$: In the frame of mirror :



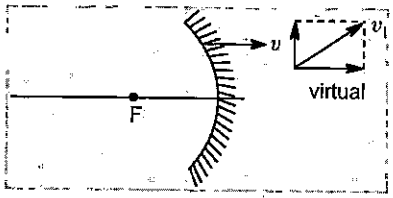
In the ground frame



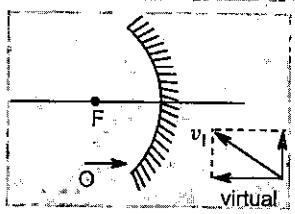
(q) In the frame of mirror



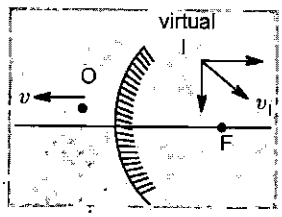
In the ground frame



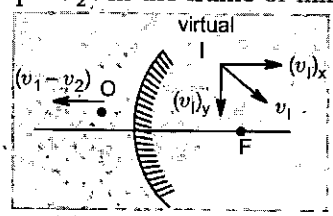
(r)



(s)

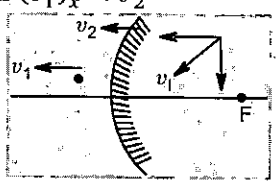


(t) $v_1 > v_2$: In the frame of mirror

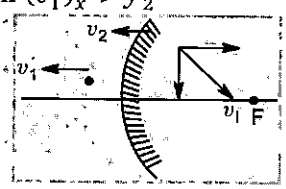


In the frame of ground

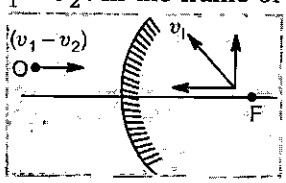
if $(v_I)_x < v_2$



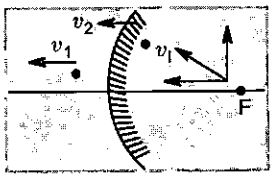
if $(v_I)_x > y_2$



$v_1 < v_2$: In the frame of mirror



In the ground frame



3. $\vec{v}_{I/M} = -m^2 \vec{v}_{O/M}$

9. Diameter of the aperture

$O = \sqrt{I_1 I_2} = \sqrt{4 \times 2.25} = 3\,\text{cm}$

Magnification, $m_1 = \frac{v_1}{u_1} = \frac{-4}{3} \Rightarrow v_1 = \frac{-4u_1}{3}$

$$\frac{1}{f} = -\frac{1}{u_1} - \frac{3}{4u_1} = \frac{-7}{4u_1} \Rightarrow f = \frac{-4u_1}{7}$$

$$u_1 = -\text{ve}, |v_1| = \frac{4u_1}{3}, f = \frac{4u_1}{7}$$

$$|v_1| - |u_1| = \frac{4u_1}{3} - u_1 = \frac{1}{3}u_1 = 17.5\,\text{cm}$$

$u_1 = -52.5\,\text{cm}$; $v_1 = 70\,\text{cm}$

Distance between the aperture and screen

$= |u_1| + |v_1| = 122.5\,\text{cm}$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

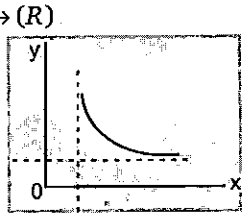
$$\frac{1}{70} - \frac{1}{-52.5} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{122.5}{70 \times 52.5} = 30\,\text{cm}$$

Focal length of the lens = 30 cm

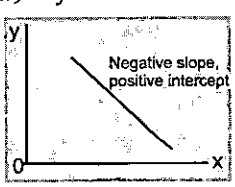
Power $= \frac{1}{f} = \frac{100}{30} = 3.3\,\text{D}$

11. (a)

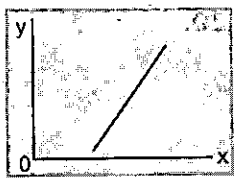
$v/u \longrightarrow (R)$



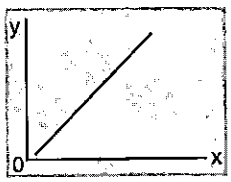
(b) $\frac{1}{v} = \frac{1}{(-u)} + \frac{1}{f}$



(c) $|m| = \left|\frac{f-v}{f}\right| = \left|1 - \frac{v}{f}\right| = \frac{v}{f} - 1$



(d) $|m| + 1 = \frac{v}{f}$



12. (a) Velocity of fish in air $= 4 \times \frac{3}{4} = 3\uparrow$

Velocity of fish w.r.t. bird $= 3 + 6 = 9\uparrow$

(b) Velocity of image of fish after reflection from mirror in air $= 4 \times \frac{3}{4} = 3\downarrow$

w.r.t. bird $= -3 + 6 = 3\uparrow$

(c) Velocity of bird in water $= 6 \times \frac{4}{3} = 8\downarrow$

w.r.t. fish $= 8 + 4 = 12\downarrow$

(d) Velocity of bird in water after reflection from mirror $= 8\uparrow$

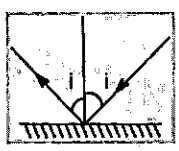
w.r.t. fish $= 8 - 4 = 4\uparrow$

Level-1: Assertion and Reason

2. (d) $t \sin i \left(1 - \frac{1}{\mu}\right) < t$

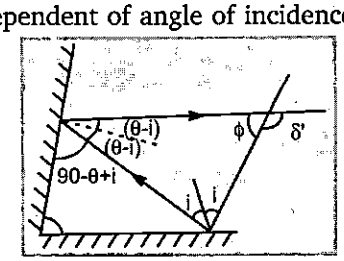
8. (b)

Due to reflection at single mirror, deviation produced $\delta = 180° - 2i$. Due to freflection's from two inclined plane mirrors, the deviation produced



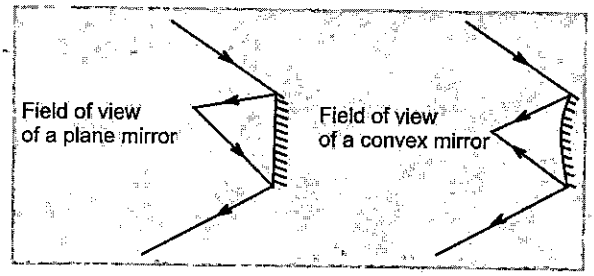
$\delta' = 180° - \phi = [180° - (180° - 2(\theta - i) - 2i] = 2\theta$

δ' is independent of angle of incidence.



10. (a)

Convex mirror gives a wider field of view than plane mirror



11. (c)

Due to refraction in the atmosphere the rays from the rising sun are bent and reach the eye of observer before it actually reaches. Horizon and exactly in the same way even after it has gone down the horizon rays from the setting sun continue to reach the observer.

12. (d)

The coating of optical fibers of a light pipe is of a material having refractive index less than that of the fiber, to optically insulate from each other. It work on the principle of total internal refraction.

13. (b)

Due to blackening of ball, some water droplets (in the form of thin film) would be there on ball's surface, as a result, light gets TIR from water-air interface and hence the ball appears bright. Statement-2 is also true but doesn't explain 1.

14. (b)

Here statement 2 is a standard concept and is true. Statement 1 is also true but not explained by Statement 2. The correct explanation for Statement I is : the different colours of water arise due to specular and diffuse reflections. The smooth areas of the water will reflect the light specularly from the sun, which is somewhat pink in colour at the beginning of sunset and hence water appears pink. While the light is reflected diffusively from areas where ripples are present, as the beginning of sunset sky is almost blue and hence reflected light from all parts of the sky make some part of the water to be appeared as blue.

15. (b)

As θ increases, incident angle approaches the critical angle. $\mu_V > \mu_R$ and as $i_C = \sin^{-1}\left(\frac{1}{\mu}\right), i_C$ for violet colour is least and for red colour maximum. So, violet colour gets TIR first, followed by other colours and red would be the last one to get TIR.

17. (a)

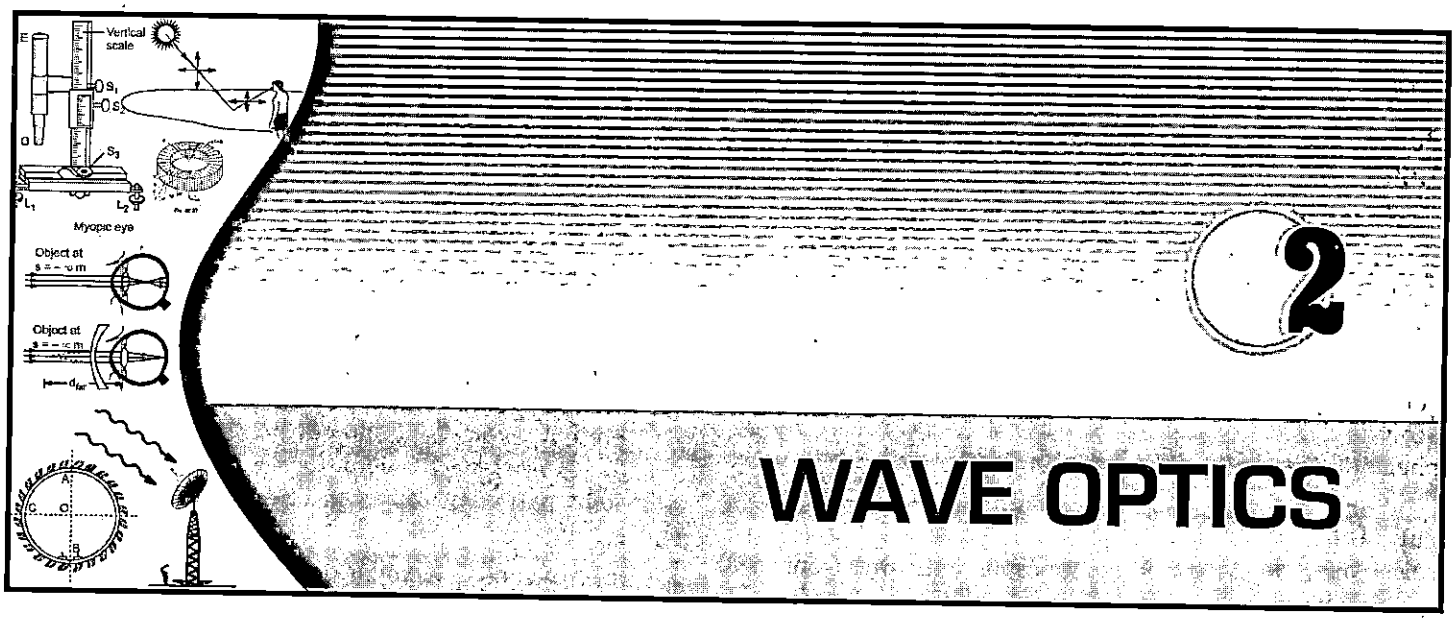
When light rays fall on dry sand, due to smooth surface, they are reflected back and hence dry sand appears bright.

19. (a)

Diffraction (bending of waves) occurs when obstacle size is comparable to wavelength.

20. (a)

Both statements are correct and statement 2 is correctly explains the Statement 1



2

WAVE OPTICS

INTERFERENCE OF LIGHT WAVES

Consider a ripple tank in which there are two needles oscillating in phase, each producing expanding circular wave- fronts of equal magnitude (Fig. 2.1). Recall that *a line or a surface connecting points of constant phase on a wave is called a* ***wave front.*** The solid lines represent crests and dashed lines represent troughs. We assume that the amplitude of the waves does not decrease with distance. When a crest from one source meets a trough from the other, the surface of water has zero displacement.

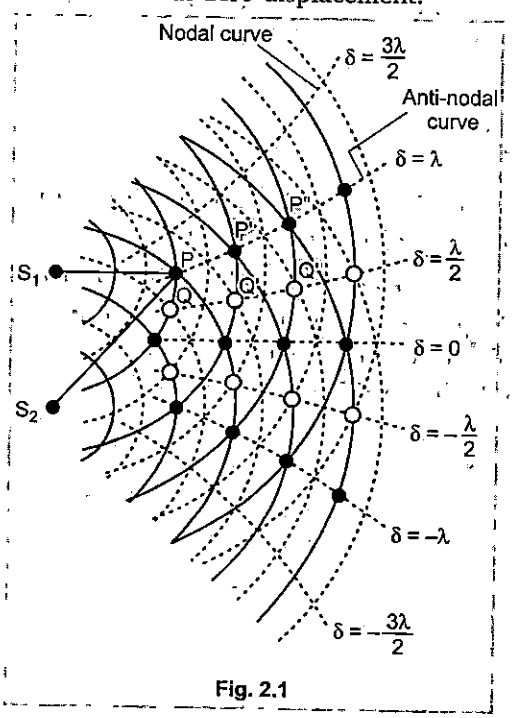


Fig. 2.1

At such point the permanent, **destructive interference** is indicated by open circles on the diagram. Similarly, where two crests or two troughs overlap, the oscillations produced have twice the amplitude. These points of permanently **constructive interference** are indicated by solid circles in the diagram.

The principle of superposition say that at points where a crest (trough) of one ripple system meets a crest (trough) of another ripple system the resultant (observed) amplitude of the water surface will be equal to the *sum* of the individual amplitudes. Constructive interference is said to occur because the ripple systems are *in phase* with each other at these points. Similarly, destructive interference occurs at points where a crest meets a trough, and the resultant amplitude is now the *difference* between the individual amplitudes.

Depicst wave crests by solid curves. Troughs, lying midway between the crests, are not shown. The antinodal curves are formed by joining up points of constructive interference and the nodal curves by joining up points of destructive interference.

Although the ripples travel outwards, the antinodal and nodal curves will stay in the same place so long as the 'dipping' frequency of the source is the same. *Stationary* antinodal and nodal curves can also be obtained if the needles are attached to *separate* bars provide they vibrate at the same frequency and there is a *constant* phase difference between them.

Conditions for Obtaining a Stationary Interference Pattern: Coherent Sources

The rules for achieving this condition are:

The frequency of the waves must remain constant.

There must be a constant phase difference between the waves leaving the sources.

Interference phenomena can be produced by waves generated by different kinds of sources: radio, microwave, acoustic and so on. Sources giving rise to these interference phenomena are said to be **coherent.**

Stationary interference patterns may be produced by any number of sources acting together, so long as the above rules are satisfied.

Interference using Ordinary Light Sources

The antinodal and nodal produced by two distinct ordinary light sources, such as two sodium discharge lamps, do not stay in the same place at all times, and a stationary interference pattern is not obtained. The light leaving the two sodium lamps is not coherent because it is impossible to correlate the light emission processes in the two sources.

The initial phase ϕ of a light wave can be related to the atomic processes occurring in a light source.

However, it is impossible to say exactly when an atom will radiate because the emission process is completely random. Fig. 2.2 pictorially describes the emission of light by a single atom in terms of wave packets. Other atoms in the source behave similarly, but with different emission times.

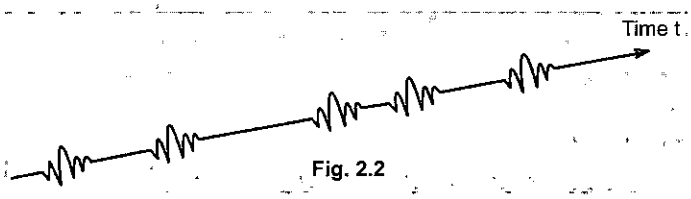


Fig. 2.2

Adding together the wave packets generated by all atoms in the light source produces a succession of wavetrains (see Fig. 2.3). Each wavetrain has a finite length. In passing from one wavetrain to the next there is an abrupt change in phase and plane of polarization. Thus, it is not possible to relate the phase at a point in wavetrain *B* to a point in wavetrain *A*.

There are, in fact, about 100 million breaks in phase every second, which means that each wavetrain lasts for about 10 ns, the so called **coherence time** t_c; the *longitudinal coherence length* l_c is equal to vt_c.

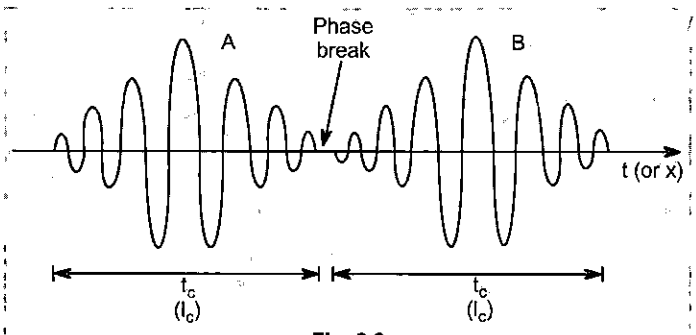


Fig. 2.3

Monochromatic light has a unique wavelength and is represented by a perfect sine wave. Laser light is an extremely good approximation to monochromatic light, possessing a coherence length of several kilometers.

Quasimonochromatic light can be thought of as a mixture of monochromatic waves of slightly different wavelength and can be represented by the wavetrain. Light from a sodium discharge lamp has a coherence length of about 2-3 cm. White light, on the other hand, is a mixture of monochromatic waves of widely different wavelengths which add to produce a light pulse: the wave are only in phase within a very narrow region where constructive interference occurs.

Light is a vector phenomenon; superposition the electric and magnetic fields are vector fields.

In accordance with the principle of superposition, the electric field intensity $\vec{\mathbf{E}}$, at a point in space, arising from the separate fields $\vec{\mathbf{E}}_1, \vec{\mathbf{E}}_2, \ldots$ of various contributing source is given by

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \ldots$$

The optical disturbance, or light field $\vec{\mathbf{E}}$, varies in time at an exceedingly rapid rate, roughly

$$4.3 \times 10^{14}\ \text{Hz} \text{ to } 7.5 \times 10^{14}\ \text{Hz}$$

making the actual field an impractical quantity to detect. On the other hand, the irradiance I can be measured directly with a wide variety of sensors (*e.g.*, photocells, bolometers, photographic emulsions, or eyes).

Consider two point sources, S_1 and S_2, emitting monochromatic waves of the same frequency in a homogeneous medium. Let their separation a be much greater than λ. Locate the point of observation P far enough away from the sources so that at P the wavefronts will be planes. For the moment, consider only linearly polarized waves of the form

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}, t) = \vec{\mathbf{E}}_{01} \cos(\vec{\mathbf{K}}_1 \cdot \vec{\mathbf{r}} - \omega t + \varepsilon_1)$$

and $$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}, t) = \vec{\mathbf{E}}_{02} \cos(\vec{\mathbf{K}}_2 \cdot \vec{\mathbf{r}} - \omega t + \varepsilon_2)$$

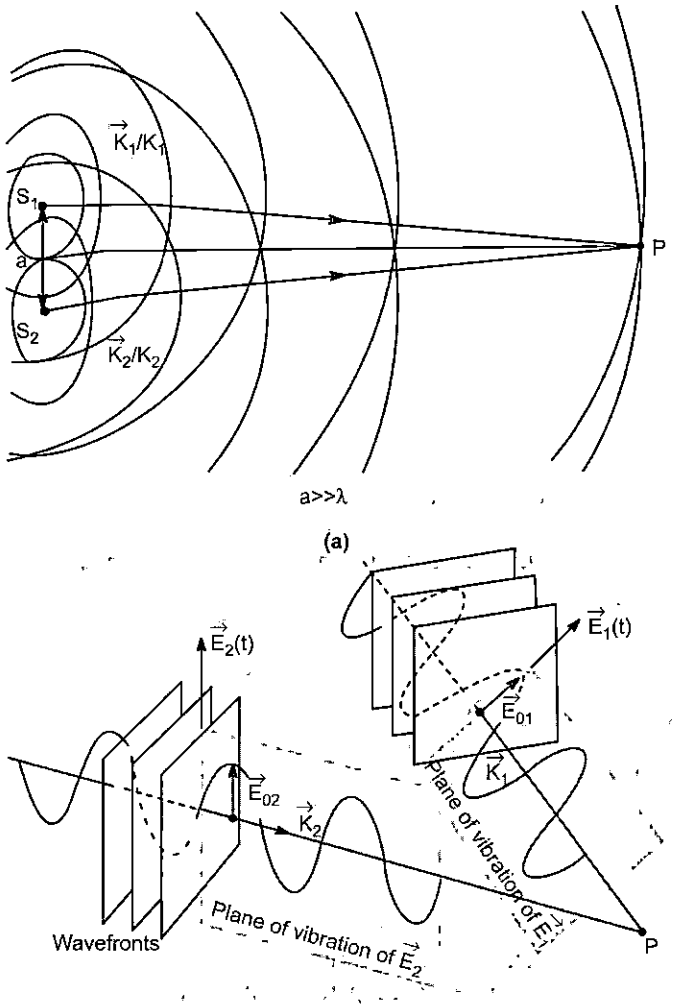


Fig. 2.4

Let us assume that two waves of the same frequency, being superposed on each other, produce oscillations of the same direction, namely,

$$A_1 \cos(\omega t + \alpha_1); \quad A_2 \cos(\omega t + \alpha_2)$$

at a certain point in space. The amplitude of the resultant oscillation at the given point is determined by the expression

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\delta$$

where $\delta = \alpha_2 - \alpha_1$

If the phase difference δ of the oscillations set up by the waves remains constant in time, then the waves are called coherent.

The phase difference δ for incoherent waves varies continuously and takes on any values with an equal probability. Hence, the time averaged value of cosδ equals zero. Therefore

$$|A^2| = |A_1^2| + |A_2^2|$$

We thus conclude that the **intensity observed upon the superposition of incoherent waves equals the sum of the intensities produced by each of the waves individually :**

$$I = I_1 + I_2 \qquad ...(1)$$

For coherent waves, cosδ has a time-constant value (but a different one for each point of space), so that

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \qquad ...(2)$$

At the points of space for which $\cos\delta > 0$, the intensity I will exceed $I_1 + I_2$; at the points for which $\cos\delta < 0$, it will be smaller than $I_1 + I_2$. Thus, the superposition of coherent light waves is attended by redistribution of the light flux in space. As a result, maxima of the intensity will appear at some spots and minima at others. This phenomenon is called the interference of waves. Interference manifests itself especially clearly when the intensity of both interfering waves is the same : $I_1 = I_2$. Hence, according to eq. (2), at the maxima $I = 4I_1$, while at the minima $I = 0$. For incoherent waves in the same condition, we get the same intensity $I = 2I_1$ everywhere [see eq. (1)].

It follows from what has been said above that when a surface is illuminated by several sources of light (for example, by two lamps), an interference pattern ought to be observed with a characteristic alternation of maxima and

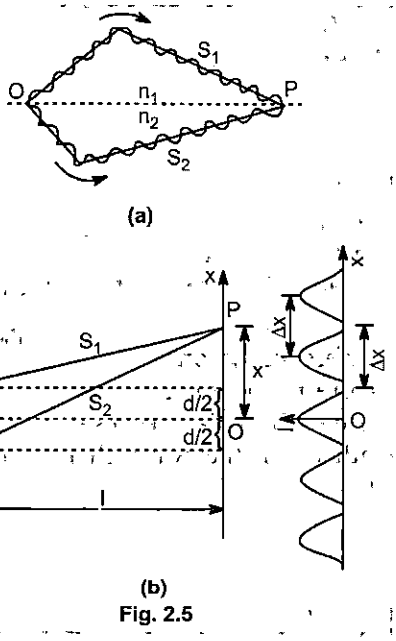


Fig. 2.5

minima of intensity. We know from our everyday experience, however, that in this case the illumination of the surface diminishes monotonously with an increasing distance from the light sources, and no interference pattern is observed. The explanation is that natural light sources are not coherent.

The incoherence of natural light sources is due to the fact that the radiation of a luminous body consists of the waves emitted by many atoms. The individual atoms emit wave trains with a duration of about 10^{-8} s and a length of about 3 m. The phase of a new train is not related in any way to that of the preceding one. In the light wave emitted by a body, the radiation of one group of atms after about 10^{-8} s is replaced by the radiation of another group, and the phase of the resultant wave undergoes random changes.

Coherent light waves can be obtained by splitting (by means of reflections or refractions) the wave emitted by a single source into two parts. It these waves are made to cover different optical paths and are then superposed onto each other, interference is observed. The difference between the optical paths covered by the interfering waves must not be very great because the oscillations being added must belong to the same resultant wave train. If this difference will be of the order of one metre, oscillations corresponding to different trains will be superposed, and the phase difference between them will continuously change in a chaotic way.

When two displacements are **in phase,** then the resultant amplitude will be the sum of the two amplitudes; this is **constructive interference.**

Similarly if $\theta = \pi, 3\pi, 5\pi, \ldots$, then

$$a = a_1 - a_2$$

When superposing waves are in **opposite phase** the resultant amplitude is the difference of the two amplitudes; this is **destructive interference.**

The difference in the distances from S_1 and S_2 to a given point is called the **path difference.**

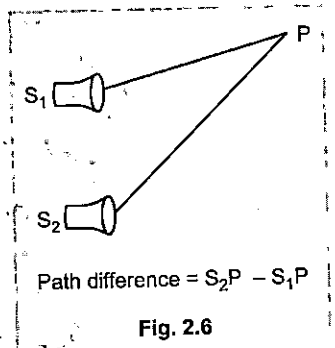


Fig. 2.6

A path difference of one wavelength corresponds to a phase difference of 2π radian because a shift in position of one wavelength along the wave changes its phase by a complete cycle.

For a path difference Δx the phase difference in radians is

$$\delta = \frac{2\pi}{\lambda}(\text{path difference}) = \frac{2\pi}{\lambda}(\Delta x)$$

The factor $\frac{\Delta x}{\lambda}$ is number of wavelengths corresponding to path difference. Each wavelength corresponds to a phase difference of π.

❒ The intensity of a wave is proportional to square of amplitude, therefore eqn. (8) reduces to

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

where $\delta = \frac{2\pi}{\lambda}(\Delta x)$

As the maximum and minimum values of $\cos\delta$ are +1 and –1 respectively. The maximum and minimum values of I are given by

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

The conditions for maximum intensity are

$$\delta = 2n\pi, \qquad n = 0, 1, 2, \ldots$$

or $\Delta x = n\lambda$

The conditions for minimum intensity are

$$\delta = (2n-1)\pi, \qquad n = 1, 2, 3, \ldots$$

or $\Delta x = (n - 1/2)\lambda, \qquad n = 1, 2, 3, \ldots$

When $I_1 = I_2 = I_0$, then

$$I = 4I_0 \cos^2 \delta/2$$

Clearly if $\delta = \pm\pi, \pm 3\pi, \ldots$, the resultant intensity will be zero and we will have a **minima.** When $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$, the intensity will be maximum ($I = 4I_0$).

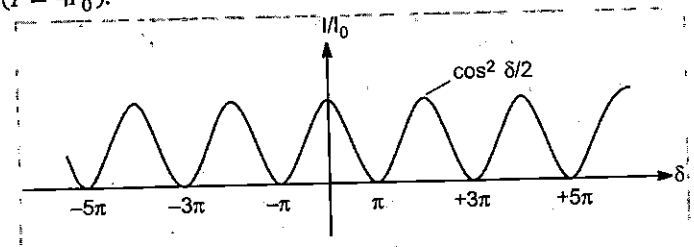


Fig. 2.7

❒ Fig. 2.8(a) shows the waves emitted by sources S_1 and S_2. The waves from the source start in phase and arrive in phase, leading to constructive interference at the point.

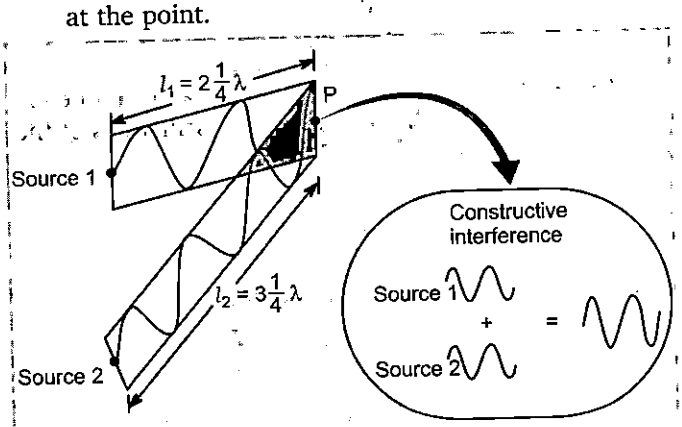


Fig. 2.8 (a)

Fig. 2.8 (b) shows two waves starting in phase but arriving in opposite phase.

In Fig. 2.8 (b), the distances travelled by waves differ by odd integer number of wavelengths.

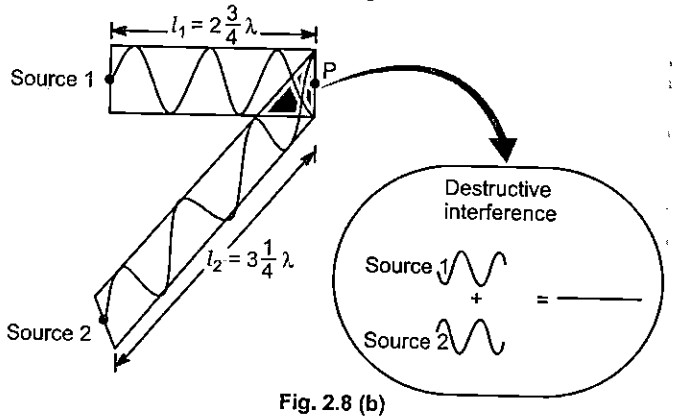


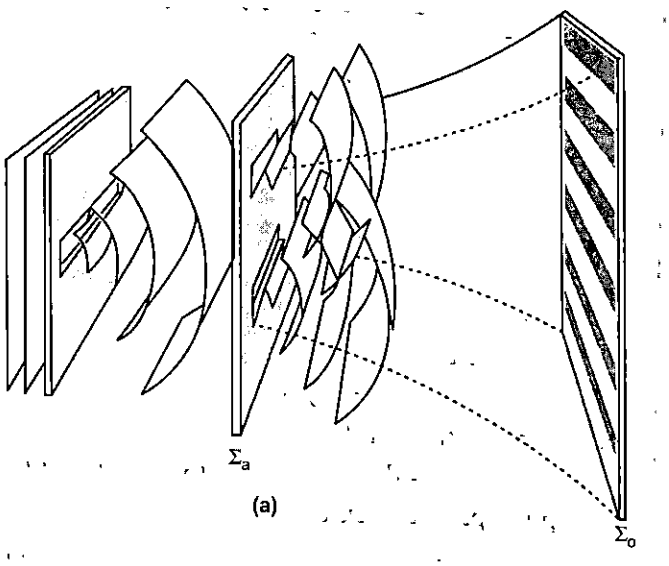
Fig. 2.8 (b)

In Fig 2.8 (a), the distances travelled by waves differ by any integer number of wavelengths :

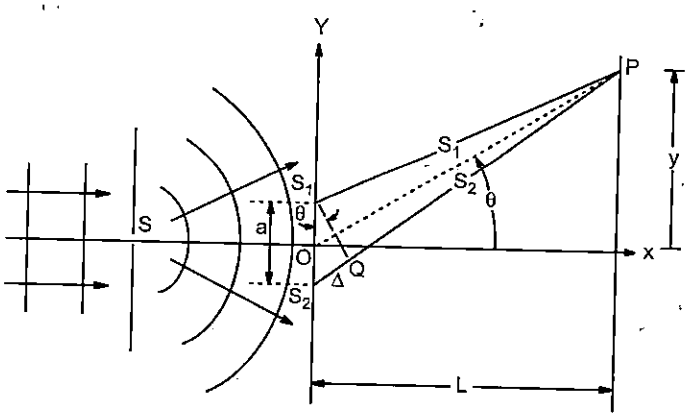
$$x_2 - x_1 = \lambda,\ 2\lambda,\ 3\lambda, \ldots, n\lambda$$

Young's Double Slit Experiment (YDSE)

The decisive experiment performed by Thomas Young in 1802 is shown schematically in Fig. 2.9. Monochromatic light is first allowed to pass through a single small hole in order to approximate a single point source S. The light spreads out in spherical waves from the source S according to Huygens' principle and is allowed to fall on a plane with two closely spaced holes, S_1 and S_2. In a modern version of this experiment, a laser is typically used to illuminate the two holes. In either case, the holes become two coherent sources of light, whose interference can be observed on a screen some distance away.



(a)



(b)

Fig. 2.9

The geometry of the experiment is simple. Parallel wavefronts of a monochromatic wave are incident on two identical narrow slits, each of width a separated by centre-to-centre distance d as shown in the figure. The slit widths and their separation are of the order of the wavelength of the incident monochromatic light. Monochromatic light after passing through two slits S_1 and S_2 acts as **coherent** sources of light waves that interfere constructively and destructively at different points on the screen to produce a pattern of alternating bright and dark fringes.

Fig. 2.10 (a) , (b) and (c) show two top views of the double slit and the screen. Fig. 2.10 (a) shows how a bright fringe arises directly opposite the midpoint between the slits. At this point the distances x_1 and x_2 to the slits are equal, each containing the same number of wavelengths. The waves arrive in same phase. Therefore constructive interference takes place, leading to a bright fringe. Fig. 2.10 (b) shows another bright fringe where x_2 is larger than x_1 by exactly one wavelength. The waves arrive in same phase. A bright fringe also occurs symmetrically on the other side of the midpoint where x_1 exceeds x_2 by one wavelength. Fig. 2.10 (c) shows the formation of a dark fringe. At this position x_2 exceeds x_1 by exactly half a wavelength; the waves arrive in opposite phase thereby interfering destructively.

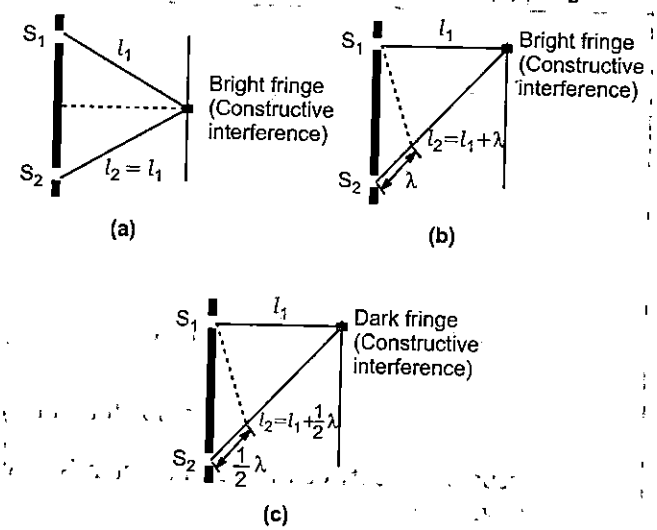


Fig. 2.10

Now we consider a point P on the distant screen, at a distance D from slits with $D >> d$. The small arc of the circle from P is almost a straight line. From the Fig. 2.11, path difference $\Delta x \simeq d \sin\theta$.

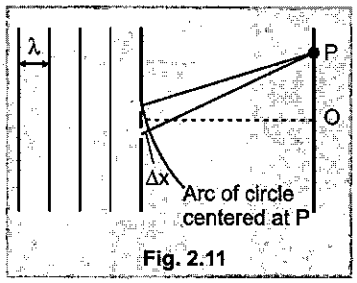


Fig. 2.11

The condition for constructive interference is

$\Delta x = S_2P - S_1P = \pm n\lambda$, where $n = 0, 1, 2, \ldots$

$d \sin\theta = \pm n\lambda$ [Condition for maxima]

The condition for destructive interference is

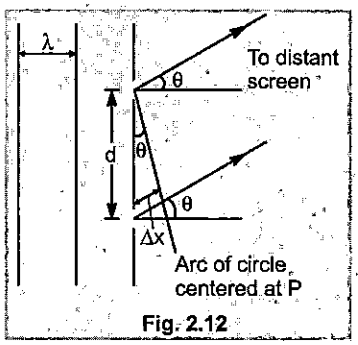


Fig. 2.12

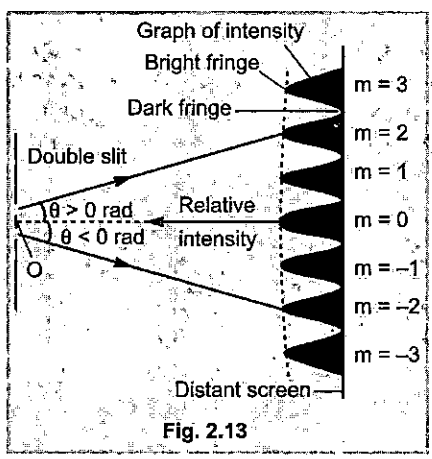


Fig. 2.13

$$\Delta x = S_2P - S_1P = \pm\left(n - \frac{1}{2}\right)\lambda, \text{ where } n = 1, 2, \ldots$$

$$d \sin\theta = P \pm \left(n - \frac{1}{2}\right)\lambda, \quad \text{[Condition for minima]}$$

❒ If the separation between screen and slits is large, $D >> d$, we have

$$\sin\theta \simeq \tan\theta \simeq \theta = \frac{y}{D}$$

where y is the vertical distance from the centre of the pattern. Position of nth bright fringes and dark fringes are respectively

$$\frac{y_n d}{D} = n\lambda \quad \text{or} \quad y_n = n\lambda\left(\frac{D}{d}\right)$$

and $$\frac{y_n d}{D} = \left(n - \frac{1}{2}\right)\lambda \quad \text{or} \quad y_n = \left(n - \frac{1}{2}\right)\lambda\frac{D}{d}$$

Each value of n corresponds to particular bright or dark fringe.

The absolute value of n is called the **order of interference.**

❒ The nth and $(n+1)$th maxima are

$$y_n = \frac{n\lambda D}{d} \quad \text{or} \quad y_{n+1} = \frac{(n+1)\lambda D}{d}$$

Separation between adjacent maxima is defined as **fringe width** β.

$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}$$

Similarly separation between minima is same. As long as d and θ are small, the separation between interference fringes is independent of n (order of fringe), *i. e.*, the fringes are evenly spaced.

As vertical distance y is related to θ by $\theta = y/D$, so $\Delta\theta = \Delta y/D$ referred to as **angular fringe width.**

$$\beta_\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

Concepts: ***1. Determination of Interference at Points Close to the Central Maximum:*** *Optical path difference corresponding any point P on the screen in a YDSE is d sin θ (Fig. 2.14).*

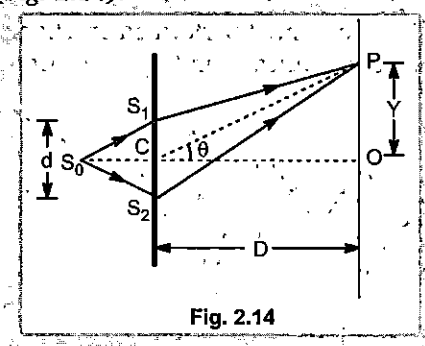


Fig. 2.14

If P is close to the central maximum, at a distance y from it (y of the order of a millimeter).

Then θ is small and hence,

$$\theta \approx \tan\theta = y/D.$$

However, $\delta l = d \sin\theta$

$\approx d\theta$. *[Since $\sin\theta \approx \theta$]*

∴ Optical path difference

$$\delta l = y\left(\frac{d}{D}\right). \quad \ldots(1)$$

2. Bright fringe : *For an n^{th} order bring fringe, the path difference δl should be $n\lambda$; $(n = 0, 1, 2, 3, \ldots)$.*

$$\therefore \quad y_n \frac{d}{D} = n\lambda$$

or $$y_n = n\left(\frac{\lambda D}{d}\right). \quad \ldots(2)$$

Putting $n = 0, 1, 2, 3,$ etc., we get

$y_0 = 0$,

$$y_1 = \frac{\lambda D}{d},$$

$$y_2 = \frac{2\lambda D}{d},$$

$$y_3 = \frac{3\lambda D}{d} \text{ and so on.}$$

3. Dark fringe : *For the nth ordered dark fringe, the path difference δl should be $(2n-1)\frac{\lambda}{2}$; $(n = 1, 2, 3, \ldots)$.*

$$\therefore \quad y_n \frac{d}{D} = (2n-1)\frac{\lambda}{2}$$

$$\text{or} \quad y_n = (2n-1)\frac{\lambda D}{2d}. \qquad \ldots(3)$$

Putting = 1, 2, 3, etc., we get

$$y_1 = \frac{\lambda D}{2d},$$

$$y_2 = \frac{3\lambda D}{2d},$$

$$y_3 = \frac{5\lambda D}{2d} \text{ and so on.}$$

4. *As long as y is too small compared to D (i.e., $\tan\theta \approx \theta$), the fringe separation is independent of the order and they are spaced evenly, with a bright fringe alternating the dark one and vice versa. The fringe width*

$$\beta = \frac{\lambda D}{d}. \qquad \ldots(4)$$

5. *The central bright fringe corresponds to the one which is a result of the interference of waves with no path length difference. As we move away from the central bright fringe, along the screen, the optical path difference increases, and when it becomes λ, the first bright fringe is encountered; similarly, when it becomes $2\lambda, 3\lambda$ etc., the second, third, etc., bright fringes are encountered. The maximum path length difference between the waves coming from the slits and interferring on the screen can be as large as d (when $y >> D, \theta \approx \pi/2$ and $\sin\theta \approx 1$).*

Thus, the total number of bright fringes on any side of the central maximum will be $\left[\frac{d}{\lambda}\right]$ where $\left[\frac{d}{\lambda}\right]$ stands for the greatest integer less than or equal to d/λ.

Hence, total number of bright fringes (in case of linear slits) will be

$$2\left[\frac{d}{\lambda}\right] + 1 \qquad \ldots(5)$$

6. *The positions and number of fringes, for a given YDSE set up depends upon wavelength, when a YDSE set up is completely submerged in a transparent liquid, the wavelength of light decrease and hence the positions and number of fringes get affected. For fringes close to the central maximum, the fringe width is given by,*

$$\beta = \frac{\lambda D}{d} \qquad \text{(in air)}$$

$$\text{and} \quad \beta' = \frac{\lambda' D}{d} \qquad \text{(in the liquid)}$$

$$\therefore \quad \beta' = \left(\frac{\lambda'}{\lambda}\right)\beta = \beta/\mu \qquad \ldots(6)$$

where, μ is the index of refraction of the liquid.

Angular fringe width (ω) : *It is defined as the angle subtended at the slits, by one complete fringe.*

Thus, $$\omega = \frac{\beta}{D} \qquad (\text{since } \beta << D)$$

$$= \frac{\lambda}{d} \qquad \left(\therefore \beta = \frac{\lambda D}{d}\right) \qquad \ldots(7)$$

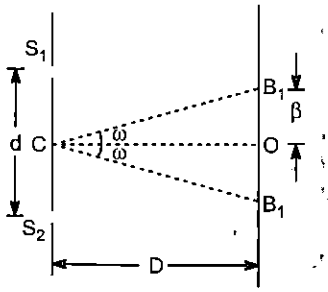


Fig. 2.15

$\omega \propto \lambda$, *hence, even the angular fringe width also decreases, when the YDSE set up is completely immersed in a liquid.*

i.e., $$\omega' = \omega/\mu$$

7. *Let us call the distance between two adjacent intensity maxima the* ***distance between interference fringes,*** *and the distance between adjacent intensity minima the* ***width of an interference fringe.*** *The distance between fringes and the width of a fringe have the same value equal to*

$$\beta = \frac{D\lambda}{d} \qquad \ldots(8)$$

According to eq. (8), the distance between the fringes grows with a decreasing distance d between the sources. If d were comparable with D, the distance between the fringes would be of the same order as λ, i.e. would be several scores of micrometers. In this case, the separate fringes would be absolutely indistinguishable. For an interference pattern to become distinct, the above-mentioned condition $d << D$ must be observed.

8. *If the intensity of the interfering waves is the same ($I_1 = I_2 = I_0$), then according to eq. $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$ the resultant intensity at the points for which the phase difference is δ is determined by the expression*

$$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

Since δ is proportional to Δ, then in accordance with δ grows proportionally to x. Hence, the intensity varies along the screen in accordance with the law of cosine square. The right-hand part of Fig. 2.16 shows the dependence of I on x obtained in monochromatic light.

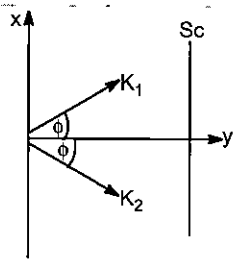


Fig. 2.16

The width of the interference fringes and their spacing depend on the wavelength λ. The maxima of all wavelengths will coincide only at the centre of a pattern when x = 0. With an increasing distance from the centre of the pattern, the maxima of different colours become displaced from one another more and more. The result is blurring of the interference pattern when it is observed in white light. The number of distinguishable interference fringes appreciably grows in monochromatic light.

9. *The separate sections of the light source produce waves whose phases are in no way related to one another. For this reason, the interference pattern appearing on the screen will be a superposition of the patterns produced by each section separately.*

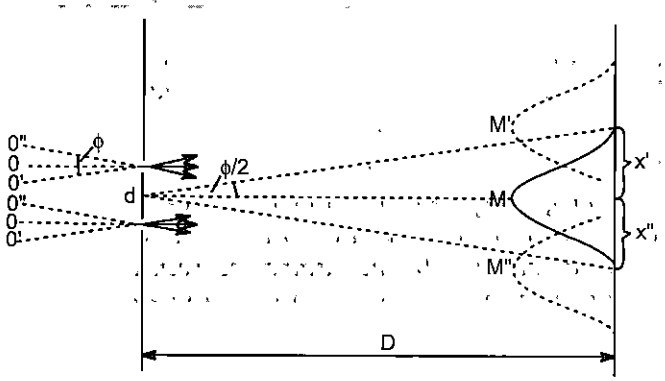


Fig. 2.17

If the displacement x' is much smaller than the width of an interference fringe Δx = Dλ/d [see Eq. (1)], then the maxima from different sections of the source will practically be superposed on one another, and the pattern will be like one produced by a point source.

When x' ≈ Δx, the maxima from some sections will coincide with the minima from others, and no interference pattern will be observed. Thus, an interference pattern will be distinguishable provided that x' < Δx, i.e.,

$$\frac{D\phi}{2} < \frac{D\lambda}{d} \qquad ...(9)$$

or

$$\phi < \frac{\lambda}{d} \qquad ...(10)$$

We have omitted the factor 2 when passing over from expression (9) to (10).

Formula (10) determines the angular dimensions of a source at which interference is observed. We can also use this formula to find the greatest distance between the slits at which interference from a source with the angular dimension ϕ can still be observed. Multiplying inequality (4) by d/ϕ, we arrive at the condition

$$d < \frac{\lambda}{\phi} \qquad ...(11)$$

10. Shape of fringe: *For the spherical waves emitted by S_1 and S_2. Such waves can be expressed as*

$$\vec{\mathbf{E}}_1(r_1,t) = \vec{\mathbf{E}}_{01}(r_1)\exp[i(kr_1 - \omega t + \delta_1)]$$

and

$$\vec{\mathbf{E}}_2(r_2,t) = \vec{\mathbf{E}}_{02}(r_2)\exp[i(kr_2 - \omega t + \delta_2)]$$

The terms r_1 and r_2 are the radii of the spherical wavefronts overlapping at P; they specify the distances from the sources to P. In this case

$$\Delta\phi = k(r_1 - r_2) + (\delta_1 + \delta_2)$$

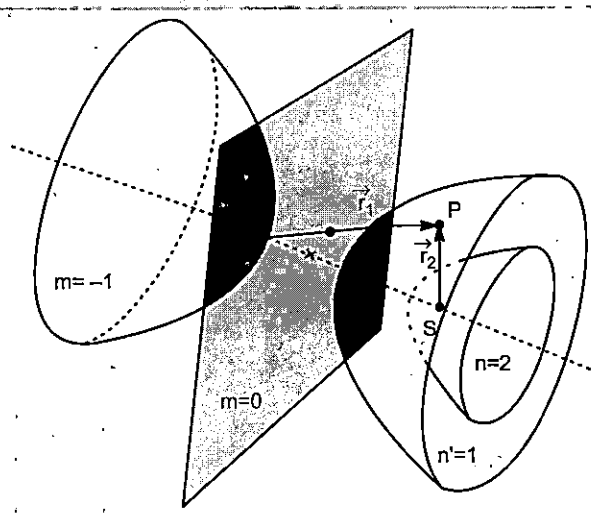
$$I = 4I_0\cos^2\frac{1}{2}[k(r_1 - r_2) + (\delta_1 - \delta_2)]$$

Maximum irradiance occurs when

$$(r_1 - r_2) = [2\pi n + (\delta_2 - \delta_1)]/k \qquad ...(12)$$

and minimum when

$$(r_1 - r_2) = [\pi n' + (\delta_2 - \delta_1)]/k \qquad ...(13)$$



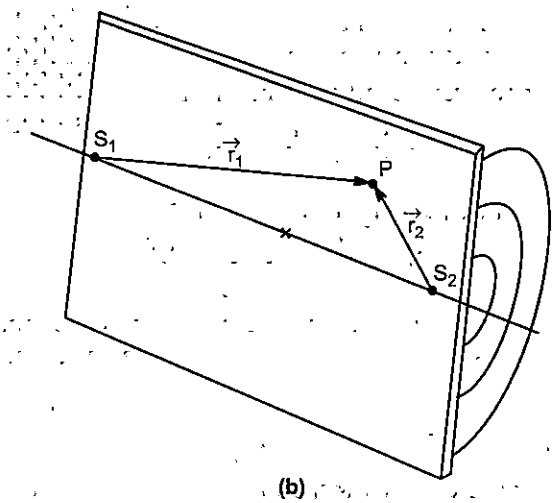


Fig. 2.18

Either one of these equations defines a family of surfaces, each of which is a hyperboloid of revolution. The vertices of the hyperboloids are separated by distance equal to the right hand sides of Eqns. (12) and (13). The foci are located at S_1 and S_2. If the waves are in phase at the emitter, $\delta_1 - \delta_2 = 0$, and Eqns. (12) and (13) can be simplified to,

$$[\text{maxima}] \quad (r_1 - r_2) = 2\pi n/k = m\lambda \qquad ...(14)$$

$$[\text{minima}] \quad (r_1 - r_2) = \pi n'/k = \frac{1}{2} n'\lambda \qquad ...(15)$$

for maximum and minimum irradiance, respectively. Fig. 2.18 (a) shows a few of the surfaces over which there are irradiance maxima. the dark and light zones that would be seen on a screen placed in the region of interference are known as ***interference fringes*** *Fig. 2.18 (b).*

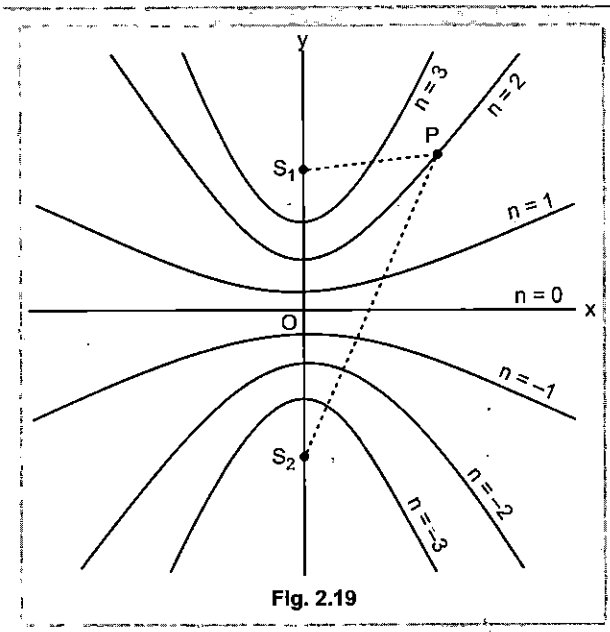


Fig. 2.19

11. *The surfaces are hyperbolic. Inasmuch as the y-axis is an axis of symmetry, the corresponding bright fringe surfaces are generated by rotating the entire pattern about the y-axis. One should then be able to visualize the intercept of these surfaces with the plane of an observational screen placed anywhere in the vicinity. In particular, a screen placed perpendicular to the ox axis, as in figure, intercepts hyperbolic arcs that appear as straight-line fringes near the axis, whereas a screen placed perpendicular to the oy axis shows concentric circular fringes centeted on the axis. Because the fringe system extends throughout the space surrounding the two sources, the fringes are said to be* ***nonlocalised.***

12. *The holes S, S_1, and S_2 of YDSE ar usually replaced by parallel, narrow slits to illuminate more fully the interference pattern. The effect of the array of point sources along the slits, each set producing its own fringe system as just described, is simply to elongate the pattern parallel to the fringes, without changing their geometrical relationships. This is true even when two points along a source slit are not mutually coherent.*

13. The effects of finite coherence length: *As P on screen is taken farther above or below the axis, path difference (which is less than or equal to S_1S_2) increases. If the primary source has a short coherence length, as the optical path difference increases, identically paired wave groups will no longer be able to arrive at P exactly together. There will be an increasing amount of overlap in portions of randomly phased wave groups, and the contrast of the fringes will decreases. It is possible for Δl_c to be less than path difference. In that case, instead of two same phase portions of the same wave groups arriving at P, only segment of different wave groups will overlap, and the fringes will vanish.*

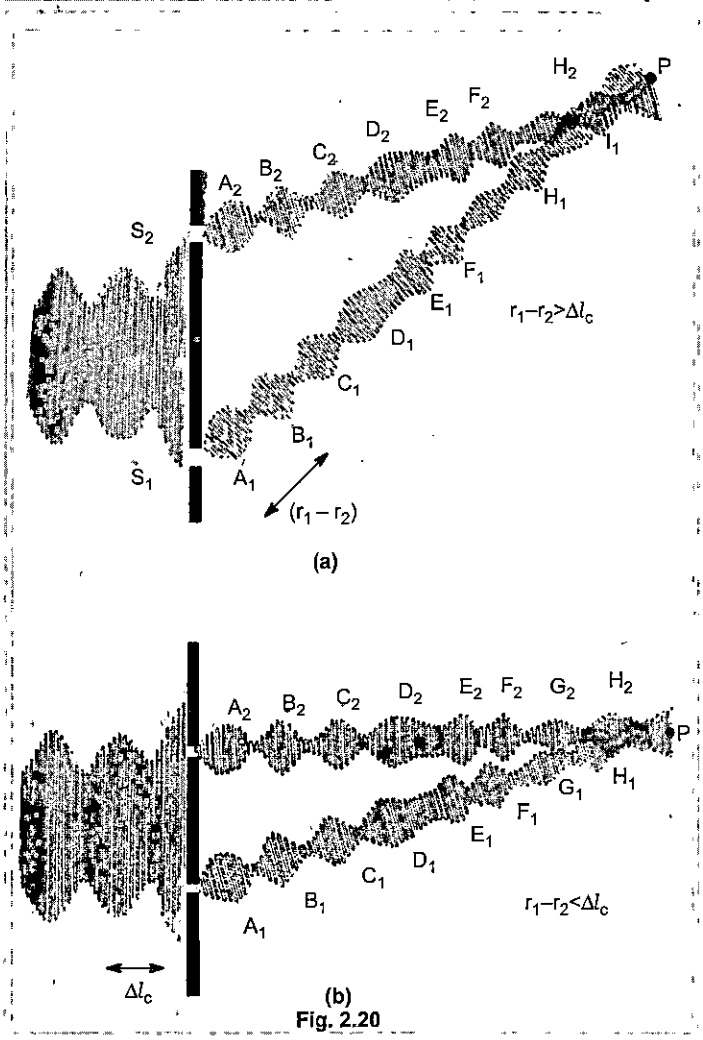


Fig. 2.20

Equivalent Optical Path Length

When a beam of light travels from one medium to another its speed changes but its frequency does not. Designating the

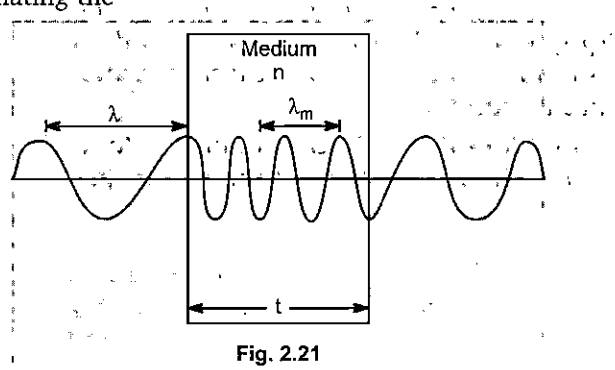


Fig. 2.21

wavelength in vacuum by λ and the wavelength in the material by λ_m, we have

$$\frac{\lambda}{\lambda_m} = \frac{C}{v} = n \quad \text{or} \quad \lambda_m = \frac{\lambda}{n}$$

The wavelength is shorter in the medium than in vacuum. If the light beam passes through a thickness t of a medium,

$$\text{Number of wavelengths in plate} = \frac{t}{\lambda_m} = \frac{t}{\lambda/n} = \frac{nt}{\lambda}$$

which shows that a thickness t of the medium has as many wavelengths as there are in a length μt of vacuum. *Therefore in terms of wavelengths, a thickness t in a medium of refractive index n is equivalent to a path length μt in vacuum.* The quantity nt is called **equivalent optical path length.**

❒ In Fig. 2.22 shown two light rays of identical wavelength and initially in phase in air travel through two different media of refractive indices n_1 and n_2, same thickness t. The wavelengths of the waves will be different in the two media; so the two waves will no longer be in phase, when they emerge.

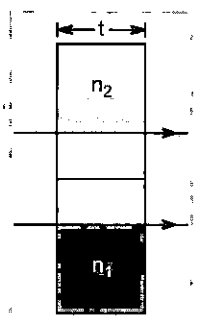


Fig. 2.22

Number of wavelengths in medium 1,

$$N_1 = \frac{t_1}{\lambda_1} = \frac{t}{\lambda/n_1} = \frac{n_1 t}{\lambda}$$

Number of wavelengths in medium 2,

$$N_2 = \frac{t}{\lambda_2} = \frac{t}{\lambda/n_2} = \frac{n_2 t}{\lambda}$$

To find a new phase difference we subtract the number of wavelengths of the waves in the two media; assuming $n_2 > n_1$, we have

$$N_2 - N_1 = (n_2 - n_1)\frac{t}{\lambda}$$

Phase difference corresponding to difference of one wavelength is 2π; hence phase difference corresponding to $N_2 - N_1$ is

$$\Delta\phi = \frac{2\pi}{\lambda}(n_2 - n_1)t$$

In other words we can say that the equivalent optical paths of wave in media 1 and 2 are $n_2 t$ and $n_1 t$ respectively. Thus the path difference, $\Delta x = n_2 t - n_1 t$.

Displacement of Fringes

When we introduce a thin transparent plate in front of one of the slits in YDSE, the fringe pattern shifts toward the side the plate is present.

The path length S_1P is

$$S_1P = (S_1P - t)_{\text{air}} + t_{\text{plate}}$$

The time required for the light to reach from S_1 to the point P is

$$\frac{S_1P - t}{c} + \frac{t}{v} = \frac{1}{c}[S_1P - t + nt]$$

$$= \frac{1}{c}\{S_1P + (n-1)t]$$

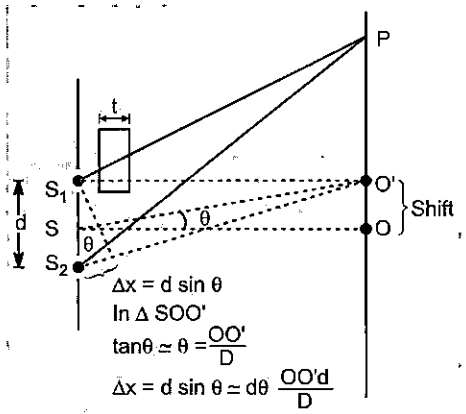


Fig. 2.23

which shows that by introduction of the plate the effective path length increases by $(n-1)t$. The position of the central bright fringe which corresponds to equal optical path length is shifted to O', where

$$S_1O' + (n-1)t = S_2O'$$

$$S_2O' - S_1O' = (n-1)t$$

From $\Delta SOO'$,

$$\tan\theta \simeq \theta = \frac{OO'}{D}$$

Also $$S_2O' - S_1O' = \Delta x = d\sin\theta \simeq d\theta = \frac{(OO')d}{D}$$

Therefore $$\frac{(OO')d}{D} = (n-1)t$$

and $$OO' = \frac{D(n-1)t}{d}$$

Example 1

Two coherent sources of light of intensity ratio β produce interference pattern. Prove that in the interference pattern

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\beta}}{1+\beta}$$

where I_{max} and I_{min} are maximum and minimum intensities in the resultant wave.

Solution : If a_1 and a_2 are amplitudes of superposing waves and I_1, I_2 are intensities,

$$\beta = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2},\ \frac{a_1}{a_2} = \sqrt{\beta}$$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a_1+a_2)^2 - (a_1-a_2)^2}{(a_1+a_2)^2 + (a_1-a_2)^2}$$

$$= \frac{4a_1a_2}{2(a_1^2 + a_2^2)}$$

$$= \frac{2\left(\frac{a_1}{a_2}\right)}{\left(\frac{a_1^2}{a_2^2} + 1\right)} = \frac{2\sqrt{\beta}}{(1+\beta)}$$

Example 2

A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in Fig. 2E.2. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25% of the light incident on it and transmits the remaining. Find the ratio of minimum and maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate.

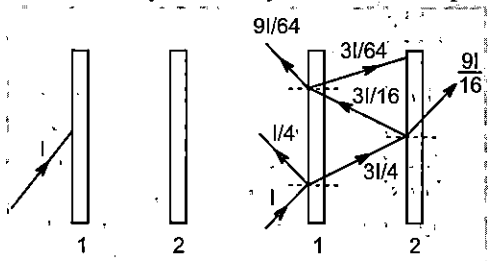


Fig. 2E.2

Solution : At each surface reflection and transmission takes place. Intensity of reflected and transmitted rays at first plate is $I/4$ and $3I/4$ respectively. Incident intensity at second plate is $3I/4$; 25% of it is reflected and 75% is transmitted. Intensity of reflected and transmitted rays at second plate is $\frac{3I}{16}, \frac{9I}{16}$ respectively. The reflected rays from second plate reaching first plate are incident at first plate.

The reflected and transmitted intensities are $3I/64$ and $9I/64$.

As $$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = I_0\left[\frac{1}{2} + \frac{3}{8}\right]^2 = \left(\frac{7}{8}\right)^2 I_0$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = I_0\left[\frac{1}{2} - \frac{3}{8}\right]^2 = \left(\frac{1}{8}\right)^2 I_0$$

Hence $$\frac{I_{max}}{I_{min}} = \left(\frac{1}{8}\right)^2\left(\frac{8}{7}\right)^2 = \frac{1}{49}$$

Example 3

Find the maximum intensity in case of interference of n identical waves each of intensity I_0 if the interference is (a) coherent and (b) incoherent.

Solution : The resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(a) The sources are said to be coherent if they have constant phase difference between them. The intensity will be maximum, when $\phi = 2n\pi$; the sources are in same phase.

Thus $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

Similarly, for n identical waves,

$$I_{max} = (\sqrt{I_0} + \sqrt{I_0} + \ldots)^2 = n^2 I_0$$

(b) The incoherent sources have phase difference that varies randomly with time

Thus $[\cos\phi]_{av} = 0$

Hence $I = I_1 + I_2$

Hence for n identical waves,

$$I = I_0 + I_0 + \ldots = nI_0$$

Example 4

Suppose that one of the slits in a Young's experiment is wider than the other; so the amplitude of the light reaching the central part of the screen from one slit, acting alone, is twice that from the other slit, acting alone. Obtain an expression for the light intensity I at the screen as a function of θ, the angular position.

Solution : The expression for resultant intensity at any point is

$$I = k(A_1^2 + A_2^2 + 2A_1A_2\cos\phi) \quad \ldots(1)$$

where k is constant of proportionality, ϕ is the path difference between interfering waves, A_1 and A_2 are amplitudes of superposing waves. As $A_1 = 2A_2 = 2A$, we have, the intensity at the central position,

$$I_0 = k(A_1 + A_2)^2 = k(2A + A)^2 = 9kA^2$$

So, $$k = \frac{I_0}{9A^2} \quad \ldots(2)$$

The path difference at any point P is

$$\Delta x = d\sin\theta$$

The phase difference $= \frac{2\pi}{\lambda} d\sin\theta \quad \ldots(3)$

Thus eqn. (1) becomes

$$I = \frac{I_0}{9A^2}\left[(2A)^2 + A^2 + 2(2A)(A)\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]$$

$$= \frac{I_0}{9A^2}\left[5A^2 + 4A^2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]$$

$$= \frac{I_0}{9}\left[1 + 8\cos^2\frac{\pi d\sin\theta}{\lambda}\right], \text{as. } \cos 2\theta = \left[\cos^2\frac{\theta}{2} - 1\right]$$

Example 5

In a Young's double slit experiment with wavelength 5890 Å there are 60 fringes in the field of vision. How many fringes will be observed in the same field of vision if wavelength used is 5460 Å ?

Solution: The fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

If the extent of field of vision is l,

$$l = N_1\beta_1$$

where N_1 is the number of fringes formed with wavelength λ.

As the field of vision is fixed,

$$l = N_1\beta_1 = N_2\beta_2$$

$$N_1\left(\frac{\lambda_1 D}{d}\right) = N_2\left(\frac{\lambda_2 D}{d}\right)$$

or $$N_1\lambda_1 = N_2\lambda_2$$

Thus $$N_2 = \frac{N_1\lambda_1}{\lambda_2} = \frac{60\times 5890}{5460} = 64.72 \approx 65$$

Example 6

A beam of light consisting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in a Young's double slit experiment. What is the least distance from the central maximum, where the bright fringes due to both the wavelengths coincide ? The distance between the slits is 2 mm and the distance between the plane of slits and the screen is 120 cm.

Solution: The position of nth bright fringe on the screen is

$$y_n = \frac{n\lambda D}{d}$$

Let the nth bright fringe of 6500 Å and the nth bright fringe of 5200 Å coincide; then

$$\frac{m\times 6500\times D}{d} = \frac{n\times 5200\times D}{d}$$

Thus the minimum values of m and n are 4 and 5 respectively.

Hence $$y = \frac{4\times 6500\times 120}{0.2} = 0.156 \text{ cm} = 1.56 \text{ mm}$$

Example 7

In a YDSE performed with wavelength $\lambda = 5890\,\text{Å}$ the angular fringe width is 0.40°. What is the angular fringe width if the entire set-up is immersed in water ?

Solution: Angular fringe width is given by

$$\beta_\theta = \frac{\lambda}{d}$$

So, $$\beta_\theta^{\text{air}} = \frac{\lambda_{\text{air}}}{d}$$

$$\beta_\theta^{\text{water}} = \frac{\lambda_{\text{water}}}{d}$$

$$\frac{\beta_\theta^{\text{air}}}{\beta_\theta^{\text{water}}} = \frac{\lambda_{\text{air}}}{\lambda_{\text{water}}}$$

$$= \frac{n_{\text{air}}}{n_{\text{water}}} = \frac{3}{4}$$

Thus $$\beta_\theta^{\text{water}} = \frac{3}{4}\beta_\theta^{\text{air}} = 0.40^\circ \times \frac{3}{4} = 0.30^\circ$$

Example 8

Find the angular separation $\Delta\theta$ between the corresponding points on the screen, if θ is small enough so that $\sin\theta \simeq \theta$. Refer to the intensity distribution curve of Fig. 2.9. There are points on the curve where the intensity of the fringe is half the maximum intensity.

Solution: For identical slits the intensity on the screen is given by

$$I = 4I_0 \cos^2\frac{\phi}{2}$$

where $$\phi = \left(\frac{2\pi}{\lambda}\right) d\sin\theta$$

where d is slit separation and λ is the wavelength.

The intensity at the centre of the interference pattern is $4I_0$. We want the angular position for which $I = 2I_0$.

So $$2I_0 = 4I_0 \cos^2\frac{\phi}{2}$$

$$\phi = 2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2}\text{ rad.}$$

Hence $$\frac{\pi}{2} = \frac{2\pi d\theta}{\lambda} \quad [\sin\theta \simeq \theta]$$

$$\theta = \frac{\lambda}{4d}$$

Another point of half intensity lies at $\theta = -\lambda/4d$, symmetrically placed relative to the central maxima. So the width of the pattern at half intensity is $2\lambda/4d = \lambda/2d$.

Example 9

In a YDSE for wavelength $\lambda = 589\,\text{nm}$, the interference fringes have angular separation of 3.50×10^{-3} radian. For what wavelength would the angular separation be 10.0% greater ?

Solution: The angular position of maxima is given by

$$d\sin\theta = n\lambda$$

$$d\theta \simeq n\lambda \qquad [\text{For small } \theta, \sin\theta \simeq \theta]$$

The angular separation of two adjacent maxima is

$$\Delta\theta = \frac{\lambda}{d}$$

Let angular separation be 10% greater for wavelength λ'.

Then $$\frac{1.10\lambda}{d} = \frac{\lambda'}{d}$$

or $$\lambda' = 1.10\lambda = (1.10 \times 589) = 648\,\text{nm}$$

Example 10

A YDSE is performed with two wavelengths simultaneously, $\lambda_1 = 480\,\text{nm}$ and $\lambda_2 = 600\,\text{nm}$. The distance between the slits is 5.0 mm and the slits are 1.0 m from the screen. What is the separation of the screen between third order ($n = 3$) bright fringes of the two interference patterns ?

Solution: Condition for maxima is

$$d\sin\theta = n\lambda$$

or $$d\theta \simeq n\lambda$$

$$\theta = \frac{n\lambda}{d}$$

The angular separation of two maxima associated with different wavelengths and same order is

$$\Delta\theta = \frac{n(\lambda_2 - \lambda_1)}{d}$$

The separation on a screen a distance D away is

$$\Delta y = D\tan(\Delta\theta) \simeq D\Delta\theta = \frac{nD(\lambda_2 - \lambda_1)}{d}$$

(as $\tan\theta \simeq \theta$ for small angles)

$$= \left[\frac{3(1.0)}{5.0 \times 10^{-3}}\right](600 - 480) \times 10^{-9}$$

$$= 7.2 \times 10^{-5}\,\text{m}$$

Example 11

Two point sources separated by 2.0 m are radiating in phase with $\lambda = 0.50$ m. A detector moves in a circular path around the two sources in a plane containing them. How many maxima are detected ?

Solution: Condition for maxima is

$$d\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d} = n\left(\frac{0.50}{2.0}\right) = 0.25n$$

As $\sin\theta$ lies between -1 and 1, so we wish to find all values of n for which

$$|0.25n| \le 1$$

These values are $-4, -3, -2, -1, 0, +1, +2, +3$ and $+4$. For each of these there are 2 different values of θ except for -4 and $+4$. A single value of θ, $-90°$ and $+90°$, is associated with $n = -4$ and $n = +4$ respectively. Thus there are 16 different angles in all and therefore 16 maxima.

Example 12

In a YDSE the incident yellow light is composed of two wavelengths 5890 Å and 5895 Å. The distance between the slits is 10^{-3} m and the screen is placed 1 m away. Up to what order can fringes be seen on the screen and how far from the centre of the screen does this occur ?

Solution: When the maxima of one wavelength coincides with minima of the other, the fringes will disappear.

So, $$\frac{n(5895)D}{d} = \frac{(n+1/2)5890D}{d}$$

or $$n = 589$$

Hence fringes will be visible up to 589 order of wavelength 5895 Å.

Position of this order on the screen is

$$Y_n = \frac{n\lambda D}{d} = \frac{589(5895)\times 1}{10^{-3}} = 0.347\text{ m}$$

Example 13

In a YDSE light of wavelength $\lambda = 5000$ Å is used, which emerges in phase from two slits a distance $d = 3\times10^{-7}$ m apart. A transparent sheet of thickness $t = 1.5\times10^{-7}$ m, refractive index $\mu = 1.17$, is placed over one of the slits. Where does the central maxima of the interference now appear ?

Solution: The path difference introduced due to introduction of transparent sheet is given by $\Delta x = (\mu - 1)t$.

If the central maxima occupies position of nth fringe, then

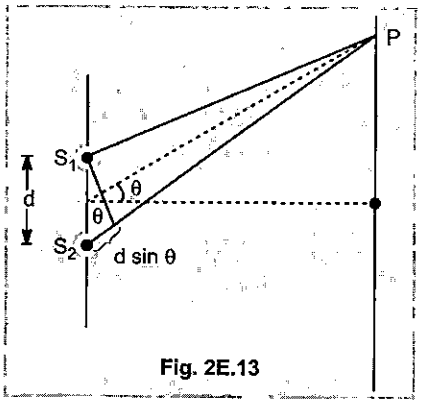


Fig. 2E.13

$$(\mu - 1)t = n\lambda = d\sin\theta$$

$$\sin\theta = \frac{(\mu-1)t}{d} = \frac{(1.17-1)\times1.5\times10^{-7}}{3\times10^{-7}}$$

$$= 0.085$$

Hence the angular position of central maxima is

$$\theta = \sin^{-1}(0.085) = 4.88°$$

For small angles $\sin\theta \simeq \theta \simeq \tan\theta$

As $$\tan\theta = \frac{y}{D}$$

so $$\frac{y}{D} = \frac{(\mu-1)t}{d}$$

Shift of central maxima is $Y = \frac{D(\mu-1)t}{d}$. This formula can be used if D is given.

Example 14

A double slit experiment is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen 1.33 mm. The slits are illuminated by a parallel beam of light whose wavelength in air is 6830 Å. Calculate the fringe width.

Solution: The experimental setup is in a liquid, therefore the wavelength of light will change.

$$\lambda_{\text{liquid}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{6300}{1.33} = \frac{6300\times10^{-10}}{1.33}\text{ m}$$

Fringe width $$\beta = \frac{\lambda_{\text{liquid}}D}{d} = \frac{\lambda_{\text{air}}D}{\mu d}$$

$$= \frac{6300\times10^{-10}}{1.33}\times\frac{1.33}{10^{-3}}$$

$$= 6.3\times10^{-4}\text{ m}$$

Example 15

Two coherent sources emit light of wavelength λ, separation between them, $d = 4\lambda$. (a) A detector Δ moves along the y-axis. What is the maximum number of maxima registered if the detector moves along the y-axis ? (b) Repeat the problem if the detector moves along the x-axis.

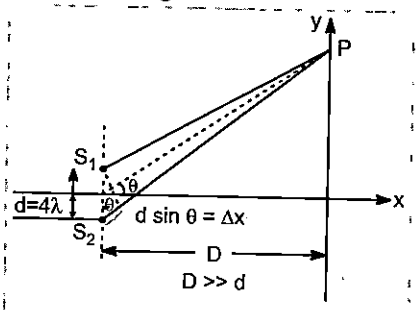


Fig. 2E.15 (a)

Solution: (a) The difference at any point on the screen is,

$$d\sin\theta = n\lambda$$

$$n = \frac{d}{\lambda}\sin\theta$$

n is maximum when $\sin\theta$ is maximum; thus

$$\text{maximum } n = \frac{d}{\lambda} = 4$$

Thus apart from central maxima at $\theta = 0$, eight other maxima, 4 on either side of central maxima are registered. Total maxima registered are 9 and 8 minima lie between them.

(b) The path difference at any point Q on the x-axis is

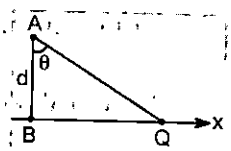


Fig. 2E.15 (b)

$$\Delta x = AQ - BQ$$

$$= \frac{AB}{\cos\theta}(1 - \sin\theta)$$

Condition for maxima is

$$\frac{d}{\cos\theta}(1 - \sin\theta) = n\lambda$$

$$n = \frac{4(1 - \sin\theta)}{\cos\theta}$$

At point B, $\theta = 0$, $n = 4$; we get fourth order maxima, at $x = \infty$, $\theta = \pi/2$, n is indeterminate at this position.

$$n = 4\frac{(1 - \sin 90°)}{\cos 90°} \to \frac{0}{0}$$

We can solve the expression for n with the help of L Hospital rule of limits.

$$\lim_{\theta \to \pi/2}\left(\frac{1 - \sin\theta}{\cos\theta}\right) = \lim_{\theta \to \pi/2}\frac{-\cos\theta}{\sin\theta} = 0$$

Thus at $\theta = \pi/2$ we have $n = 0$, *i.e.*, zeroth order maxima is formed at $x = \infty$.

Hence along x-axis, beginning from slit B three other maxima are registered.

Example 16

In YDSE the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5th bright fringe. Find the thicknesss of the plate. When the source is now changed to green light of wavelength 10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to change in wavelength.

Solution: Introduction of a glass plate shifts the central maxima by

$$\Delta x = \frac{D(\mu - 1)t}{d} = \frac{\beta(\mu - 1)t}{\lambda}$$

The central maxima shifts by five fringes; therefore

$$\frac{\beta_R(\mu_R - 1)t}{\lambda_R} = 5\beta_R \qquad ...(1)$$

Therefore, $t = \dfrac{5\lambda_R}{(\mu_R - 1)} = \dfrac{5 \times 7 \times 10^{-7}}{(1.5 - 1)} = 7\mu\text{m}$

Similarly, when green light is used, the central maxima shifts by 6 fringes; therefore

$$\frac{\beta_G(\mu_G - 1)t}{\lambda_G} = 6\beta_G \qquad ...(2)$$

Dividing eqn. (1) by (2), we get

$$\frac{(\mu_R - 1)}{(\mu_G - 1)} = \frac{5}{6},\ (\mu_G - 1) = \frac{6}{5}(1.5 - 1)$$

or $\mu_G = 1.6$

As given in the example,

$$5\beta_R = 10^{-3},\ \beta_R = 2 \times 10^{-4}$$

Also $$\frac{\beta_G}{\beta_R} = \frac{(\lambda_G D/d)}{(\lambda_R D/d)} = \frac{\lambda_G}{\lambda_R} = \frac{5}{7}$$

or $$\frac{\beta_G}{\beta_R} - 1 = \frac{5}{7} - 1 = -\frac{2}{7}$$

$$\frac{\beta_G - \beta_R}{\beta_R} = -\frac{2}{7}$$

Thus $$\Delta\beta = \beta_G - \beta_R = -\frac{2}{7} \times \beta_R = -0.57 \times 10^{-4}\text{ m}$$

The minus sign denotes that fringe width will decrease, when green light is used.

Example 17

In a YDSE the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen, where the central maxima (n = 0) lay before the glass plates were inserted, now has 3/4 the original intensity. It is further observed that what used to be the fifth maxima earlier, lies below point P while the sixth maxima lies above P. Calculate the thickness of glass plate (Absorption of light by the glass plate may be neglected).

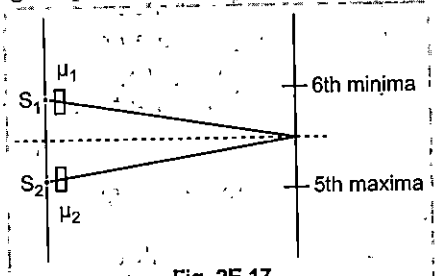


Fig. 2E.17

Solution: When both the slits are covered by glass plates the additional path difference at point P (position of central maxima in the absence of plates) is

$$\Delta x = (\mu_2 - \mu_1)t = (1.7 - 1.4)t = 0.3t$$

Path difference at fifth maxima is 5λ and at sixth minima is $\frac{11\lambda}{2}$. As the point P lies between fifth maxima and sixth minima, the path difference at P is

$$\Delta x = 5\lambda + \Delta n'$$

The resultant phase difference is $\Delta\phi = \left(10\pi + \frac{2\pi}{\lambda}\Delta x'\right)$

The resultant intensity is given by

$$I_R = 4I_0 \cos^2\frac{\phi}{2}$$

According to the problem

$$I_R = \frac{3I_0}{4} = 4I_0\cos^2\frac{\phi}{2}$$

or $$\cos^2\frac{\phi}{2} = \frac{3}{4}$$

$$\phi = 2\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

Path difference $\Delta x'$ corresponding to phase difference

$$\frac{\pi}{3} = \left(\frac{\pi}{3}\times\frac{\lambda}{2\pi}\right) = \frac{\lambda}{6}$$

Thus $$\Delta x = 0.3t = 5\lambda + \frac{\lambda}{6} = \frac{31\lambda}{6}$$

or $$t = \frac{31\lambda}{6\times 0.3} = \frac{31\times 54\times 10^{-7}}{0.3\times 6} = 9.3\mu m$$

Example 18

A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in Fig. 2E.18 (a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen. (b) If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in the figure), find the y-coordinates of the first minima on either side of the central maximum.

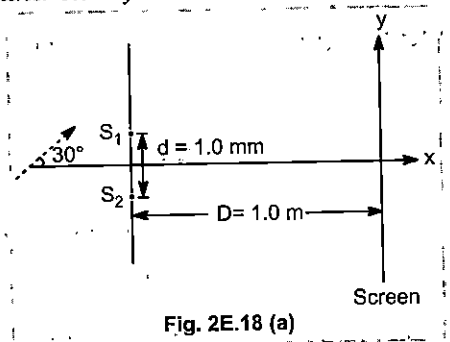


Fig. 2E.18 (a)

Solution: Position of a point on a screen is

$$y = D\tan\theta = 1\tan\theta$$

For first minima, $n = 1$, $\sin\theta_1 = \frac{1}{4}$, $\tan\theta_1 = \frac{1}{\sqrt{15}}$

For second minima, $n = 2$, $\sin\theta_2 = \frac{3}{4}$, $\tan\theta_2 = \frac{3}{\sqrt{7}}$

So the positions of minima are

$$y_1 = \tan\theta_1 = \frac{1}{\sqrt{15}} = 0.258\,m$$

and $$y_2 = \tan\theta_2 = \frac{3}{\sqrt{7}} = 1.13\,m$$

The minima are symmetrically placed on either side of central maxima; therefore there will be 4 minima at positions ± 0.258 m and ± 1.13 m on the screen.

(a) When incident rays are incident normally, the waves arriving at slits are in phase, zero path difference before slits. Pathdifference after slits, at point P, is $d\sin\theta$. Condition for minima at y-axis is

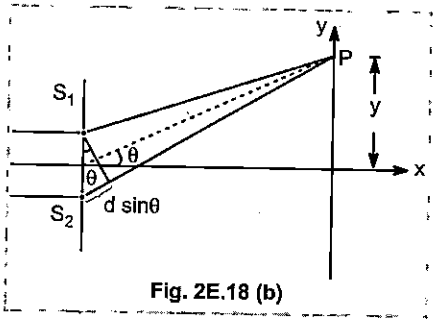


Fig. 2E.18 (b)

$$d\sin\theta = (2n-1)\frac{\lambda}{2}$$

$$\sin\theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)(0.5)}{2\times 1} = \frac{(2n-1)}{4}$$

As $\sin\theta \le 1$; $\left(\frac{2n-1}{4}\right) \le 1$ or $n \le 2.5$, as $n = 1, 2, 3, \ldots$

Hence only first order and second order minima are possible.

(b) Path difference before slits = $d\sin\phi$

Path difference after slits = $d\sin\theta$

As path of rays before slits is longer at S_1 and $S_2P > S_1P$ after slits, so net path difference for first minima is

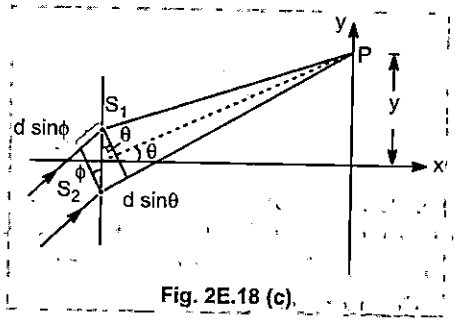


Fig. 2E.18 (c)

$$d\sin\theta - d\sin\phi = \pm\frac{\lambda}{2}$$

$$\sin\theta = \sin\phi \pm \frac{\lambda}{2d}$$

$$= \sin 30° \pm \frac{0.5}{2\times 1}$$

$$= \frac{3}{4} \quad \text{or} \quad \frac{1}{4}$$

Thus $\tan\theta = \frac{3}{\sqrt{7}}$ or $\frac{1}{\sqrt{15}}$

So the position of first minima on either side of central maxima is

$$y = D\tan\theta = \frac{3}{\sqrt{7}}\text{m} \quad \text{and} \quad \frac{1}{\sqrt{15}}\text{m}$$

Example 19

A YDSE is performed in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 nm separation. The lower slit S_2 is covered by a thin glass plate of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in Fig. 2E. 19. (a). Find the location of the central maximum (bright fringe with zero path difference) on the y-axis. (b) Find the light intensity at point O relative to maximum fringe intensity. (c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelength of the light that forms maxima exactly at point O. (All the wavelengths in this problem are for the given medium of refractive index 4/3. Ignore absorption.)

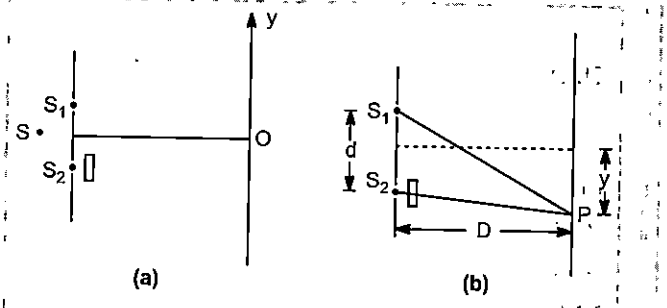


Fig. 2E.19

Solution: (a) Path difference at point P on the screen,

$$\Delta x = \frac{yd}{D}$$

At the position of central maxima the optical path lengths S_2P and S_1P are equal,

$$i.e.; \quad \frac{(S_2P - t)}{c/\mu_1} + \frac{t}{c/\mu_2} = \frac{S_1P}{c/\mu_1},$$

where $\mu_1 = 4/3$, $\mu_2 = 3/2$

or $\mu_1(S_1P - S_2P) = (\mu_2 - \mu_1)t$

or $\mu_1\left(\frac{yd}{D}\right) = (\mu_2 - \mu_1)t$

$$y = \frac{(\mu_2 - \mu_1)tD}{\mu_1 d}$$

$$= \frac{[(3/2) - (4/3)]\times 10.4\times 1.5}{(4/3)\times 0.45\times 10^{-3}}$$

$$= 4.33\,\text{mm}$$

(b) At point O net path difference

$$\Delta x = \left(\frac{\mu_2}{\mu_1} - 1\right)t$$

Net phase difference $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$$= \frac{2\pi}{6\times10^{-7}}\left(\frac{1.5}{4/3}-1\right)(10.4\times10^{-6})$$

$$= \left(\frac{13}{3}\right)\pi$$

Thus intensity $I = I_{max}\cos^2(\phi/2)$

$$= I_{max}\cos^2\left(\frac{13\pi}{6}\right)$$

$$= \frac{3}{4}I_{max}$$

(c) For maximum intensity at point O

$\Delta x = n\lambda$ (where $n = 1, 2, 3, \ldots$)

Path difference at point O

$$\Delta x = \left(\frac{1.5}{4/3}-1\right)(10.4\times10^{-6}) = 1300\,\text{nm}$$

Thus maximum intensity will correspond to

$$\lambda = 1300\,\text{nm},\ \frac{1300}{2}\,\text{nm},\ \frac{1300}{3}\,\text{nm},\ldots$$

In the given range required values are 650 nm and 433.33 nm.

Example 20

In a YDSE performed with light of wavelength 6000 Å, the screen is placed 1 m from the slits. Fringes formed on the screen are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes. If they subtend an angle more than 1 minute of arc, calculate the maximum distance between the slits so that the fringes are clearly visible. Using this information calculate the position of 3rd bright and 5th dark fringe from the centre of the screen.

Solution: The angular fringe width, *i.e.*, the angle subtended by a fringe at the centre of slits is given by

$$\beta_\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

According to the given condition,

$$\beta_\theta = \frac{\lambda}{d} \geq \frac{\pi}{180\times60}$$

As $1' = \dfrac{\pi}{180\times60}$ rad.

So $$d \leq \frac{6\times10^{-7}\times180\times60}{\pi}$$

$$d \leq 2.06\times10^{-3}\,\text{m}$$

Thus $d_{max} = 2.06\times10^{-3}\,\text{m} = 2.06\,\text{mm}$

Position of 3rd bright fringe,

$$y_3 = \frac{3\lambda D}{d} = \frac{3\times6\times10^{-7}\times1}{2.06\times10^{-3}}$$

$$= 8.74\times10^{-4}\,\text{m}$$

Position of 5th dark fringe,

$$y_5 = \frac{\left(n-\frac{1}{2}\right)\lambda D}{d} = \frac{9\lambda D}{2d}$$

$$= \frac{9\times6\times10^{-7}\times1}{2\times2.06\times10^{-3}} = 13.1\times10^{-4}\,\text{m}$$

$$= 1.31\,\text{mm}$$

Example 21

In a modified YDSE source S is kept in front of slit S_1. Find the phase difference at a point O that is equidistant from slits S_1 and S_2 and a point P that is in front of slit S_1 in the following situations :

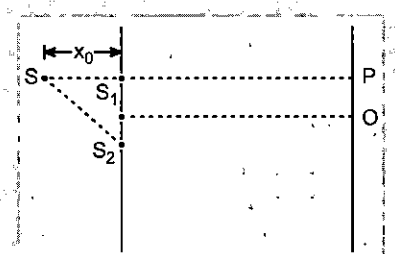


Fig. 2E.21

(a) A liquid of refractive index μ is filled between the screen and slits.

(b) Liquid is filled between the slit and the source S.

Solution: While calculating path difference it must be remembered that it is equal to difference of optical path lengths from source S to point on the screen.

Total path difference is

$$\Delta x_{total} = [\Delta x]_{\text{before slits}} + [\Delta x]_{\text{after slits}}$$

$$= (SS_2 - SS_1) + (S_2P - S_1P)$$

(a) When liquid is filled between slits and screen,

$$[S_2P]_{liquid} = [\mu S_2P]_{air}$$

and $$[S_1P]_{liquid} = [\mu S_1P]_{air}$$

At point *O*: $\mu S_2O = \mu S_1O$, no path difference is introduced after slits.

So, $$\Delta x_{total} = SS_2 - SS_1 = \sqrt{d^2 + x_0^2} - x_0$$

Thus, phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}(\sqrt{d^2 + x_0^2} - x_0)$$

At point *P* :

$$[\Delta x]_{\text{before slits}} = \sqrt{d^2 \times x_0^2} - x_0$$

$$[\Delta x]_{\text{after slits}} = \mu S_2P - \mu S_1P = \mu(S_2P - S_1P)$$

$$= \frac{\mu y d}{D}$$

$$= \frac{\mu d^2}{2D} \quad \text{as} \quad y = \frac{d}{2}$$

Therefore $[\Delta x]_{\text{total}} = [\sqrt{d^2 + x_0^2} - x_0] + \frac{\mu d^2}{2D}$

Thus phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\left[[\sqrt{d^2 + x_0^2} - x_0] + \frac{\mu d^2}{2D}\right]$$

(b) When liquid is filled between source and slits.

At point *O* :

$$(\Delta x)_{\text{before slits}} = (SS_2 + SS_1)_{\text{liquid}}$$
$$= (\mu SS_2 + \mu SS_1)_{\text{air}}$$
$$= \mu(\sqrt{d^2 + x_0^2} - x_0)$$
$$(\Delta x)_{\text{after slits}} = S_2O - S_1O = 0$$
$$(\Delta x)_{\text{total}} = \mu(\sqrt{d^2 + x_0^2} - x_0)$$

Thus phase difference at P,

$$\Delta\phi = \frac{2\pi}{\lambda}[\mu\sqrt{d^2 + x_0^2} - x_0]$$

At point *P* :

$$(\Delta x)_{\text{before slits}} = (SS_2 + SS_1)_{\text{liquid}}$$
$$= (\mu SS_2 + \mu SS_1)_{\text{air}}$$

$$(\Delta x)_{\text{after slit}} = (S_2P - S_1P)_{\text{air}} = \frac{yd}{D} = \frac{d^2}{2D}$$

$$(\Delta x)_{\text{total}} = \left[\mu(\sqrt{d^2 + x_0^2} - x_0) + \frac{d^2}{2D}\right]$$

Thus phase difference at P,

$$\Delta\phi = \frac{2\pi}{\lambda}\left[\mu\sqrt{d^2 + x_0^2} - x_0 + \frac{d^2}{2D}\right]$$

Example 22

A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel to illuminate the slits as shown in Fig. 2E.22. (a) Calculate the position of the central bright fringe on the other wall CD w.r.t. the line OQ. (b) Now a liquid is poured into the vessel and filled up to OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid.

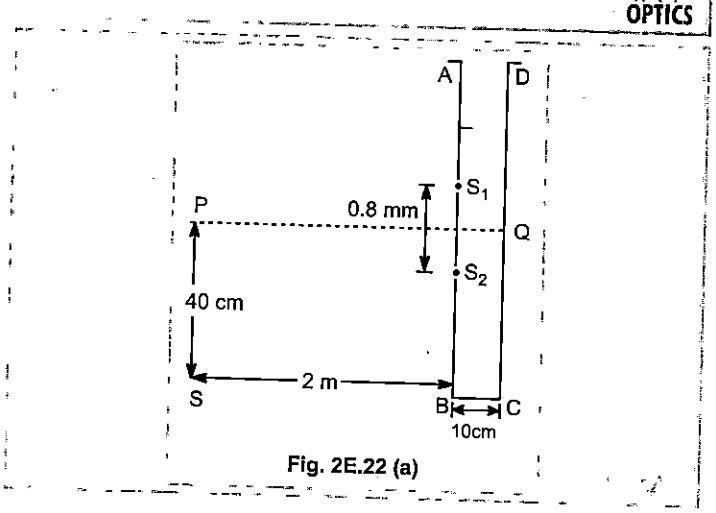


Fig. 2E.22 (a)

Solution: (a) Path difference before slits,

$$SS_1 - SS_2 = d\sin\phi$$

Path difference after slits,

$$S_2P - S_1P = d\sin\theta$$

For point P to central maxima, net path difference at P must be zero, *i.e.*,

$$d\sin\phi = d\sin\theta \quad \ldots(1)$$

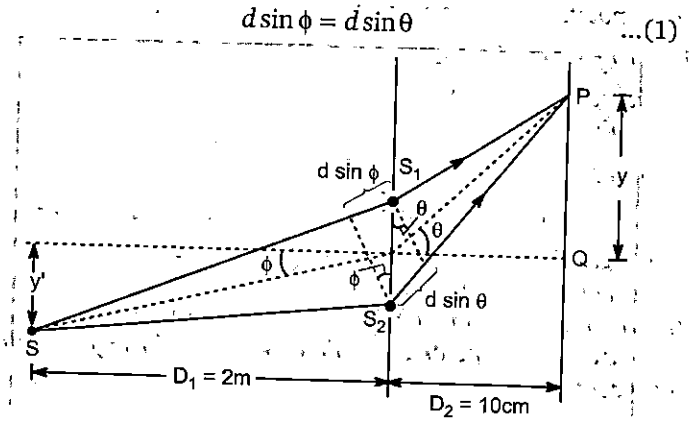


Fig. 2E.22 (b)

Note that $SS_1 > SS_2$ and $S_2P > S_1P$, *i.e.*, before slits the path up to S_1 is longer and after slits the path S_2P is longer.

For small angles $\sin\theta \approx \tan\theta$, $\sin\phi \approx \tan\phi$

So, from eqn. (1), $\tan\phi = \tan\theta$

$$\frac{y'}{D_1} = \frac{y}{D_2}$$

or $$y = \frac{y'D_2}{D_1} = 2\,\text{cm}$$

(b) When liquid is filled up to OQ, the central maxima will be formed at Q if optical path difference before slits, produced due to asymmetry of source S_1 is nullified by optical path difference after slits.

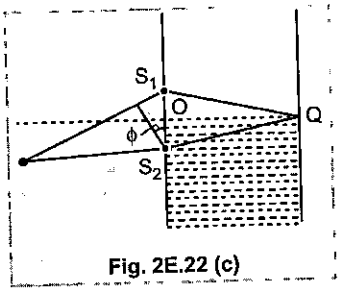


Fig. 2E.22 (c)

$$(\mu S_2Q - S_1Q) = d\sin\phi$$

$$(\mu - 1)D_1 = d \times \frac{y'}{D_1}$$

As $S_2Q = S_1Q \simeq D_2$ and $\sin\phi \simeq \tan\phi = \frac{y'}{D_1}$,

So $$(\mu - 1)10 = 8 \times 10^{-2}\left(\frac{40}{200}\right)$$

Thus, $$\mu = 1.0016$$

Example 23

In a modified YDSE a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $(10/\pi)$ W/m^2 is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m, respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A (see Fig. 2E.23). Calculate the power in watts received at the focal spot F of the lens. The lens is symmetrically placed w.r.t. the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.

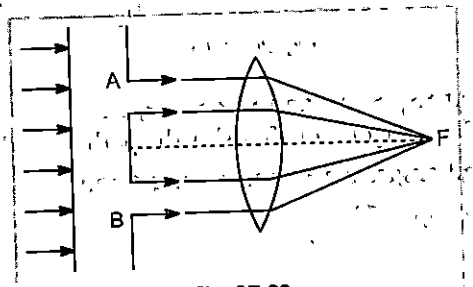


Fig. 2E.23

Solution: The power transmitted by apertures A and B are

$$P_A = \left(\frac{10}{\pi}\right)[\pi(0.001)^2] = 10^{-5}\,W$$

$$P_B = \left(\frac{10}{\pi}\right)[\pi \times (0.002)^2] = 4 \times 10^{-5}\,W$$

Only 10% of transmitted power reaches the focus.

$$P'_A = 10^{-5} \times \frac{10}{100} = 10^{-6}\,W$$

$$P'_B = 4 \times 10^{-5} \times \frac{10}{100} = 4 \times 10^{-6}\,W$$

The resultant power at the focus after superposition of two waves is

$$P = P'_A + P'_B + 2\sqrt{P'_A P'_B}\cos\phi$$

where ϕ is phase difference.

The introduction of mica sheet in the path of A creates a path difference $(\mu - 1)t$.

$$(\mu - 1)t = (1.5 - 1) \times 2000\,\text{Å} = 1000\,\text{Å}$$

Phase difference $\Delta\phi = \frac{2\pi}{\lambda}(\mu - 1)t = \frac{2\pi}{6000} \times 1000 = \frac{\pi}{3}$

Therefore, $P = 10^{-6} + 4 \times 10^{-6} + 2\sqrt{(10^{-6})(4 \times 10^{-6})}\cos\frac{\pi}{3}$

$$= 7 \times 10^{-6}\,W$$

FRESNEL'S MIRRORS

Fig. 2.24 shows Fresnel's bimirrors apparatus to produce interference by *division of the wavefront*. Light from a slit S is *reflected* by two place mirrors *slightly* inclined to each other. The mirrors produce two virtual images S_1 and S_2 of the slit, the interference fringes are observed in the region BC, where the *reflected* beams overlap.

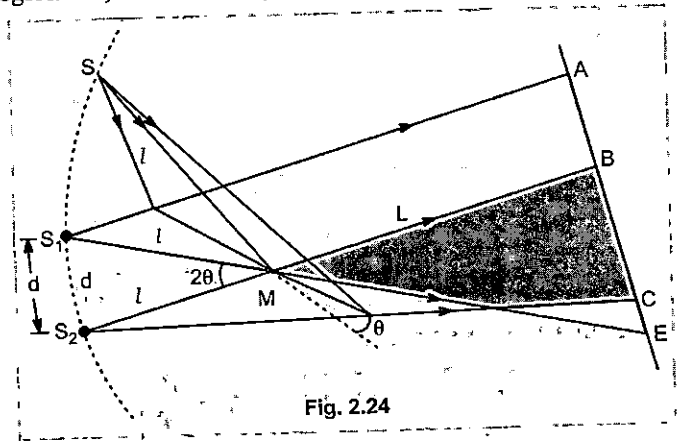


Fig. 2.24

If θ is the angle between the planes of the mirrors, then S_1 and S_2 subtend angle 2θ at the point of intersection M between the mirrors.

If l is the distance between the slit and the mirrors intersection and L is the distance between the screen and the mirrors intersection, then the separation between the images S_1 and S_2 is

$$d = l(2\theta) = 2l\theta$$

and $$D = l + L$$

Thus, the *fringe width* is given by

$$\beta = \frac{\lambda D}{d} \quad \text{or} \quad \beta = \frac{\lambda(l + L)}{2l\theta}$$

or $$\beta = \frac{\lambda}{2\theta}\left[1 + \frac{L}{l}\right]$$

Fresnel's Biprism

Fig. 2.25 shows the Fresnel's biprism experiment schematically. The thin prism P refracts light from the slit source S into two beams AC and BE. When a screen MN is placed as shown in the figure, the interference fringes are observed *only* in the region BC. If the screen MN is removed, the two beams will overlap over the whole region AE.

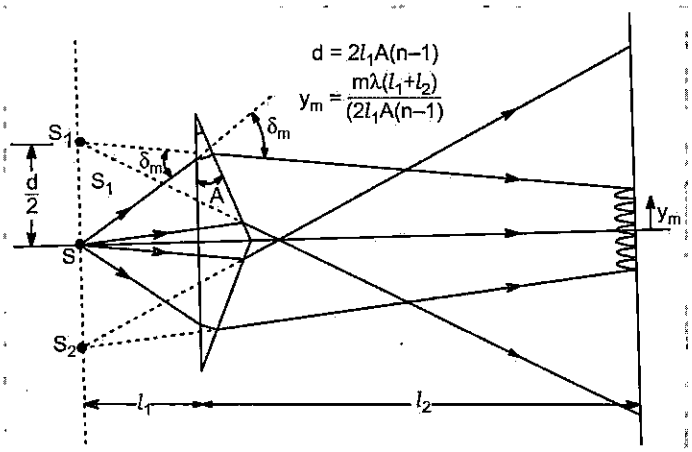


Fig. 2.25: Interference with Fresnel's biprism. Coherent sources are the virtual images S_1 and S_2 of source S, formed by refraction in the two halves of the prism.

If A is the angle of refraction of the thin prism and μ is the refractive index of its medium, then the angle of deviation produced by the prism is

$$\delta = A(\mu - 1)$$

If l_1 is the distance between the *source* and the *prism,* then the separation between virtual sources is

$$d = 2\delta l_1 = 2A(\mu - 1)l_1$$

If l_2 is the distance between the *prism* and the *screen,* then the distance between *virtual sources* and the *screen* is given by

$$D = l_1 + l_2$$

Thus, by using the results of Young's experiment, the *fringe width* is given by

$$\beta = \frac{\lambda D}{d} \quad \text{or} \quad \beta = \frac{\lambda(l_1 + l_2)}{2\delta l_1}$$

or $$\beta = \frac{\lambda}{2\delta}\left[1 + \frac{l_2}{l_1}\right]$$

or $$\beta = \frac{\lambda}{2A(\mu - 1)}\left[1 + \frac{l_2}{l_1}\right]$$

Fringes observed in the Fresnel's biprism experiment are *vertical straight lines.*

The Lloyd's Mirror Experiment

In this arrangement the light reflected from a long mirror and the light coming directly from the source without reflection produce interference on a screen. An important feature of this experiment lies in the fact that when, the screen is placed in contact with the end of the mirror the edge O of the reflecting surface comes at the centre of a dark fringe instead of a bright fringe. The direct beam does not suffer any phase change. This means that the reflected beam undergoes a phase change of π radian. Hence at a point P on the screen the conditions for minima and maxima are

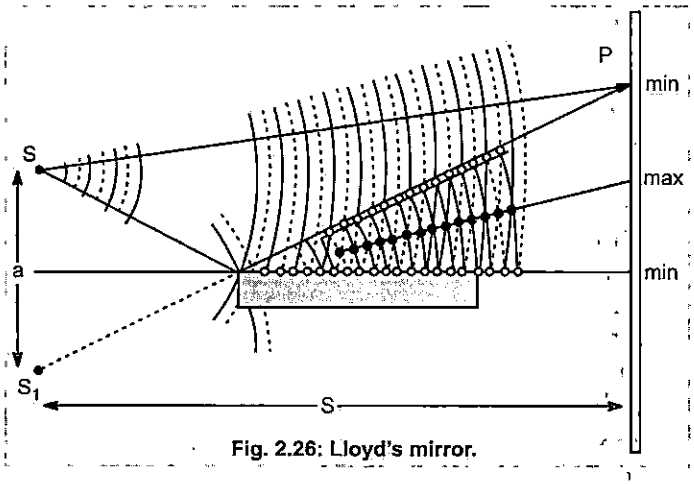


Fig. 2.26: Lloyd's mirror.

$$S_2P - S_1P = n\lambda, \quad n = 0, 1, 2, 3, \ldots \text{ [minima]}$$

$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda \quad \text{[maxima]}$$

The Phase Change on Reflection

A ray of light is incident on air-water interface; let the amplitude reflection and transmission coefficients be r_1 and t_1 respectively. The amplitudes of reflected and transmitted waves are ar_1 and at_1 respectively. From the principle of reversibility of light, the system retraces its whole previous motion. The wave of amplitude ar_1 gives a reflected wave of amplitude ar_1^2 and the transmitted wave of amplitude ar_1t_1. The wave of amplitude at_1

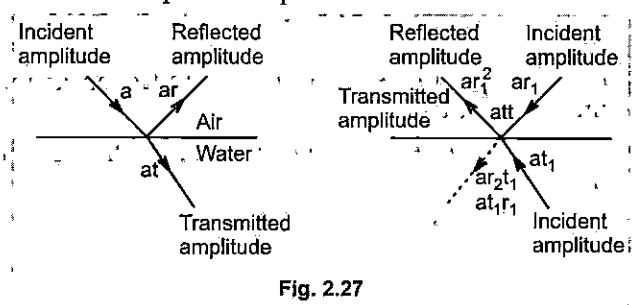


Fig. 2.27

gives a reflected wave of amplitude ar_2t_1 and transmitted wave of amplitude at_1t_2.

So, $$ar_1^2 + at_1t_2 = a$$

$$t_1t_2 = 1 - r_1^2 \quad \ldots(1)$$

Further, the waves of amplitudes at_1r_2 and ar_1t_1 must cancel each other.

$$at_1r_2 + ar_1t_1 = 0$$

$$r_2 = -r_1 \quad \ldots(2)$$

Eqn. (2) shows a difference of phase of π between the two cases; a reversal of sign means a displacement in the opposite sense. If there is no change of phase on reflection from above, there must be a phase change of π from below and *vice-versa.*

When light gets reflected from a denser medium there is an abrupt phase change of π; no phase change occurs when reflection takes place from rarer medium.

Thin Film Interference

When light passes the boundary between two transparent media, same light is reflected at the boundary. As shown in the figure some light is reflected from first interface and some from second interface. If we consider a monochromatic incident light the two reflected waves are also monochromatic and coherent because, they arise from the same monochromatic incident light wave *via amplitude division.* These waves interfere, since they are superposed along the same normal line.

The phase difference between two interfering waves is due to :

(1) Optical path difference (due to distances travelled),

(2) Reflection from a denser medium.

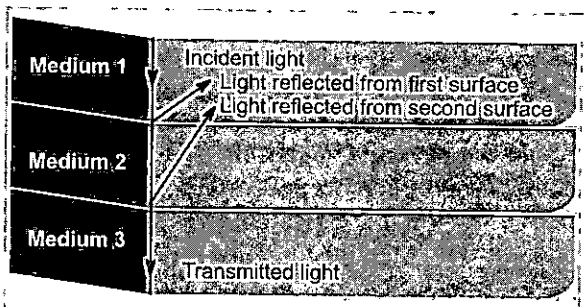


Fig. 2.28

The second factor is irrelevant for reflection at rarer medium. Three situation may arise :

(1) Neither wave experiences a phase change upon reflection.

(2) Both the waves suffer a phase change upon reflection.

In either of these two cases the phase change due to reflection is irrelevant; no difference in phase results due to reflection.

In either of these cases phase change is determined solely from optical path difference.

Condition for constructive interference :

$$2\mu t = m\lambda$$

Condition for destructive interference :

$$2\mu t = \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2, \ldots$

(3) One of the reflected waves experiences a phase change of π radian upon reflection and the other wave does not.

It is material which wave suffers a phase change; the conclusions in the previous case are first reversed.

Condition for destructive interference :

$$2\mu t = m\lambda$$

Condition for constructive interference :

$$2\mu t = \left(m + \frac{1}{2}\right)\lambda$$

where $m = 0, 1, 2, \ldots$

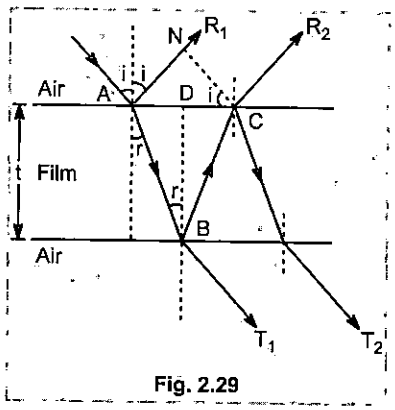


Fig. 2.29

❒ If the rays of light are incident obliquely on the thin film, the optical path difference is $2\mu t \cos r$, where r is angle of refraction.

$$\Delta = \mu(AB + BC) - AN \left[AN = (AD + DC)\sin i = \frac{2t\sin i}{\tan r}\right]$$

$$= \mu\left(\frac{2t}{\cos r}\right) - \frac{2t\sin i}{\tan r}$$

From Snell's law, $\sin i = \mu \sin r$; on substituting it and solving we get

$$\Delta = 2\mu t \cos r$$

THE INTERFERENCE CONDITIONS BY DIVISION OF WAVEFRONT AND AMPLITUDE

The phase difference δ at a point P between waves leaving two light sources is given by

$$\delta = k(x_2 - x_1) + (\phi_2 - \phi_1)$$

The resultant amplitude keeps the same value if (1) the waves have the same frequency, and (2) $(\phi_2 - \phi_1)$ remains constant. However, the presence of the phase breaks shown in Fig. 2.31 makes it impossible for $((\phi_2 - \phi_1)$ to stay constant for times longer than about 10 ns. Away of circumventing this problem is to divide the light from a single source into two parts. The standard techniques for doing this are called **division of wavefront** and **division of amplitude.**

In division of wavefront, the expanding wavefront from a *primary* light source S falls on two narrow, closely spaced apertures, S_1 and S_2, cut in an opaque screen, as shown in Fig. 2.30 (a). If the apertures are equidistant from the primary source, as in the simplest experimental set-up. the phase of the wave at S_1 will be the same as the phase at S_2 because they were emitted by S at the same instant. Further, waves leaving S_1 and S_2 have the same frequency as the primary source. Hence, sources S_1 and S_2 satisfy the interference conditions; they act as *secondary coherent* sources. The secondary sources are real in Fig. 2.30 (a) but they may also be virtual for a biprism.

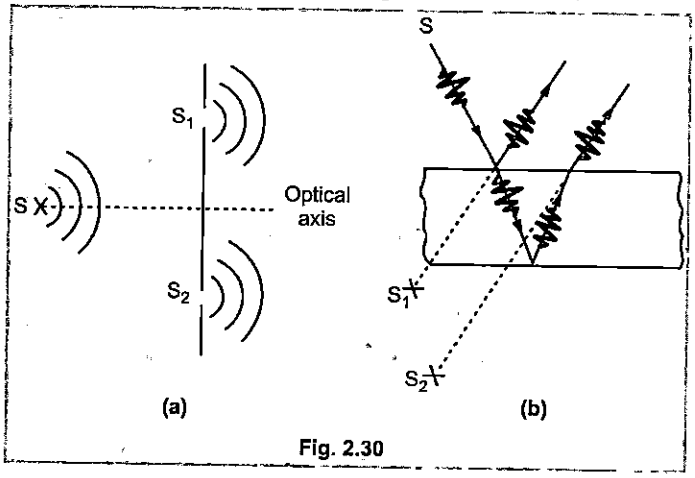


Fig. 2.30

In division of amplitude, each wavetrain is split into two. Each part has the same initial phase as the primary wavetrain and change its values in step with it, thus keeping $(\phi_2 - \phi_1)$ constant. the wavetrains appear to come from virtual secondary coherent sources. These are S_1 and S_2 in Fig. 2.30 (b).

Before considering interference by division of wavefront and amplitude in detail, it is essential to discuss Huygen's principle of secondary sources.

Interference by Division of Wavefront

Coherent waves can be obtained from a 'point source' in two ways. In the wavefront division approach, the spherical wavefront emanating from the point source is split and then recombined after introducing an appropriate path difference. This is what happens in Young's two slit arrangement. Alternatively, the amplitude of the incident wave is split at an interface between two media (division of amplitude) to generate two waves which interfere upon recombination.

Some of the standard arrangements used to observe two-wave interference by division of the wavefront are shown in Fig. 2.31.

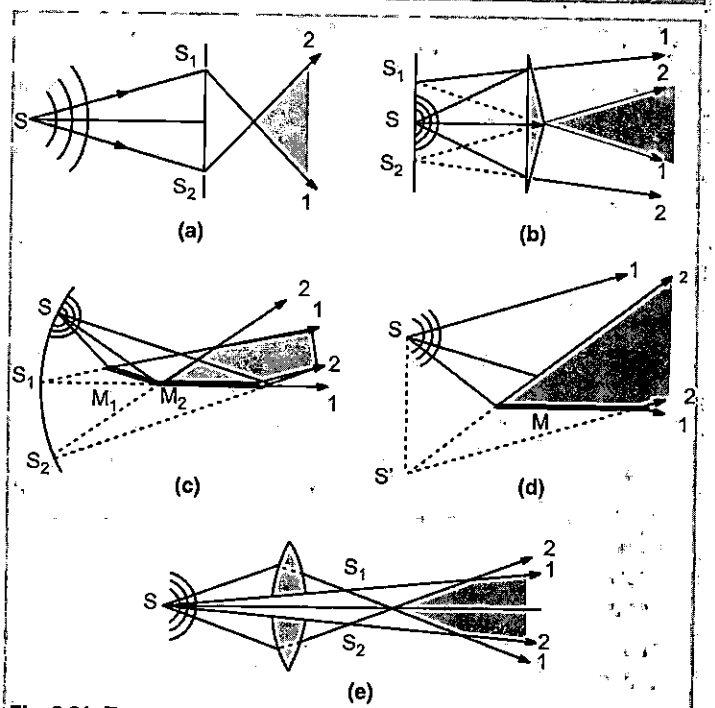


Fig. 2.31: Two-wave interference; (a) Young's two-slit arrangement, (b) Fresnel's biprism, (c) Fresnel's two mirrors, (d) Loyd's mirror, (e) Billet's split lens.

The narrow slits S_1 and S_2 in Young's double slit arrangement [Fig. 2.31 (a)] intercept portions of the spherical wavefront and act as real mutually coherent point sources, diffracting light in the forward direction. Interference among the diffracted waves can be observed anywhere in the region of overlap (shaded portion) behind the plane of the Fig. 2.31.

BILLET'S LENS

Consider a lens of focal length f cut into equal halves along any diameter and separated away by a distance a with an opaque substance filling the gap as shown in Fig. (2.32). Let S be a single slit at a distance of b from the lenses $(b > f)$. Evidently the two lens halves form two distinct images of S.

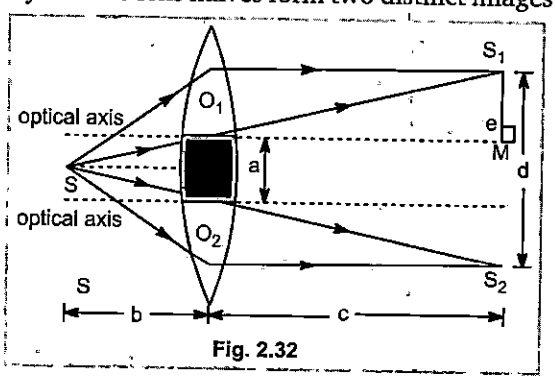


Fig. 2.32

From the lens equation we have,

$$c = \frac{bf}{b - f}$$

$\therefore$ From similar $\Delta S_1 SS_2$ and $\Delta O_1 SO_2$

we have, $\frac{S_1S_2}{b+c} = \frac{O_1O_2}{b}$

or $\frac{d}{b+c} = \frac{a}{b}$

$$d = \left(1+\frac{c}{b}\right)a$$

$$= \left[1+\frac{f}{(b-f)}\right]a$$

$$d = \frac{ab}{b-f}$$

For suitable values of a, b and f it is possible to obtain the value of $d \approx 1mm$ comparable to the order of the slit separation adopted in Young's double slit experiment, so that interference fringes can be observed on a screen placed at a distance of 1 to 2m from the images.

Two lens pieces : If a convex lens be cut in such a way that its central portion of a width $\delta(\approx mm)$ is cut out and the two halves are tightly fitted against each other, then corresponding to a single slit placed symmetrically in front of the combination yields two image sources. Usually, the distance 'a' of the slit S is less than the focal length f of the lens pieces.

Example 24

Fig. 2E.24 (a) shows three equidistant slits S_1, S_2 and S_3 each emitting light of intensity I_0, and separated by a distance $d = \sqrt{2\lambda D/3}$ where λ is the wavelength of the monochromatic light illuminating the slits. Find the intensity at point P directly infront of the slit S_1. (Here $\lambda << D$).

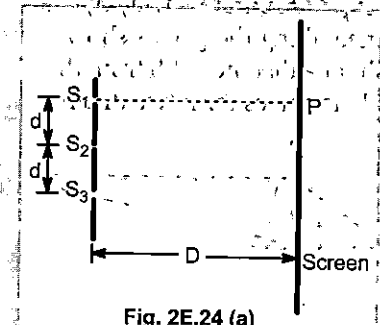


Fig. 2E.24 (a)

Solution: First let us find the optical path difference between the waves interfering at P.

$$S_2P = [(S_1P)^2 + d^2]^{1/2} = (D^2 + d^2)^{1/2} = D\left[1+\frac{d^2}{D^2}\right]^{1/2}$$

Since, $d = \sqrt{2\lambda D/3}$

$\therefore$ $d^2 = 2\lambda D/3$

$\therefore$ $$S_2P = D\left[1+\frac{2\lambda}{3D}\right]^{1/2}$$

$$= D\left[1+\frac{\lambda}{3D}+\ldots\right].$$

[Expanding by binomial theorem, and ignoring higher powers of $\frac{\lambda}{D}$]

$\therefore$ $S_2P - S_1P = \lambda/3.$

$\therefore$ Phase difference $= \frac{2\pi}{3}$

Similarly, $S_3P = [(S_1P)^2 + (2d)^2]^{1/2} = (D^2 + 4d^2)^{1/2}$

Using $d = \sqrt{2\lambda D/3}$, $S_3P - S_1P = 4\lambda/3$

(effectively)

$\therefore$ Corresponding phase difference $= 2\pi/3$.

Applying vector diagram method, [See Fig. 2E.24 (b)] the resultant amplitude $= \sqrt{3}\,A_o$.

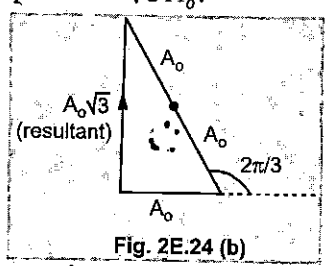


Fig. 2E.24 (b)

$\therefore$ Resultant intensity $I \propto 3A_o^2$

i.e., $I = 3I_o$.

Example 25

In Young's double slit experiment the screen is at a distance D from the plane of the slit and slits are illuminated by plane monochromatic light of wavelength λ. P is a point on the screen at a distance y from the central maximum. If, by some special arrangement, the slits be moved symmetrically apart with a relative velocity v, estimate the number of fringes crossing the point P per unit time.

Solution: Let l be the distance between the two slits, at any instant, then

it is given that $v = \frac{dl}{dt}$. ...(i)

The path difference reaching the point P from the slits is evidently $\Delta = y\frac{l}{D}$.

Differentiating both sides with respect to time t, we get

$$\frac{d\Delta}{dt} = \frac{y}{D}\cdot\frac{dl}{dt} = \frac{yv}{D}$$ ...(ii)

[From eqn. (i)]

Since, a change in optical path difference of λ, corresponds to one fringe, so the number of fringe crossing point P, per unit time is

$$\left(\frac{d\Delta}{dt}\right)\frac{1}{\lambda}=\frac{yv}{\lambda D}.$$

Example 26

Fig. 2E.26 (a) shows an arrangement to observe interference pattern, by two coherent sources S_1 and S_2 separated by a distance 4λ. A screen is placed at a large distance D from the sources normal to the line joining them.

Determine the distance y ($y \neq 0$ or ∞) such that the intensity at point P is same as that at point O.

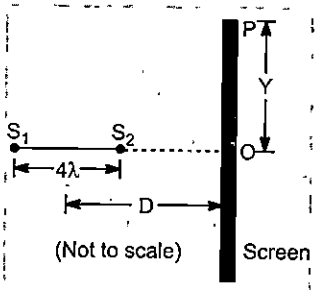


Fig. 2E.26 (a)

Solution: Since $D \gg \lambda$, so D can be taken to be the distance of the screen from the mid-point of the sources S_1 and S_2.

Obviously, the path difference between the waves interfering at point O on the screen is 4λ; hence, point O is point of maximum intensity. [Fig. 2E.26(b)]

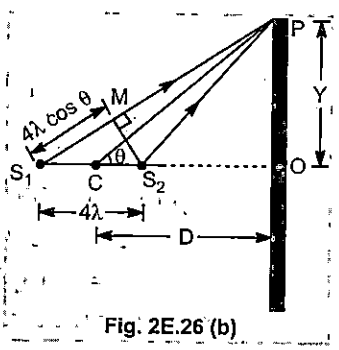


Fig. 2E.26 (b)

Let θ be the angular position of a maximum (point P) with respect to the central line S_1S_2O. The path difference between the interfering waves at P will, evidently be

$$\Delta l = 4\lambda\cos\theta.$$

$(\because \Delta l = S_1P - S_2P = (S_1M + MP - S_2P) = S_1M)$

Since $|\cos\theta| \leq 1$,

$\therefore$ $|\Delta l| \leq 4\lambda$.

Also, for an intensity maximum at point P

$$\Delta l = n\lambda;\ n \in I$$

$\therefore$ $\Delta l = 3\lambda, 2\lambda, \lambda$.

(so that $n < 4$ and $n \neq 4$ for points other than O)

$\therefore$ $4\lambda|\cos\theta| = 3\lambda, 2\lambda, \lambda$

or $|\cos\theta| = \frac{3}{4}, \frac{1}{2}, \frac{1}{4}$.

But $|\cos\theta| = \frac{D}{CP} = \frac{D}{\sqrt{D^2+y^2}}$

$\therefore$ $|\cos\theta|^2 = \frac{D^2}{D^2+y^2} = \frac{9}{16}, \frac{4}{16}, \frac{1}{16}$

or $\frac{D^2+y^2}{D^2} = \frac{16}{9}, \frac{16}{4}, 16$

$\therefore$ $y^2 = \frac{7D^2}{9}, 3D^2, 15D^2$

$\therefore$ $y = \pm\sqrt{\frac{7D}{9}}, \pm\sqrt{3}D$ and $\pm\sqrt{15}D$.

In case of linear slits, the positive and negative values of y respectively stand for points lying above and below O. In case of point sources, the three values of y denote the radii of the circular fringes.

Example 27

Two parallel beams of light P and Q (separation d) containing radiation of wavelength 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in Fig. 2E.27. The refractive index of prism as a function of wavelength is given by the relation

$$\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$$

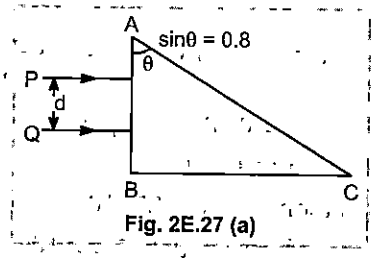


Fig. 2E.27 (a)

where λ is in Å and b is a positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wave and is not satisfied for the other. (a) Find the value of b. (b) Find the deviations of the beam transmitted through the face AC.

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and lower beams immediately, after transmission from the face AC are $4I$ and I respectively, find the resultant intensity at focus.

Solution: (a) At face AB rays pass without deviation. Both the rays are incident at angle θ at the face AC. Critical angles for the two wavelengths are

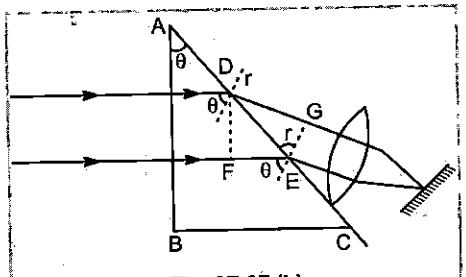


Fig. 2E.27 (b)

$$\sin\theta_C^1 = \frac{1}{\mu_1},\ \sin\theta_C^2 = \frac{1}{\mu_2}$$

where $\mu_1 = 1.2 + \dfrac{b}{(4000)^2}$

$$\mu_2 = 1.2 + \frac{b}{(5000)^2}$$

Since b is a constant, $\mu_1 > \mu_2$; hence

$\sin\theta_C^1 < \sin\theta_C^2$ or $\theta_C^1 < \theta_C^2$

If angle of incidence $\theta = \theta_C^1$, the ray of wavelength λ_1 will graze the surface AC and the ray of wavelength λ_2 will refract at angle less than 90°, because angle of incidence θ is less than critical angle θ_C^2.

Therefore, $\mu_1 = \dfrac{1}{\sin\theta_C^1} = \dfrac{1}{\sin\theta}$

or $\dfrac{1}{0.8} = 1.2 + \dfrac{b}{(4000)^2}$ or $b = 8\times10^5(\text{Å})^2$

(d) Let r be the angle of refraction of ray Q. From Snell's law,

$$\sin r = \mu_2 \sin\theta = 0.8\left[1.2 \times \frac{8\times10^5}{(5000)^2}\right]$$

$$= 0.9856$$

Hence, $r \simeq 80°20'$

Angle of deviation $= r - \theta = 27°10'$

As $\theta = \sin^{-1}(0.8) \simeq 53°10'$

(e) The transmitted beams from P and Q contain wavelength λ_2 only as wavelength λ_1 is totally internally reflected. Thus we have two parallel beams emanating from points P and Q, that are focussed by a convex lens.

The optical path difference between two waves,

$$\Delta x = (DG)_{air} - (EF)_{glass}$$
$$= DE\sin r - \mu d\tan\theta$$
$$= \frac{d\sin r}{\cos\theta} - \mu d\tan\theta$$
$$= \frac{d(\mu\sin\theta)}{\cos\theta} - \mu d\tan\theta$$
$$= 0$$

Therefore, the waves are in same phase.

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= I + 4I + 2\sqrt{(4I)(I)}$$
$$= 9I$$

Example 28

In a modified YDSE the region between screen and slits is immersed in a liquid whose refractive index varies with time as $\mu_l = (5/2) - (T/4)$ until it reaches a steady state value 5/4. A glass plate of thickness 36μm and refractive index 3/2 is introduced infront of one of the slits. (a) Find the position of central maxima as a function of time and the time when it is at point O, located symmetrically on the x-axis. (b) What is the speed of the central maxima, when it is at O ?

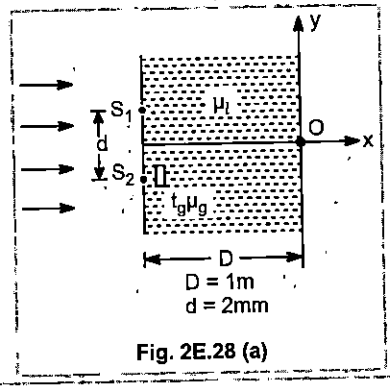


Fig. 2E.28 (a)

Solution: We consider a point P on the screen.

Optical path length,

$$[S_1P]_{liquid} = [\mu_l S_1P]_{air}$$

and $[S_2P - t]_{liquid} + t_{glass} = \mu_l[S_2P - t]_{air} + [\mu_g t]_{air}$

$$= [\mu_l S_2P + (\mu_g - \mu_l)t]_{air}$$

Hence optical path difference at P,

$$\Delta x = [\mu_l S_2P + (\mu_g - \mu_l)t] - \mu_l S_1P$$
$$= \mu_l(S_2P - S_1P) + (\mu_g - \mu_l)t$$

For a point P at the screen in the absence of liquid,

$$S_2P - S_1P = \frac{yd}{D}$$

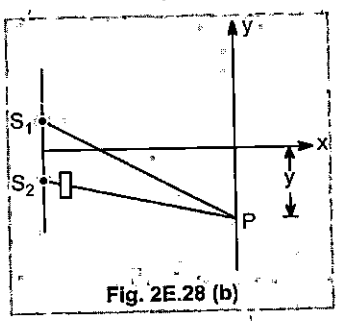


Fig. 2E.28 (b)

If a liquid is filled,

$$[S_2P - S_1P]_{liquid} = \frac{\mu_l yd}{D}$$

Thus $\Delta x = \frac{\mu_l yd}{D} + [\mu_g - \mu_l]t$

(a) For central maxima,

$\Delta x = 0$

$y = -\left[\frac{\mu_g - \mu_l}{\mu_l}\right]\frac{tD}{d}$

$= \frac{(4-T)tD}{(10-T)d}$

At point O, $y = 0$

Thus $T = 4$ sec.

(b) Speed of central maxima,

$v = \left|\frac{dy}{dt}\right| = \frac{6Dt}{(10-T)^2 d}$

Thus $v = \frac{6Dt}{36d} = \frac{1\times 36\times 10^{-6}}{6\times 2\times 10^{-3}} = 3\times 10^{-3}$ m/sec

Example 29

In a modified YDSE the source S of wavelength 5000 Å oscillates about axis of setup according to the equation $y = 0.5\sin\pi t$, where y is in millimetre and t in second. (a) Find the position of maxima as a function of time. (b) At what time t will the intensity at P, a point exactly in front of slit S_1, be maximum for the first time ?

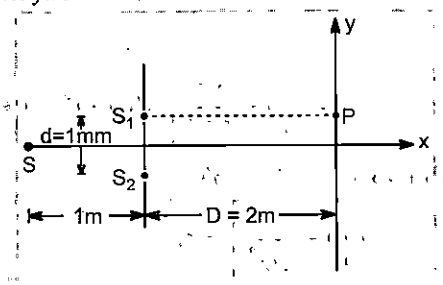


Fig. 2E.29 (a)

Solution: (a) Let the source be at a displacement Y and Q be the position of central maxima. At point Q the waves from S_1 and S_2 arrive in same phase, thus

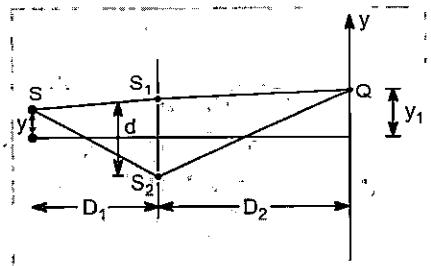


Fig. 2E.29 (b)

$(SS_2 + S_2Q) - (SS_1 + S_1Q) = 0$

or $(SS_2 - SS_1) + (S_2Q - S_1Q) = 0$

or $\frac{dy}{D_1} + \frac{dy_1}{D} = 0$

$y_1 = -\frac{yD_2}{D_1}$

$y_1 = -\frac{(0.5\sin\pi t)(2)}{1} = -(\sin\pi t)$ mm

(b) The path difference at point P,

$\Delta x = (SS_2 - SS_1) + (S_2P - S_1P)$

$= \frac{dy}{D_1} + \frac{d(d/2)}{D_2}$

For constructive interference,

$\Delta x = \frac{dy}{D_1} + \frac{d^2}{2D_2} = n\lambda$

$\frac{(10^{-3})(0.5\sin\pi t)\times 10^{-3}}{1} + \frac{(10^{-3})^2}{2\times 2} = n\lambda$

$(0.5\sin\pi t)\times 10^{-6} + 0.25\times 10^{-6}$

$= (5000\times 10^{-10})n = 0.5\times 10^{-6}n$

$\sin\pi t = \frac{0.5n - 0.25}{0.5}$

For the minimum value of t, $n = 1$.

$\sin\pi t = \frac{1}{2}$

$\pi t = \frac{\pi}{6}$

or $t = \frac{1}{6}$ sec. $= 0.167$ sec.

Example 30

A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of t for which rays interfere constructively.

Solution: R_1 and R_2 are the two rays considered for interference. R_1 is the result of reflection at denser medium; hence it suffers an additional path difference $\lambda/2$ and phase difference π. Ray R_2 originates after reflection at lower surface; this reflection takes place at rarer medium. Net path difference,

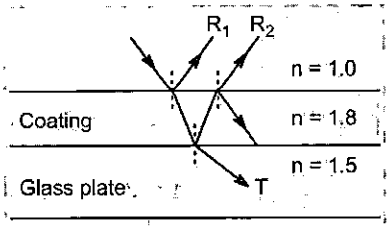


Fig. 2E.30

$$2\mu t + \frac{\lambda}{2} = n\lambda \quad \text{[for constructive interference]}$$

$$2\mu t = \frac{(2n-1)\lambda}{2}$$

t will be minimum for $n = 1$.

$$t_{min} = \frac{\lambda}{4\mu} = \frac{648}{4\times 1.8}\,\text{nm} = 90\,\text{nm}$$

Example 31

In a YDSE using monochromatic visible light the distance between the plane of slits and the screen is 1.7 metre. At a point (P) on the screen which is directly in front of the upper slit, path maximum is observed. Now the screen is moved 50 cm closer to the plane of slits. Point P now lies between third and fourth minima above the central maxima and the intensity at P is one-fourth of the maximum intensity on the screen. Find the :

(a) value of n,

(b) wavelength of light if the separation of slits is 2 mm.

Solution: (a) For point P infront of upper slit, $y = d/2$.

Initially, path difference, $\frac{(d/2)d}{D} = n\lambda$ ...(1)

Intensity at any point on the screen is

$I = I_{max}\left(\frac{1-\cos\phi}{2}\right)$ (ϕ being the phase difference)

When the screen is displaced,

$$I = \frac{I_{max}}{4}$$

Hence, $\cos\phi = \frac{1}{2}$ ...(2)

Also as P lies between third and fourth minima,

$5\pi < \phi < 7\pi$ ...(3)

From eqns. (2) and (3), we get

$$\phi = \frac{17\pi}{3} \text{ or } \frac{19\pi}{3}$$

But $\phi = \frac{(d/2)d}{D'}\frac{2\pi}{\lambda} = \frac{(n\lambda D)}{D'}\frac{2\pi}{\lambda} = \frac{2n\pi D}{D'}$

[using eqn. (1)]

$= \frac{17n\pi}{6}$ (putting the values of D and D')

$\therefore$ $n = 2$ or $(38/17)$. (but n is an integer)

$n = 2$ is the only valid answer.

(b) Putting the value of n and d in eqn. (1), we get

$$\lambda = 5.9\times 10^{-7}\,\text{m}$$

Example 32

In Fig. 2E.32 shown, a screen is placed normal to the line joining the two point coherent sources S_1 and S_2. The interference pattern consists of concentric circles.

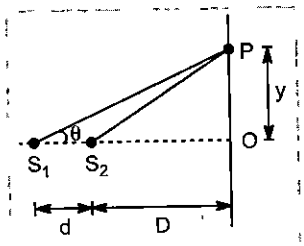


Fig. 2E.32

(a) Find the radius of the nth bright ring.

(b) If $d = 0.5$ mm, $\lambda = 5000\,\text{Å}$ and $D = 100$ cm, find the radius of the closest second bright ring.

(c) Also, find the value of n for this ring.

Solution: The optical path difference at P is

$$\Delta x = S_1P - S_2P = d\cos\theta$$

Since $\cos\theta \simeq 1 - \frac{\theta^2}{2}$ for small θ,

Therefore, $\Delta x = d\left(1-\frac{\theta^2}{2}\right)$

$= d\left[1-\frac{y^2}{2D^2}\right]$ where $D + d \approx D$

For nth maxima, $\Delta x = n\lambda$

or $d\left[1-\frac{y^2}{2D^2}\right] = n\lambda$

(a) y = radius of the nth bright ring

$= D\sqrt{2\left(1-\frac{n\lambda}{d}\right)}$

(b) At the central maxima, $\theta = 0$.

$\Delta x = d = n\lambda$

or $n = \frac{d}{\lambda} = \frac{0.5}{0.5\times 10^{-3}} = 1000$

$\therefore$ for the closest second bright ring, $n = 998$.

(c) Putting values, $r = 6.32$ cm.

Example 33

A narrow slit S allows monochromatic light of wavelength $\lambda = 4000$ Å to fall on a prism of very small angle as shown in the figure. A screen is placed at a distance $l = 150$ cm from the source to obtain an interference pattern. To determine the distance between the virtual images formed by the prism an experiment is done. The prism and screen are kept fixed and a converging lens is moved between the prism and the screen. For two positions of the lens (between the prism and the screen) we get two sharp point images on the screen in each case. The images are separated from each other by distance 4.5 mm in one case and 2 mm in the other. Now the lens is removed and interference pattern is obtained on the screen. Calculate:

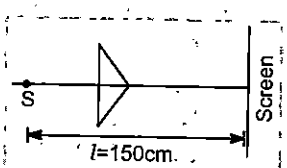


Fig. 2E.33 (a)

(i) The fringe width of the pattern on the screen.

(ii) Focal length of the lens.

Solution: $I_1 = 4.5$ mm

$I_2 = 2$ mm

$O = \sqrt{I_1 I_2} = 3\text{mm} = d$

$$\beta = \frac{\lambda D}{d} = \frac{4\times10^{-7}\times1.5}{3\times10^{-3}}$$

$$= 2\times10^{-4} = 0.2 \text{ mm}$$

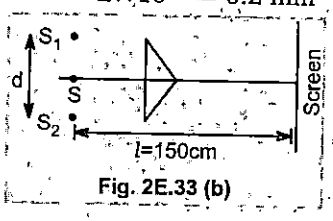


Fig. 2E.33 (b)

(ii) $$|\mu_1| = \frac{v}{150 - v} = \frac{4.5}{3} = 1.5$$

$\Rightarrow$ $v = 225 - 1.5v$

$\Rightarrow$ $$v = \frac{225}{2.5} = 90 \text{ cm}$$

$\Rightarrow$ $u = 60$ cm

$$\frac{1}{f} = \frac{1}{90} + \frac{1}{60} = \frac{2+3}{180} \Rightarrow f = \frac{180}{5} = 36\text{cm}$$

Example 34

Two radio transmitter each transmitting a radio of frequency 12 MHz in phase with each other are placed at a distance $l = 100$ m on either side of the runways on the towers as shown in the figure. An aircraft with a ground speed of v is flying towards the airport such that its velocity makes an angle θ with the runway. The aircraft is heading directly for the mid-point of the towers.

(i) At the aircraft's current position, the intensity of the signal from each tower separately would be I_0. Find the intensity of the combined signal from both towers received by the aircraft for heading of $\theta = 0$ and $\theta = \pi/2$.

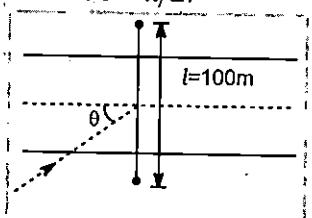


Fig. 2E.34 (a)

(ii) For what heading would the aircraft receive no signal. Assuming distance of plane much greater than l.

(iii) If the aircraft is heading from large distance directly towards on of the transmitter in the direction perpendicular to 'l', find the position of the aircraft from the nearest tower when the signal dropped to a minimum first time.

Solution:

(i) For $\theta = 0$, $\Delta x = 0$

For $\theta = \pi/2$

$\Delta x = 100\text{m} = 4\lambda$

$\{\lambda = 25\text{m} ; v = \lambda f\}$

$\therefore$ $I = 4I_0$

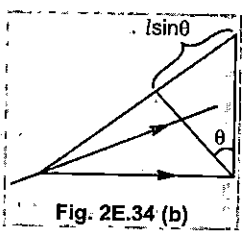


Fig. 2E.34 (b)

(ii) Assuming aircraft for from poles.

For minima

$\Rightarrow$ $$l\sin\theta = (2n+1)\frac{\lambda}{2}$$

$\Rightarrow$ $$\sin\theta = \frac{(2n+1)}{8};\ n = 0,\ 1,\ 2,\ 3$$

(iii) $\Delta x = \sqrt{D^2 + x^2} - x$

For minima first time

$\Rightarrow$ $\sqrt{D^2 + x^2} - x = \lambda/2$

$\therefore$ $x = 393.75$m

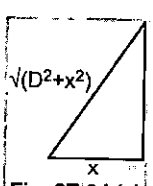


Fig. 2E.34 (c)

Example 35

In the figure shown A, B and C are three slits each of them individually producing the same intensity I_0 at P_0 when the are illuminated by parallel beam of light of wavelength 'λ'. It is given that $BP_0 - AP_0 = \frac{\lambda}{2}$. Also given that $d << D$. Find:

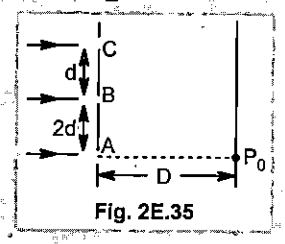


Fig. 2E.35

(i) λ is terms of d and D

(ii) Resultant intensity at P_0.

Solution: (i) $2d \sin\theta = \frac{\lambda}{2}$

$$2d \times \frac{d}{D} = \frac{\lambda}{2}$$

$$\lambda = \frac{4d^2}{D}$$

(ii) Since $BP_0 - AP_0 = \frac{\lambda}{2}$

∴ Waves from A and B cancel

∴ Resultant intensity = I_0

Example 36

A convex lens of focal length 50 cm is cut along the diameter into two identical halves A and B and in the process a layer C of the lens thickness 1 mm is lost. Then the two halves A and B are put together to form a composite lens. Now in front of this composite lens a source of light emitting wavelength $\lambda = 6000 Å$ is placed at a distance of 25 cm as shown in the figure. Behind the lens there is a screen at a distance 50 cm from it. Find the fringe width of the interference pattern obtained on the screen.

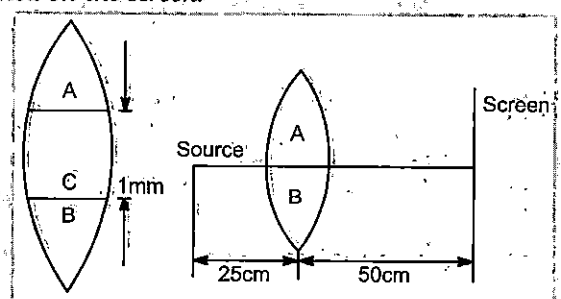


Fig. 2E.36 (a)

Solution: $u = -25$

$$\frac{1}{v} - \frac{1}{-25} = \frac{1}{50}$$

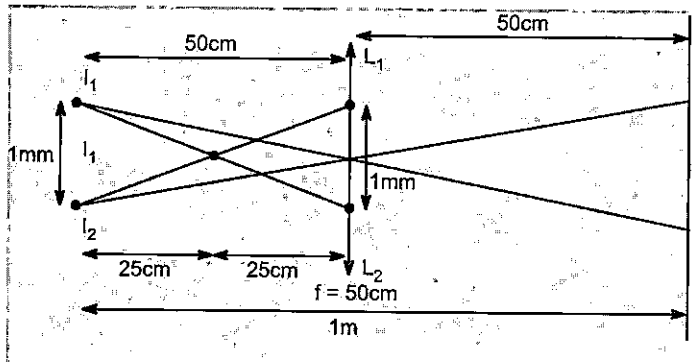


Fig. 2E.36 (b)

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{50} \Rightarrow v = -50$$

$$\beta = \frac{6 \times 10^{-7} \times 1}{10^{-3}}$$

$$= 6 \times 10^{-4} = 0.6 \text{ mm}$$

Example 37

(a) Two flat mirrors form an angle close to 180°. A source of light S is placed at equal distances 'a' from the mirrors. Find the fringe width on a screen placed at a distance b from the point of intersection of the mirrors. A shield C does not allow the light to pass directly on to the screen. The wavelength of light wave is λ and α is the acute angle between the mirrors.

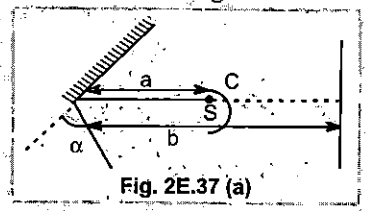


Fig. 2E.37 (a)

(b) Two point coherent sources are on a straight line $d = n\lambda$ apart. The distance of a screen perpendicular to the line of the sources is $D (>> d)$ from the nearest source. Calculate the width of the central maxima.

Solution: (a) The fringe width is given by $\beta = \frac{D\lambda}{d}$

Here $D = a + b$

$d = 2a\alpha$

(The reflected ray rotates by 2α, when the mirror rotates by α.)

$$\Rightarrow \quad \beta = \frac{\lambda . (a+b)}{2a.\alpha}$$

(b) Consider any point P on the screen at a distance y from O. Then $D_1^2 = D^2 + y^2$

$$\Rightarrow \quad D_1 = D\left[1+\frac{y^2}{D^2}\right]^{\frac{1}{2}}$$

$$= D\left[1+\frac{y^2}{2D^2}\right] = D+\frac{y^2}{2D}$$

Similarly, $D_2 = (D+d)+\frac{y^2}{2(D+d)}$

$$D_2 - D_1 = d - \frac{d\cdot y^2}{2D[D+d]}$$

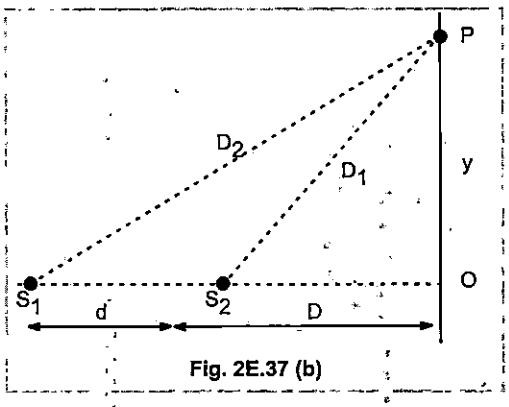


Fig. 2E.37 (b)

For the point $O, D_2 - D_1 = d = n\lambda$ (given)

Thus there is brightness at O of n^{th} order. Since the path difference decreases, the adjacent dark fringe or minima will be at path difference of $(n-1/2)\lambda$

$$\Rightarrow \quad D_2 - D_1 = \left(n-\frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad d - \frac{d\cdot y^2}{2[D(D+d)]} = \left(n-\frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad n\lambda\left[1-\frac{y^2}{2D(D+d)}\right] = n\lambda\left[1-\frac{1}{2n}\right]$$

$$\Rightarrow \quad \frac{y^2}{2D(D+d)} = \frac{1}{2n}$$

$$\Rightarrow \quad y = \left[\frac{D(D+d)}{n}\right]^{\frac{1}{2}}$$

Width of central maxima

$$= 2y = 2\left[\frac{D(D+d)}{n}\right]^{\frac{1}{2}}$$

Example 38

In the Young's double slit experiment light of wavelength 400 nm falls normal to the plane of the slits. Interference pattern is observed on a large screen placed at a distance D parallel to the plane of the slits. When a transparent strip of thickness 4 μm and refractive index μ_1 is placed in front of one of the slits then central maxima occupies the position of 12th maxima prior to the introduction of the strip. When another strip of same thickness and refractive index μ_2 is placed in front of other slit the central maxima comes to the position of 5th maxima on the same side of the 12th maxima prior to the introduction of the strips. Now the light is replaced by another light of wavelength λ. It is found that central maxima is formed at a distance 2 cm (which is four times the new fringe width) from the centre of the screen. It is given that separation of the slits is 1 mm. Find the value of μ_1, μ_2, λ and separation between the plate of the slits and the screen D.

Solution: When the first strip is placed in front of the one of the slits no. of fringes that will shift

$$= \frac{(\mu_1 - 1)t}{\lambda} = 12 \text{ (given)}$$

$$\Rightarrow \quad \mu_1 - 1 = \frac{12\lambda}{t} = \frac{12\times 400\times 10^{-9}}{4\times 10^{-6}} = 1.2$$

$$\Rightarrow \quad \mu_1 = 2.2$$

When the second strip is placed infront of the another slit, number of fringes that will shift

$$= \frac{(\mu_1 - \mu_2)t}{\lambda} = 5 \text{ (given)}$$

$$\Rightarrow \quad \mu_1 - \mu_2 = \frac{5\times 400\times 10^{-9}}{4\times 10^{-6}} = 0.5$$

$$\Rightarrow \quad \mu_2 = 1.7$$

Now the present light is replaced by another light of wavelength λ then the position of central maximum from the centre of screen will be

$$\frac{D}{d}(\mu_1 - \mu_2)t = 2\times 10^{-3}\text{ m (given)}$$

$$\Rightarrow \quad D = \frac{2\times 10^{-3}\times 1\times 10^{-3}}{0.5\times 4\times 10^{-6}} = 1\text{ m}$$

Since, it is given that four time of the new fringe width is equal to 2 cm

$$\Rightarrow \quad \frac{4\lambda_2 D}{d} = 2\times 10^{-3}$$

$$\Rightarrow \quad \lambda_2 = \frac{2\times 10^{-3}\times 1\times 10^{-3}}{4\times 1}$$

$$= 5\times 10^{-7}\text{ m} = 500\text{ nm}$$

Example 39

The figure shows an Young's double slit experiment with parallel wavefronts inclined at an angle ϕ to the plane of the slits.

(a) Find the position of central maxima.

(b) Locate the region in which fringe pattern is observed

(c) Find the number of fringe widths.

(d) If the central maximum is to be shifted at the origin by introducing a thin film of refractive index μ in front of any one of the two slits. What should be its thickness and where should it be placed.

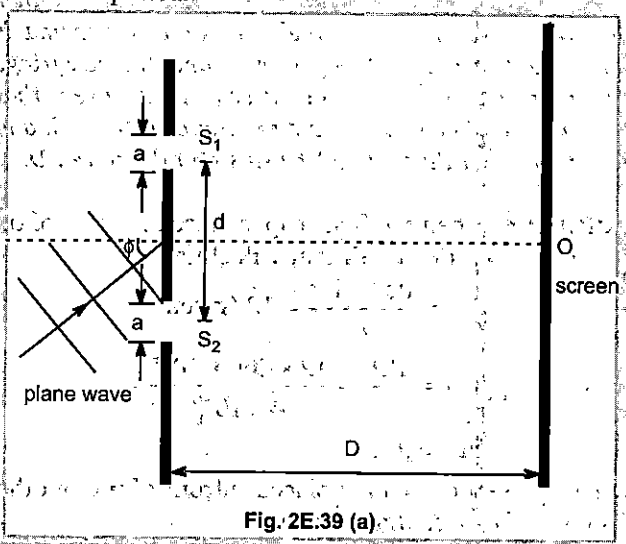


Fig. 2E.39 (a)

Solution: Let n the maxima occurs at the point P such that optical path difference $= d\sin\theta - d\sin\phi = n\lambda$

Since θ and ϕ are small, therefore

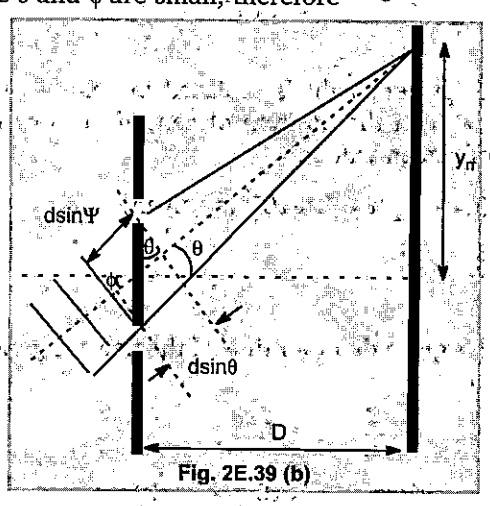


Fig. 2E.39 (b)

$$\sin\theta \approx \tan\theta = \frac{y_a}{D} \text{ and } \sin\phi = \phi$$

Now, $$\frac{y_a}{D} - \phi d = n\lambda$$

or $$y_a = \frac{n\lambda D}{d} - \phi D$$

(a) Position of the central maxima is given by

$$y_0 = \phi D \qquad (n = 0)$$

(b) In case of Fraunhoffere diffraction, the half angular width of the central maxima is given by $\alpha = \frac{\lambda}{a}$ where a = slit width

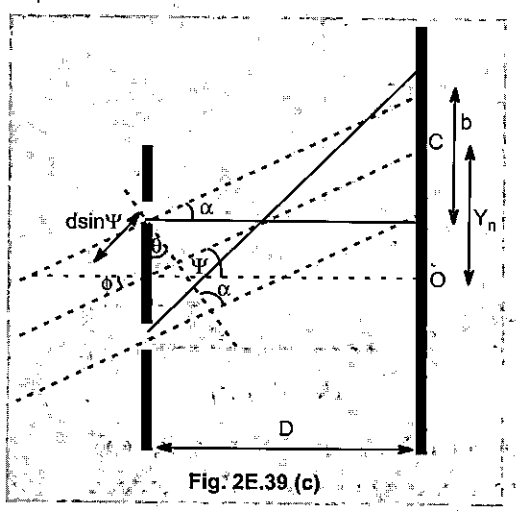


Fig. 2E.39 (c)

Fringe pattern is observed in the region of width, as shown in figure.

$$b = \frac{2\lambda D}{d} = d$$

The region is symmetrically located about the central maxima. The ordinates of the lower and upper fringes are

$$y_1 = y_0 - \frac{b}{2}$$

$$y_1 = y_0 + \frac{b}{2}$$

(c) Number of fringe widths are $N = \frac{b}{w} = \frac{bd}{\lambda D}$

(d) the film should be placed in front of the lower slit.

$$\text{o.p.d.} = d\sin\theta + (\mu - 1)t\frac{D}{d}$$

Since, central maxima shifts to the origin, therefore,

$$(\mu - 1)t\frac{D}{d} = y_0$$

or $$t = \frac{y_0 d}{(\mu - 1)D}$$

Example 40

(a) A thin glass plate of thickness t and refractive index μ is inserted between one of the slits in a Young's experiment. If the intensity at the centre of the screen is I, what was the intensity at the same point prior to the introduction of the sheet.

(b) One slit of a Young's experiment is covered by a glass plate ($\mu = 1.40$) and the other by another glass plane ($\mu_2 = 1.7$) of the same thickness. The point of central maxima on the screen, before the plates were introduced is now occupied by the third bright fringe. Find the thickness of the plates, when the wavelength of the light used is 4000Å.

Solution: (a) Let the original intensity prior to the introduction of the glass plate = I_0.

The path difference introduced by the glass plate $(\mu - 1)t$

(δ) phase difference = $\frac{2\pi}{\lambda}(\mu - 1)t$

$$I = 4I_0 \cos^2\left[\frac{\delta}{2}\right] = 4I_0 \cos^2\left[\frac{\pi[\mu - 1]t}{\lambda}\right]$$

$$\Rightarrow \quad I_0 = \frac{1}{4}\sec^2\left[\frac{\pi[\mu - 1]\cdot t}{\lambda}\right]$$

(b)

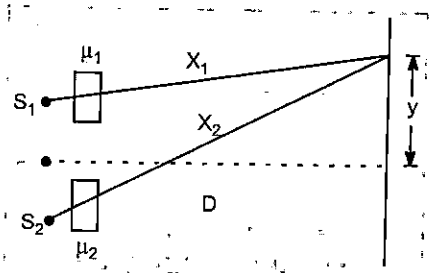


Fig. 2E.40

Optical path difference

$$[(S_2P - t) + \mu_2 t] - [(S_1P - t) + \mu_1 t]$$
$$= S_2P - S_1P + (\mu_2 - \mu_1)t$$
$$= \frac{y \cdot d}{D} + (\mu_2 - \mu_1)\cdot t$$

Let P be the point where optical path difference is zero

$$\Rightarrow \quad \frac{y \cdot d}{D} + (\mu_2 - \mu_1)\cdot t$$

$$\Rightarrow \quad y = \frac{D\cdot(\mu_1 - \mu_2)\cdot t}{d}$$

But $y = 3\omega$ (where ω is the fringe width)

$$y = 3\frac{D\lambda}{2} \Rightarrow 3\frac{D\lambda}{d} = \frac{D}{d}(\mu_1 - \mu_2)\cdot t$$

$$\Rightarrow t = \frac{3\lambda}{(\mu_1 - \mu_2)} = \frac{3\times 4000\times 10^{-10}}{(1.7 - 1.4)} = 4\mu m$$

Example 41

A monochromatic light of $\lambda = 5000$Å is incident on the slits separated by a distance of 5×10^{-4} m. The interference pattern is seen on a screen placed at a distance 1 meter from the slit. A thin glass plate of thickness 3×10^{-6}m and of refractive index 1.5 is placed between one of the slits and screen. Find intensity at the centre of the screen if the intensity there is I_0 in absence of plate.

Solution: Path difference due to introduction of thin glass plate

$$x = (\mu - 1)t$$

$$\phi = \frac{2\pi}{\lambda}\times 1.5\times 10^6 = 6\pi$$

$$I' = \sqrt{I_1} + \sqrt{I_2} + 2\sqrt{I_1 I_2}\cos 6\pi = 4I = I_0, \text{ here}$$

$$I_1 = I_2 = 1$$

$$I_0 = 4I \text{ (given)}$$

$$I' = 4I = I_0$$

i.e., the intensity remains I_0.

Example 42

In the figure shown S is a narrow slit illuminated by a monochromatic light with wavelength $\lambda = 589$nm. A and B are identical tubes with air length $l = 10$cm each. D is a diaphragm with two slits. After the air in tube A was replaced with another gas, the interference pattern on the screen was displaced by $N = 20$ fringes. The refractive index of air is equal to $n = 1.00002777$. Determine the refractive index of the other gas.

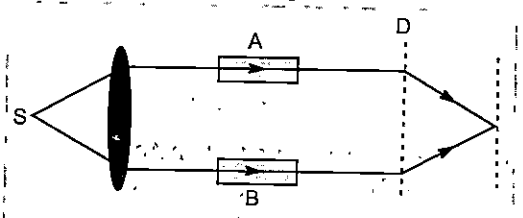


Fig. 2E.42

Solution: Shift in figure pattern = $\frac{D}{d}t(\mu_A - \mu_B)$

β = fringe width

$$= \frac{\beta t}{\lambda}(\mu_A - \mu_B) = N\beta$$

$$\therefore \quad \mu_A - \mu_B = \frac{N\lambda}{t} \quad \Rightarrow \quad \mu_A - \mu_B + \frac{N\lambda}{t}$$

Given $\mu_B = 1.00002777$, $N = 20$, $\lambda = 589\times10^{-9}$m, $l = 10\times10^{-2}$

$$\therefore \quad \mu_A = 1.00002777 + 0.0001178 = 1.00014557$$

Example 43

Light of wavelength $\lambda = 5000\,nm$ falls on two narrow slits placed a distance $d = 50\times10^{-4}\,cm$ apart, at an angle $\phi = 30°$ relative to the slits shown in figure. On the lower slit a transparent slab of thickness 0.1 mm and refractive index 3/2 is placed. The interference pattern observed on screen at a distance $D = 2m$ from the slits. Then

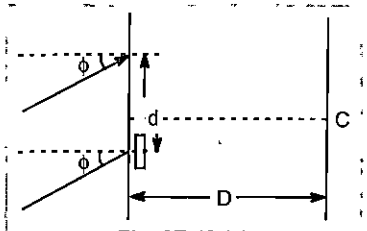


Fig. 2E.43 (a)

Calculate

(a) Position of the central maxima?

(b) The order of minima closest to centre C of screen?

(c) How many fringes will pass over C, if we remove the transparent slab from the lower slit?

Solution: Phase difference ϕ *kds* in

$$\phi + kd\sin\theta - kb(1-m)$$

where $k = \dfrac{2\pi}{\lambda}$

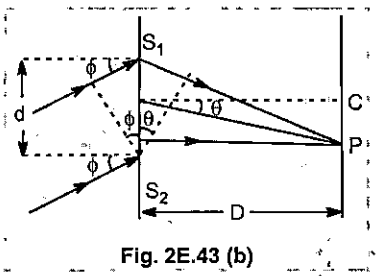


Fig. 2E.43 (b)

Central maxima is obtained when $\Delta\phi = 0$

$$kd\sin\phi + kd\sin\theta - kb(1-n) = 0$$

$$\sin\theta = \frac{b(n-1)}{d} - \sin\phi$$

$$= \frac{1}{2} \Rightarrow \text{therefore } \theta = 30°$$

At $c, \theta = 0\ \Delta\phi = kd\sin\theta - kb(n-1) = \dfrac{kd}{2}\left[1 - \dfrac{2d}{d}(n-1)\right]$

$$= \frac{2\pi d}{2\lambda}\left[1 - \frac{2b}{d}(n-1)\right]$$

Substituting these values, we get,

$$\Delta\phi = -100\pi = 2m\pi \Rightarrow m = -50$$

As $I \propto \cos^2\left(\dfrac{\Delta\phi}{2}\right)$

Hence at C there will be maxima

For, minima $\dfrac{\Delta\phi}{2} = (2m-1)\pi/2$

or $\Delta\phi = (2m-1)\pi$

for $m = -50$ and -49 we get $\Delta\phi = -101\pi$ and -99π respectively

Hence the minima closest to $\theta = 0$ are of order $m = -49$ and -50 on both side

The phase difference at C when we remove the slab

$$\Delta\phi = 100\pi \quad i.e. \quad m = 50$$

Hence number of fringes passed over = 150

Example 44

A double slit is illuminated normally with coherent light. The interference pattern is observed on a screen. The apparatus is shown.

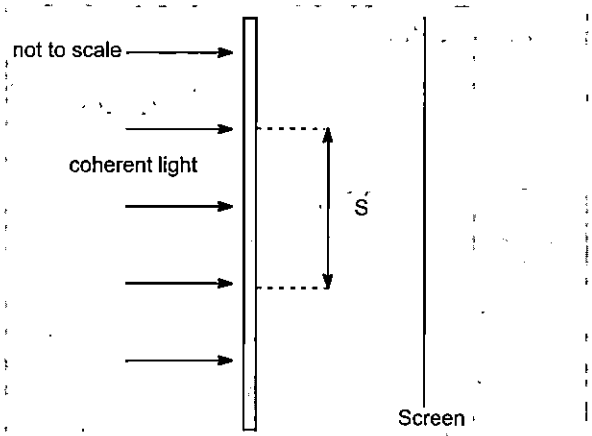


Fig. 2E.44 (a)

The width of both slits in the double slit arrangement is increased without altering the separation s.

Describe and explain the effect, if any, of this change on

(a) The fringe width.

(b) The intensity of the fringes.

(c) A Young's double slit experiment for red light is set up as shown.

An interference pattern of light and dark fringes is observed on the screen.

(i) The red filter is now replaced by a blue filter. State and explain the change in appearance, other than change of colour, of the fringes on the screen.

(ii) The filter in (i) is removed. Describe the pattern on screen.

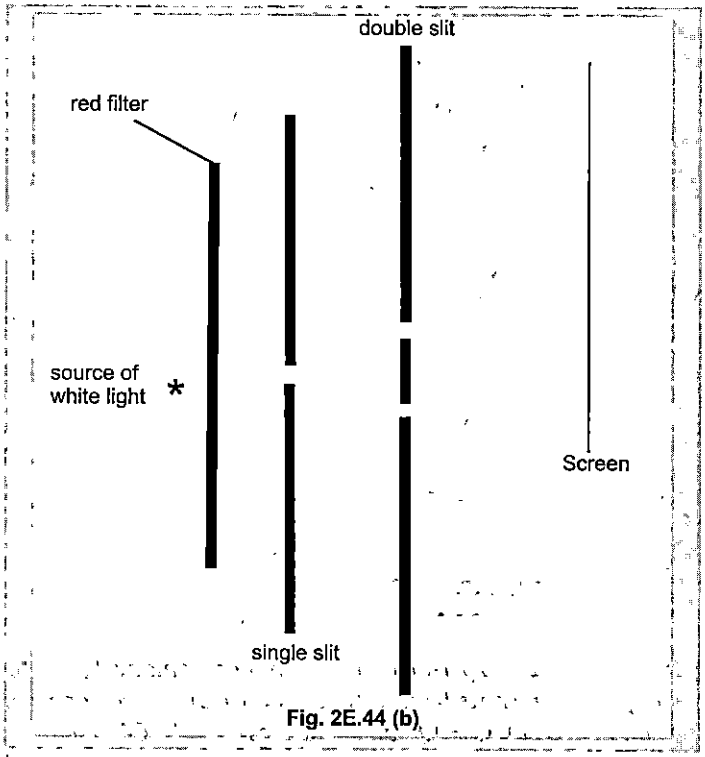


Fig. 2E.44 (b)

Solution: (a) There is no change in the fringe width.

(b) Increases but overlapping increases and interference pattern clarity decreases.

(c) (i) Fringes width decreases $\left(B = \frac{\lambda D}{d}\right)$ and no.of fringes ↑.

(ii) Only central maxima is of white colour. Other parts are mixture of different colours maximum and minimas.

Example 45

Three sources S_1, S_2 and S_3 emitting light of the same amplitude and wavelength λ and in the same phase are kept as shown; the source S_1 at $\left(0, \frac{d}{2}\right)$, S_2 at $\left(0, -\frac{d}{2}\right)$ and S_3 at $\left(0, -\frac{d}{2}d\right)$. A large screen is placed at $x = D$, perpendicular to the x-axis. Assuming $d << D$, find the intensity at a point $P(D, y)$ on the screen, if $y << D$.

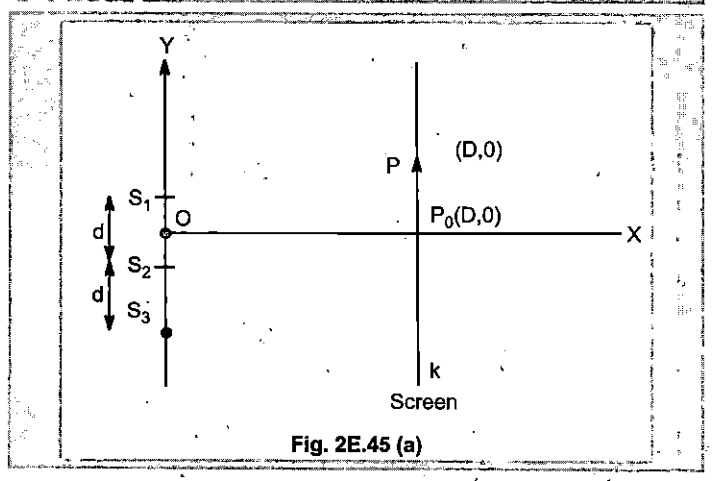


Fig. 2E.45 (a)

Solution: The path difference $S_2P - OP = \frac{d}{2}\sin\theta$,

$S_1P - OP = -\frac{d}{2}\sin\theta$, $S_3P - OP = \frac{3d}{2}\sin\theta$

The relative phases of the waves from S_1, S_2, S_3 when they interfere at P are

$$\phi_1 = -\frac{\pi d\sin\theta}{\lambda}, \phi_2 = +\frac{\pi d\sin\theta}{\lambda}, \phi_3 = \frac{4\pi d\sin\theta}{\lambda}$$

Since we calculate relative phases only, we can add

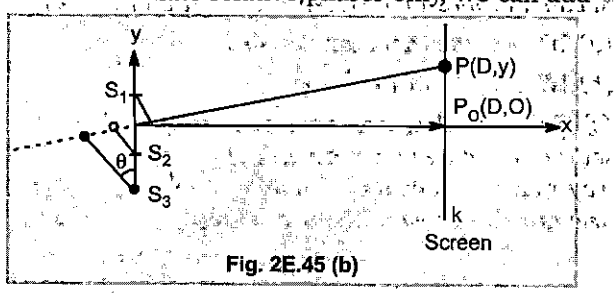


Fig. 2E.45 (b)

$\phi = +\frac{\pi d\sin\theta}{\lambda}$ to ϕ_1, ϕ_2, ϕ_3 to get

$$\phi_1' = 0, \phi_2' = \frac{2\pi\sin\theta}{\lambda}, \phi_3' = +\frac{4\pi\sin\theta}{\lambda}$$

In order to add the vibrations we use the phasor method, net amplitude at P,

$$A_p = A + 2A\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right) = A(1+\cos\phi)$$

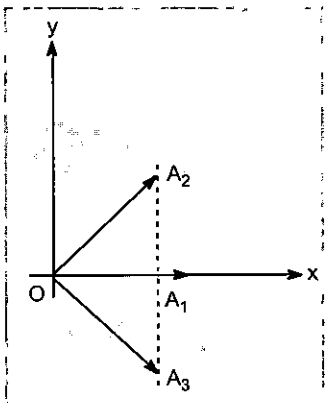


Fig. 2E.45 (c)

where $\phi = \dfrac{2\pi d \sin\theta}{\lambda}$

$\therefore \quad I_p = (1+2\cos\phi)^2 I_0$

Example 46

Three radio sources of equal power and same phase are positioned collinearly as shown in the figure. The frequency of each source is 10^8 Hz and the distance between two adjacent source is $d = 1$ km. The distance between two adjacent maxima formed on a screen near O at a distance D is 10 km.

(a) What is the radiation wavelength?

(b) What is the distance D?

(c) The central source stops radiating, and the two other sources are moved (symmetrically) to new positions in such a way that the maximum points are unchanged. What is the new distance between the two sources ?

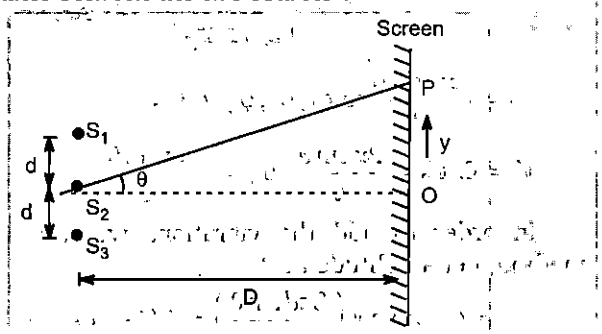


Fig. 2E.46

Solution: (a) Equation of the three waves arriving at *P* are

$$y_1 = a\sin(kx - \phi)$$

$$y_2 = a\sin kx$$

$$y_3 = a\sin(kx + \phi) \text{ where } \phi = kd\sin\theta$$

The resultant amplitude of the three waves is

$$y = y_1 + y_2 + y_3$$

$$A = a[1 + 2\cos(kd\sin\theta)]$$

If $I \propto A^2$ and $I_0 \propto a^2$, then

$$I = I_0[1 + 2\cos(kd\sin\theta)]^2$$

For maxima $\cos(Kd\sin\theta) = 1$

or $kd\sin\theta = 2n\pi$

$$\sin\theta = \frac{2n\pi}{kd} = \frac{n\lambda}{d}$$

Since $\tan\theta \approx \sin\theta \approx \dfrac{y_n}{D}$, therefore,

$$y_n = \frac{n\lambda D}{d}$$

Fringe width $\omega = \dfrac{\lambda D}{d}$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^8} = 3\,\text{m}$$

Example 47

In the Young's experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a micasheet of $\mu = 1.6$ and thickness $t = 1.964$ micron is introduced in the path of one of the slits. The mica sheet is now removed and the distance between the slits and screen is doubled. It is found that the distance between successive maxima (or minima) is the same as the observed fringe shift upon the introduction of mica. Calculate λ.

Solution: Shift produced due to introduction of mica sheet

$$\Delta = \frac{(\mu-1)tD}{d}$$

Fringe width when screen is displaced

$$\beta = \frac{\lambda(2D)}{d}$$

$$\frac{(\mu-1)tD}{d} = \frac{\lambda(2D)}{d}$$

or $\lambda = \dfrac{(\mu-1)t}{2} = \dfrac{(1.6-1)\times1.964\times10^{-6}}{2} = 5892\,\text{Å}$

Example 48

A cylindrical shell of radius 1 m has two slits S_1 and S_2 separated by a distance of 1 mm. Light having a wavelength $\lambda = 4000$ Å is incident on the double slit and produces a fringe pattern within the shell. Assume that the intensity does not very substantially as one moves from O to P.

(a) Find the fringe width of the pattern at the point O.

(b) How does the fringe width vary as one moves to an angular position P, such that $\angle PCO = 60°$?

(c) Find the total number of fringes in the angular range of 60° (region OP).

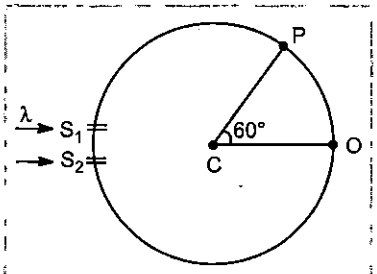


Fig. 2E.48 (a)

Solution: The distance $S_1S_2 = d = 1\,\text{mm}$

The path difference,

$$S_2P - S_1P = d\sin\theta, 2\theta = \phi$$

For the n^{th} maximum at P,

$$d\sin\theta_n = n\lambda \quad \text{or} \quad d\sin\frac{\phi_n}{2} = n\lambda \qquad ...(1)$$

At the point P, the angular fringe width $\delta\phi$ is given by

$$d\sin\frac{\phi_{n+1}}{2} - d\sin\frac{\phi_n}{2} = \lambda$$

or
$$\frac{d}{2}\cos\frac{\phi_n}{2}\delta\phi = \lambda,\ \delta\phi = \frac{2\lambda}{d\cos\left(\frac{\phi_n}{2}\right)} \qquad ...(2)$$

At the point P, the angular fringe width $\delta\phi$ is given by:

$$d\sin\frac{\phi_{n+1}}{2} - d\sin\frac{\phi_n}{2} = \lambda \quad \text{or} \quad \frac{d}{2}\cos\frac{\phi_n}{2}\delta\phi = \lambda$$

or
$$\delta\phi = \frac{\lambda}{d\cos\left(\frac{\phi_n}{2}\right)} \qquad ...(3)$$

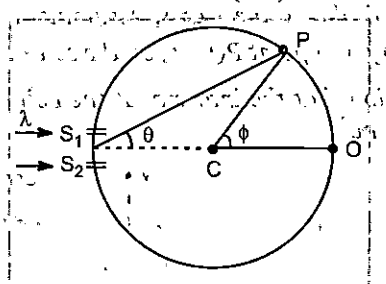


Fig. 2E.48 (b)

At the point O, the linear width $W_0 = \frac{2\lambda R}{d}$

and at P, $W_p = \frac{2\lambda R}{d\cos\left(\frac{\phi}{2}\right)}$ ...(4)

Putting the values, $W_0 = 0.8\,\text{mm}$, $W_p = 0.92\,\text{mm}$

Number of fringes = 1250 [from eqn. (1)]

Example 49

A Young's double slit experimental setup shown in the figure consists of two slits S_1 and S_2 separated by a small distance 'd' and a screen of mass 'm' at distance D from the plane containing the slits. A thin glass plate of thickness 'K' and refractive index 'n' is placed in front of one of the slits.

(a) If light of intensity I is emitted by the slits and the glass plate allows only 64% of the energy to pass through, find the ratio of the intensity of the central maxima and the first minima on the screen.

(b) If a force $F = Mgt/t_0$ is applied on the screen in the direction as shown in the figure, draw the graph of the acceleration of central maxima with respect to time.

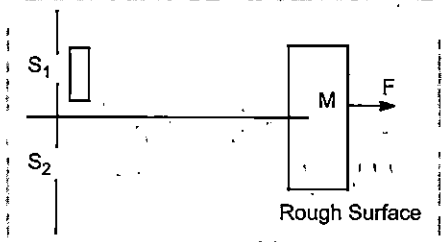


Fig. 2E.49 (a)

Solution: (a) $I_1 = I,\ I_2 = 0.641$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos 0°$$
$$= 1 + 0.641 + 2I\sqrt{0.64}$$
$$= 1.641 + 1.61 = 3.24I$$
$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1I_2}$$
$$= 1 + 0.64I - 2I\sqrt{0.64} = 0.04I$$
$$\frac{I_{\max}}{I_{\min}} = \frac{3.24}{0.04} = 81$$

(b) Height of central maxima is given by

$$y = \frac{D}{d}K(\mu - 1)$$

$$\frac{dy}{dt} = \frac{K}{d}(\mu - 1)\frac{d(D)}{dt} \text{ and } \frac{d^2y}{dt^2} = \frac{K(\mu - 1)}{d}\frac{d^2(D)}{dt^2}$$

Now $\frac{d^2D}{dt^2} = 0$, for $t \le t_0$

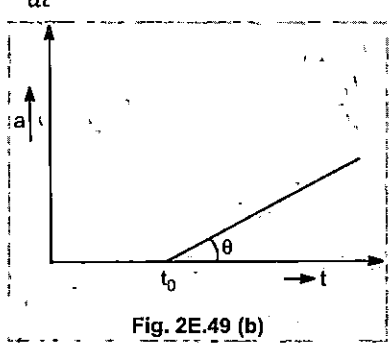


Fig. 2E.49 (b)

$$\frac{d^2(D)}{dt^2} = g\left[\frac{t}{t_0} - \mu\right], \text{ for } t \geq \mu t_0$$

$$\therefore \quad \frac{d^2y}{dt^2} = 0, \text{ for } t \leq t_0$$

$$\frac{d^2y}{dt^2} = \frac{K}{d}(\mu - 1)\left[\frac{t}{t_0} - \mu\right]g$$

$$\tan\theta = \frac{K(\mu-1)}{d}\frac{g}{t_0}$$

Example 50

In the arrangement shown in Fig. 2E.50, light of wavelength 6000 Å is incident on slits S_1 and S_2. Slits S_3 and S_4 have been opened such that S_3 is the position of first maximum above the central maximum and S_4 is the closest position, where intensity is same as that of the light used, below the central maximum. The point O is equidistant from S_1 and S_2 and O′ is equidistant from S_3 and S_4. The intensity of incident light is I_0.

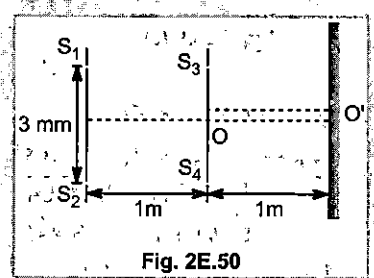


Fig. 2E.50

Find :
(a) the intensity at O′ (on the screen) and
(b) the intensity of the brightest fringe.

Solution: From the given condition,

$$OS_3 = \frac{D\lambda}{d} = \frac{1\times 6\times 10^{-7}}{3\times 10^{-3}} = 2\times 10^{-4}\,\text{m}$$

Let light reaching from S_1 and S_2 to S_4 has phase difference ϕ and intensity of incident light is I_0.

Resultant intensity at S_4, $I = 4I_0\cos^2\frac{\phi}{2}$

As $I = I_0$,

Hence $\frac{I_0}{4I_0} = \cos^2\frac{\phi}{2}$ or $\cos\frac{\phi}{2} = \frac{1}{2} = \cos 60°$

or $\phi = \frac{2\pi}{3}$

For $\phi = \frac{2\pi}{3}$, $OS_4 = \frac{D\lambda}{3d}$

Therefore, $S_3S_4 = OS_3 + OS_4 = \frac{4}{3}\frac{d\lambda}{d} = \frac{8}{3}\times 10^{-4}\,\text{m}$

Now resultant wave coming out of S_3 has intensity $4I_0$ and waves coming out of S_4 have intensity I_0.

(a) Phase difference at $S_3 = 2\pi$

Phase difference at $S_4 = 2\pi/3$.

These phase differences are relative to the light incident on slits S_1 and S_2.

Now S_3 and S_4 are secondary sources of light.

Phase difference at $O' = \frac{4\pi}{3}$, equal to initial phase difference between the light reaching at $O' = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$.

Let intensity at O' be I'.

$$I' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{4\pi}{3}$$

$$= 5I_0 + 4I_0\cos\left(\pi + \frac{\pi}{3}\right) = 3I_0$$

(b) For brightest fringe, phase difference $= 2n\pi$, $n = 0, \pm1, \pm2, \ldots$

Let I'' be the intensity of brightest fringe.

$$I'' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\phi \quad (\text{where } \cos\phi = 1)$$

$$= 9I_0$$

Example 51

Three monochromatic, coherent (same phase) sources S_1, S_2 and S_3 emitting light of the same amplitude A and wavelength λ are kept as shown : the source S_1 at $\left(0, \frac{d}{2}\right)$, S_2 at $\left(0, -\frac{d}{2}\right)$ and S_3 at $\left(0, -\frac{3}{2}\right)$. A large screen is placed at $x = D$, perpendicular to the x-axis. Assuming $d << D$, find :
(i) the intensity at a point P(D, y) on the screen, if $y << D$,
(ii) the condition that the intensity at the point P is a maximum / minimum.

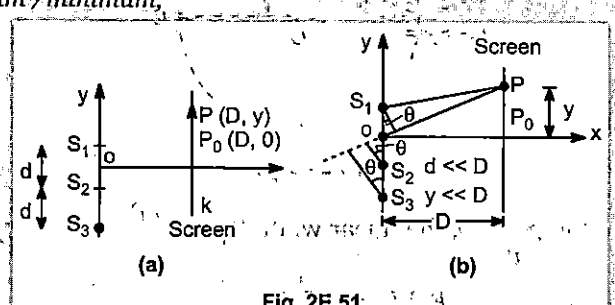


Fig. 2E.51

Solution: (i) The path difference,

$$S_2P - OP = \frac{d}{2}\sin\theta,$$

$$S_1P - OP = -\frac{d}{2}\sin\theta,$$

$$S_3P - OP = \frac{3d}{2}\sin\theta$$

The relative phases of the waves from S_1, S_2, S_3 when they interfere at P are

$$\phi_1 = -\frac{\pi d\sin\theta}{\lambda},\ \phi_2 = +\frac{\pi d\sin\theta}{\lambda},\ \phi_3 = \frac{3\pi d\sin\theta}{\lambda}$$

As we are concerned with relative phases only, we can add $\phi_0 = +\frac{\pi d\sin\theta}{\lambda}$ to ϕ_1, ϕ_2, ϕ_3 to simplify the calculations

$$\phi_1' = 0,\ \phi_2' = \frac{2\pi d\sin\theta}{\lambda},\ \phi_3' = +\frac{4\pi d\sin\theta}{\lambda}$$

We use the phasor method to get resultant amplitude.

Resultant amplitude at P,

$$A_P = A + 2A\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)$$

$$= A(1 + 2\cos\phi)$$

where $\phi = \frac{2\pi d\sin\theta}{\lambda}$.

Intensity at

$P, I_P = (1 + 2\cos\phi)^2 I_0$ ...(i)

(ii) For a maximum/minimum at P,

$$\frac{dI_P}{d\phi} = 0$$

$1 + 2\cos\phi = 0$ (minimum)

$$\cos\phi = -\frac{1}{2}$$

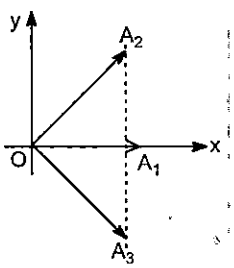


Fig. 2E.51 (c)

$$\phi = 2n\pi \pm \frac{2\pi}{3} \quad \text{or} \quad \frac{2\pi d\sin\theta}{\lambda} = 2n\pi \pm \frac{2\pi}{3}$$

or $d\sin\theta = \left(n \pm \frac{1}{3}\right)\lambda$ for a minimum

and $d\sin\theta = n\lambda\left(n + \frac{1}{2}\right)\lambda$ for a maximum

Example 52

Two coherent sources A and B emitting light of wavelength λ are placed at positions $(-D, 0)$ and $(-D, 3\lambda)$ respectively, $D >> \lambda$. Find:

(a) the number of maxima on y-axis and their positions,

(b) the number of maxima on x-axis and their positions.

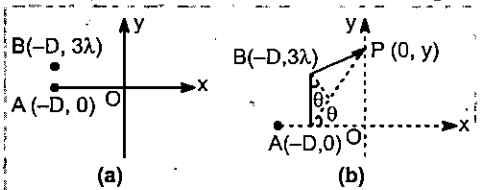


Fig. 2E.52

Solution: (a) At a point $P(0, y)$, the path difference is

$$\Delta x \doteq AB\sin\theta = 3\lambda\sin\theta$$

For minima, the permissible values are (as $-1 < \sin\theta < 1$)

$$3\lambda\sin\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$$

or $$\sin\theta = \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$$

From geometry of figure,

$$\sin\theta = \frac{y}{\sqrt{y^2 + D^2}}$$

Hence, $$y = \pm\frac{D}{\sqrt{5}}, \pm\frac{D}{\sqrt{3}}, \pm\frac{5D}{\sqrt{11}}$$

The number of minima on y-axis is six.

(b) For point $Q(x, 0)$ on x-axis,

Path difference, $\Delta x = AB\cos\theta = 3\lambda\cos\theta$

For maxima, $3\lambda\cos\theta = 0, \lambda, 2\lambda, 3\lambda$

$$\cos\theta = 0, \frac{1}{3}, \frac{2}{3}, 1$$

Acceptable values of θ are

$$\theta = \cos^{-1}\frac{1}{3}, \cos^{-1}\frac{2}{3}, 0$$

From geometry of the figure, we have

$$\tan\theta = \frac{(D + x)}{3\lambda}$$

$$\therefore \quad x = 0, \pm 3\sqrt{2}\lambda, \pm\frac{3\sqrt{13}}{2}\lambda$$

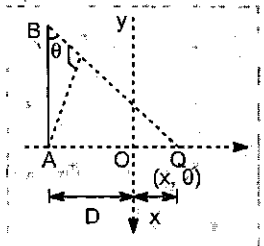


Fig. 2E.52 (c)

Example 53

A lens of focal length $f = 40$ cm is cut along the diameter into two identical halves. In this process, a layer of the lens $t = 1$ mm in thickness is lost, then the halves are put together to form a composite lens. In between the focal place and the composite lens, a narrow slit is placed, near the focal plane. The slit is emitting monochromatic light with wavelength $\lambda = 0.6\mu$m. Behind the lens a screen is located at a distance $L = 0.5$ m from it. Find the fringe width and number of visible maxima.

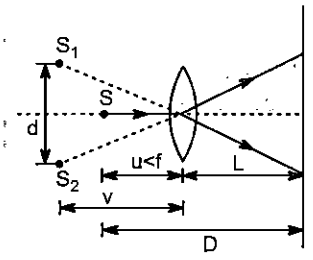


Fig. 2E.53 (a)

Solution: From lens equation, we have

$$v = \frac{uf}{f-u} \qquad (v \text{ lines on the left side})$$

$$d = 2\left(\frac{t}{2}\right)\left[\frac{v}{u}-1\right] = t\left[\frac{u}{f-u}\right]$$

$$D = L + v = L + \frac{uf}{f-u}$$

Fringe width,

$$\omega = \frac{\lambda D}{d} = \frac{\lambda\left[L+\frac{uf}{f-u}\right]}{t\left[\frac{u}{f-u}\right]}$$

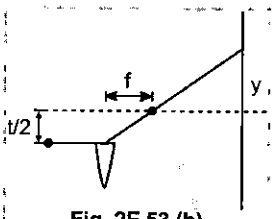


Fig. 2E.53 (b)

or $\omega = \frac{\lambda}{t}\left[f + \frac{L(f-u)}{u}\right]$

as $u \to f, \qquad \omega = \frac{\lambda f}{t}$

From figure, $\frac{y}{L-f} = \frac{t/2}{f} \Rightarrow y = \frac{t/2}{f}(L-f)$

Distance of the point upto which interference occurs,

$$y_0 = \frac{t/2}{f}(L-f) + \frac{t}{2} = \frac{tL}{2f}$$

Number of visible maxima $= \frac{tL \cdot t}{2f \times \lambda f} = \frac{Lt^2}{2\lambda f^2}$

Example 54

A plane light wave of wavelength $\lambda = 6000$ Å falls normally on the base of a biprism made of glass ($n = 1.52$) and refracting angle $\alpha = 3°$. Behind the biprism (see Fig. 2E.54) there is a plane-parallel plate with the space filled up with benzene ($n' = 1.50$). Find the width of a fringe on a screen placed behind this system.

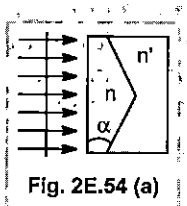


Fig. 2E.54 (a)

Solution: We will first determine the deviation produced by the system. From the figure, we see that

$$n \sin\alpha = n' \sin\theta',$$

$$n' \sin\theta'' = 1 \cdot \sin\theta'''.$$

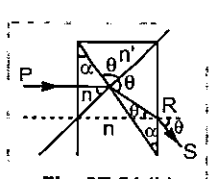


Fig. 2E.54 (b)

Now,

deviation of ray $\delta = (\theta' - \alpha) + (\theta''' - \theta'')$

From triangle QRB, we have

$$\alpha + 90° + \theta'' + 90° - \theta' = 180°$$

$$\Rightarrow \qquad \theta'' = \theta' - \alpha.$$

Since angle α is small, $\theta', \theta'', \theta'''$ are also small.

$$\therefore \qquad n\alpha = n'\theta' \quad \text{and} \quad n'\theta'' = \theta'''.$$

Solving θ', θ'' and θ''' in terms of $\alpha, \theta' = \frac{n}{n'}\alpha$,

$$\theta'' = \frac{n}{n'}\alpha - \alpha = \frac{n-n'}{n'}\alpha \quad \text{and}$$

$$\theta''' = n'\frac{n-n'}{n'}\alpha = (n-n')\alpha.$$

$$\therefore \quad \delta = \frac{n}{n'}\alpha - \alpha + (n-n')\alpha - \left(\frac{n}{n'}\alpha - \alpha\right) = (n-n')\alpha$$

If the source is at a distance a, then

d (distance between virtual sources)

$$= 2a\delta = 2a(n-n')\alpha.$$

$D = a + b$, where b = distance of screen from the system.

$$\therefore \qquad \beta = \frac{\lambda(a+b)}{2a(n-n')\alpha} = \frac{\lambda(1+b/2)}{2(n-n')\alpha}$$

For plane wave $\alpha \to \infty$.

$$\therefore \quad \beta = \frac{\lambda}{2(n-n')\alpha} = \frac{6000 \times 10^{-10}}{2(1.52-1.50) \times 3 \times \frac{\pi}{180}}$$

$$= 0.2864 \text{ mm}.$$

Example 55

A radio telescope sited on the edge of a cliff overlooking the sea operates on a wavelength of 100 m. A radio star rises above the horizon and is tracked by the telescope. The first minimum of the received signal occurs, when the star is 30° above the horizontal. Explain why the minimum occurs and determine the height of the cliff, assuming that radio waves suffer a phase change of π on reflection at the water surface (see Fig. 2E.55).

Solution: The radio telescope is receiving signals direct from the star (ray XBC) and also by reflection from the water surface, (YAC), which is acting like a plane mirror. In this case the source of radiation is far enough away to be considered at infinity and the rays are descending on the earth in a parallel beam. When the path difference between rays YAC and XBC (see figure) is such that they reach the telescope exactly out of phase, destructive interference occurs and the signal at C will be a minimum.

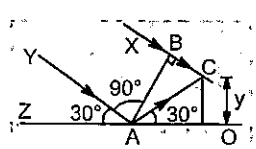


Fig. 2E.55

Consider the diagram. If AB is drawn perpendicular to XC, it will also be perpendicular to YA, and AB is a plane wavefront of the incoming beam. Thus the disturbances A and B have the same phase at all times. A phase change of π occurs at A and this is equivalent to an increase in path of $\lambda/2$. Thus the path difference between the two rays is $\delta = AC + (\lambda/2) - BC$. But by the laws of reflection, $\angle YAZ = \angle CAO = 30°$, and therefore $\angle BAC = 180° - 90° - 30° - 30° = 30°$.

$$\delta = AC + \frac{\lambda}{2} - AC \sin 30°$$

$$= \frac{\lambda}{2} + AC(1 - \sin 30°)$$

Since $\sin 30° = \frac{y}{AC}$,

$$\therefore \quad AC = \frac{y}{\sin 30°}$$

and $$\delta = \frac{\lambda}{2} + \frac{y}{\sin 30°}(1 - \sin 30°) = \frac{\lambda}{2} + y.$$

In order for minima to occur, the path difference δ must equal an odd number of half wavelengths,

or $$\delta = (2n+1)\frac{\lambda}{2}, \qquad n = 0, 1, 2, \ldots$$

Then $$(2n+1)\frac{\lambda}{2} = \frac{\lambda}{2} + y.$$

If $n = 0$, $\delta = \lambda/2$, $y = 0$, and this corresponds to the centre of the interference pattern being a minimum. Since this is the first minimum the telescope receives, C must be at the first minimum above O for the situation shown in the diagram. (Were it a minimum other than the first, a previous minimum must have occurred for an angle lower than 30°. But we are told that this doesn't happen.) Therefore, $n = 1$ and θ must be $3\lambda/2$.

Hence, $\frac{3\lambda}{2} = \frac{\lambda}{2} + y$ and $y = \lambda = 100\,\text{m}$.

FRINGES OF EQUAL THICKNESS

Soap bubbles and oil films do not have uniform thickness. The thickness of the film at any given point determines whether the reflected light has a maximum or a minimum intensity. When white light is used, each wavelength has its own fringe pattern. At a given point on the film, one wavelength may be enhanced and/or another wavelength suppressed. This is the source of the colours in soap bubbles and oil films on the road.

A wedge-shaped film of air may be produced by placing a sheet of paper or a hair between the ends of two glass plates, as in Fig. 2.33. With flat plates, one sees a series of bright and dark bands, each characteristic of a particular thickness.

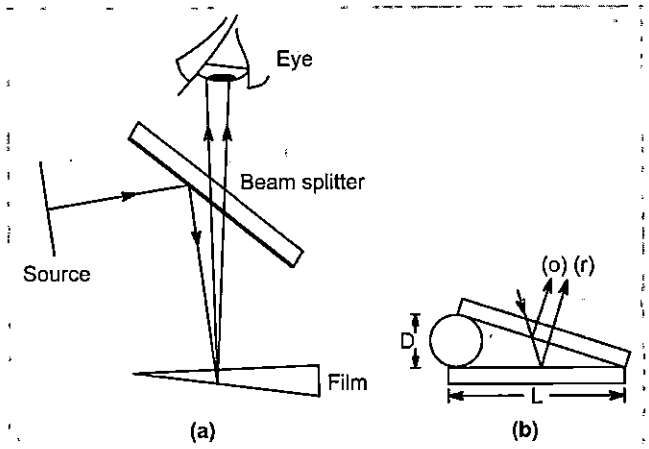


Fig. 2.33

Wedge Interference

An optical wedge consists of two surfaces inclined at a very small angle to each other, such as would be produced by placing a piece of paper between two microscope slides (see Fig. 2.34); angles of about 1 mradian (0.05°) are typical.

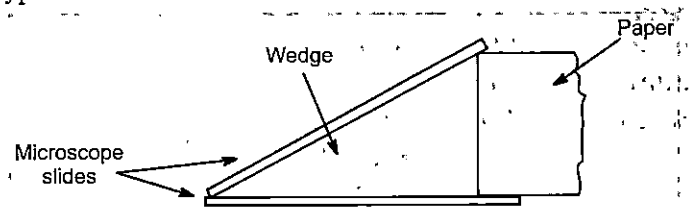


Fig. 2.34

The wedge surfaces may be considered to be practically parallel to one another because the wedge angle is very small. With an air wedge, one phase change of π radians occurs on reflection. This means that the interference conditions are identical to those for the parallel-sided glass block in air. However light is incident *normally* on the wedge. This reduces the interference conditions to

$2d = (m + 1/2)\lambda$ for a bright fringe

$2d = m\lambda$ for a dark fringe

because $\alpha = 0$, $\theta = 0$ and $\cos\theta = 1$.

Fringe Separation with an Air Wedge

Fig. 2.35 shows two neighbouring dark fringes of order m and $m+1$ formed at distances x_m and x_{m+1} from OO'. Our objective is to express $(x_{m+1} - x_m)$ in terms of d_m, d_{m+1} and the wedge angle α.

For small angles, we can write,

$$x_m = d_m/\alpha$$

$$x_{m+1} = d_{m+1}/\alpha$$

Then $$(x_{m+1} - x_m) = (d_{m+1} - d_m)\alpha$$

$$= \lambda/2\alpha$$

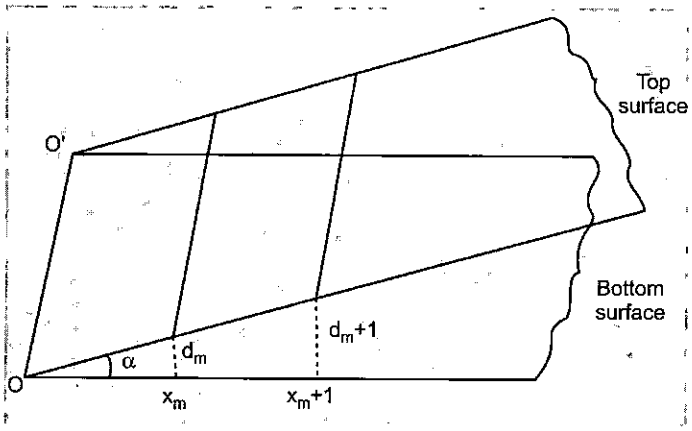


Fig. 2.35

This result also applies to bright fringes: it can be obtained directly. The fringe separation when the wedge medium is not air $\lambda/2n_m\alpha$.

Example 56

A wedge-shaped film of air is produced by placing a fine wire of diameter D between the ends of two flat glass plates of length L = 20 cm, as in Fig. 2E.56. When the air film is illuminated with light of wavelength λ = 550 nm, there are 12 dark fringes per centimetre. Find D.

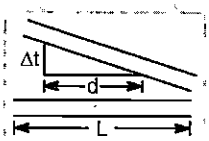


Fig. 2E.56

Solution: As indicated in Fig. 2E.56, only one of the reflected rays suffers a phase inversion. At the thin end of the wedge, where the thickness is less than $\lambda/4$, the two rays interfere destructively. This region is dark in the reflected light. The condition for destructive interference in the reflected light is

$$2t = m\lambda, \quad m = 0, 1, 2, \ldots$$

The change in thickness between adjacent dark fringes is $\Delta t = \lambda/2$. The horizontal spacing between fringes $d = 1/12\,\text{cm} = 8.3\times10^{-4}\,\text{m}$. From figure we see that $D/L = \Delta t/d$, so

$$D = \frac{\lambda L}{2d} = \frac{(5.5\times10^{-7})(0.2)}{16.6\times10^{-4}}$$

Thus $D = 6.6\times10^{-5}\,\text{m}$

Example 57

(a) White light is incident normally on a lens (n = 1.52) that is coated with a film of MgF_2 (n = 1.38). For what minimum thickness of the film will yellow light of wavelength 550 nm (in air) be missing in the reflected light ?

(b) At what minimum thickness (other than zero) is there constructive interference in the reflected light ?

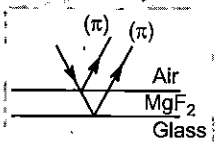


Fig. 2E.57

Solution: (a) Since the index for glass is higher than that of the film, there is a phase change for the reflection at the film-glass interface, as shown in Fig. 2E.57. There is no net phase difference introduced by the reflections. Thus, in this case the condition for destructive interference in the reflected light is

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

Notice that this differs from the expression given in the previous section. The minimum thickness occurs when $m = 0$; thus

$$t_{\min} = \frac{\lambda}{4n} = \frac{5.5\times10^{-7}}{(4)(1.38)} = 99.6\,\text{nm}$$

Both reflected rays suffer a phase inversion. In order to produce destructive interference in the reflected light, the minimum thickness of the film is $\lambda_F/4$ (so that the path difference is one-half wavelength in the film).

Although the condition for destructive interference applies only to one wavelength, the reflection for other wavelengths is also reduced. The combination of the red and violet reflected light gives such a coated lens a purple hue.

(b) In this case the condition for constructive interference is

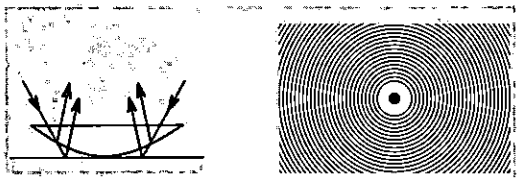
$$2t = \frac{m\lambda}{n}$$

The minimum thickness is $t = \lambda/2n = 199\,\text{nm}$.

NEWTON'S RINGS

When a lens with a large radius of curvature is placed on a flat plate as in [Fig. 2.36 (a)], a thin film of air is formed. When the film is illuminated with monochromatic light,

circular fringes, called **Newton's rings,** can be seen [Fig. 2.36 (b)]. An important feature of Newton's rings is the dark



(a) (b)

Fig. 2.36

central spot. It implied that the light wave suffers a phase inversion on reflection at a medium with a higher refractive index. If we place oil of sassafras between a lens of crown glass and a plate of flint glass, the refractive index of the oil

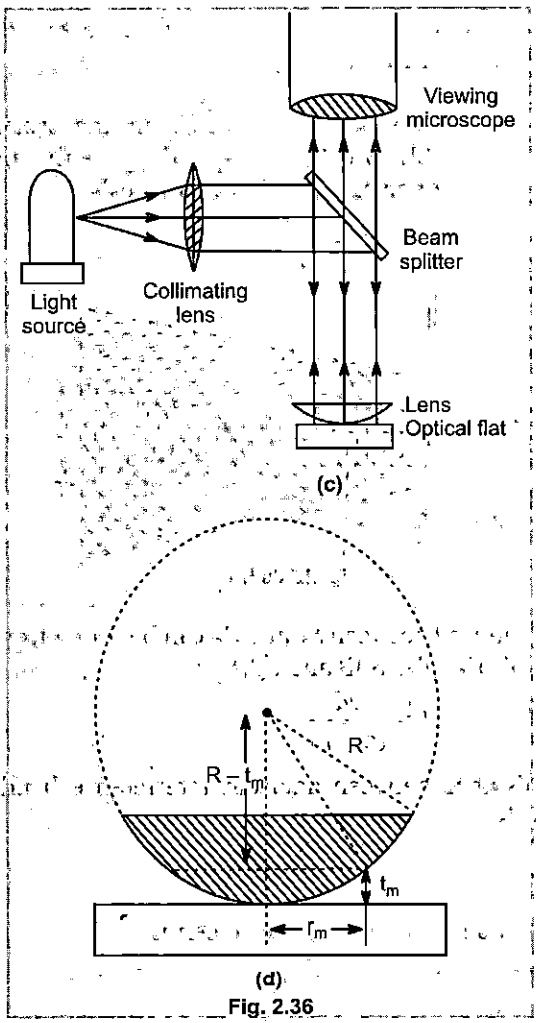


Fig. 2.36

is between the values for these two glasses. Since both reflections occur at a medium with a higher refractive index, they should both suffer a phase inversion and therefore be in phase. The central spot became bright—and undoubtedly gave Young much satisfaction.

Example 58

In an experiment on Newton's rings the light has a wavelength of 600 nm. The lens has a refractive index of 1.5 and a radius of curvature of 2.5 m. Find the radius of the 5th bright fringe.

Solution: If R is the radius of curvature of the lens, then from Fig. 2E.58, we see that $r^2 = R^2 - (R-t)^2$, where r is the radius of a fringe and t is the thickness of the film. Since t is very small, we may drop terms in t^2 to obtain

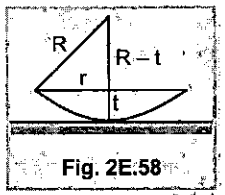


Fig. 2E.58

$$r^2 \approx 2Rt \qquad ...(1)$$

In order to find r, we must first find t. The condition for a bright fringe is

$$2t = \left(m + \frac{1}{2}\right)\lambda_F \qquad ...(2)$$

We note that $n = 1$ for the air film (the index for the glass is irrelevant) and that $m = 4$ for the fifth bright fringe. Thus, from eqn. (2),

$$t = \frac{(4.5)(6\times10^{-7})}{2} = 1.35\times10^{-6}\,\text{m}$$

Substituting this into egn. (1), we find

$$r = \sqrt{2Rt} = 2.6\times10^{-3}\,\text{m}.$$

Example 59

A thin lens of long focal length is supported horizontally a short distance above the flat polished end of a steel cylinder. The cylinder is 5 cm high and its lower end is rigidly held. Newton's rings are produced between the lens and the upper end of the cylinder, using normally incident light of wavelength 6000 Å, and viewed from above by means of a microscope. When the temperature of the cylinder is raised 25 C deg, 50 rings move past the cross-wires of the microscope. What is the coefficient of linear expansion of steel ?

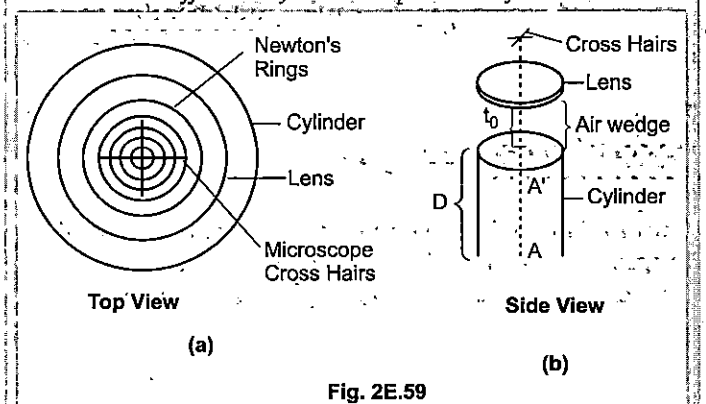


Fig. 2E.59

Solution: Initially, the cylinder has a length d and a gap of length t_0 exists between the top of the cylinder and the bottom surface of the lens in the portion of the air wedge viewed in the microscope at the same position as the cross-wires. (Technically, t_0 is the distance of the lens from the cylinder as measured along axis AA' in Fig. 2E.59 (b). However, since the lens is thin, t_0 is the distance of any point on the bottom surface of the lens from the top of the cylinder). Let us now see how interference fringes are produced.

Light is incident upon the lens from above. Light is transmitted through both lens surfaces with no phase change. However, the light reflected from the bottom lens surface and the top of the cylinder are 180° (or $\lambda/2$) out of phase. This phase change is due to the reflection from the cylinder. In addition, the wave reflected from the cylinder travels a distance $2t_0$ greater than the ray reflected from the lens in traversing the air gap. If this distance is equal to an odd number of half-wavelengths (odd because the waves are already $\lambda/2$ out of phase due to reflection), constructive interference will occur. Hence

Constructive interference

$$2t_0 = (2n+1)\frac{\lambda}{2} \qquad (n = 0, 1, 2, \ldots)$$

or $$2t_0 = \left(n + \frac{1}{2}\right)\lambda \qquad (n = 0, 1, 2, \ldots)$$

Hence, for a bright fringe to appear at the cross-wires, $2t_0 = (n + 1/2)\lambda$, where n is an unknown integer.

After one heats the cylinder through a temperature difference T, the length of the cylinder is $d(1 + \alpha T)$, where α is the coefficient of linear expansion of steel, and the gap will have been reduced to t, where $t_0 - t = d\alpha T$.

If a bright fringe is again seen at the position of the cross-wires, then, similarly, $2t = (m + 1/2)\lambda$, where m is an integer ($m \neq n$)

$$\therefore \quad 2(t_0 - t) = 2d\alpha T = (n - m)\lambda$$

During the heating process, $(n - m)$ bright fringes must have passed over the cross-wires. Thus

$$\alpha = \frac{(n-m)\lambda}{2dT} = \frac{50 \times 6 \times 10^{-5}}{2 \times 5 \times 25}$$

$$= 1.2 \times 10^{-5} \text{ per °C}.$$

Example 60

A central portion with a width of $d = 0.5$ mm is cut out of a convergent lens having a focal length of $f = 10$ cm, as shown in Fig. 2E.60(a). Both halves are tightly fitted against each other.

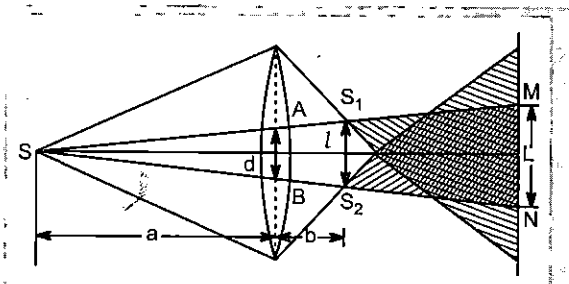


Fig. 2E.60 (a)

The lens receives monochromatic light ($\lambda = 5000$ Å) from a point source at a distance of $a = 5$ cm from it. At what distance should a screen be fixed on the opposite side of the lens to observe three interference bands on it ?

What is the maximum possible number of interference bands that can be observed in this installation ?

Solution: The path of the rays in the system is illustrated in Fig. 2E.60 (b). Here S_1 and S_2 are the images of source S in the halves of the lens. Obviously,

$$b = \frac{fa}{(a-f)}$$

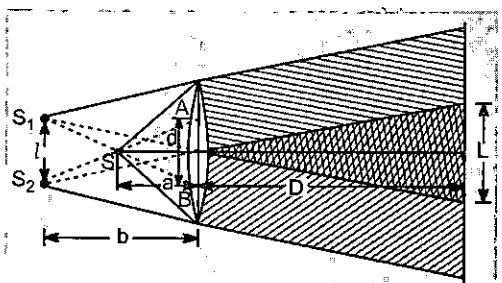


Fig. 2E.60 (b)

The distance l between S_1 and S_2 can be found from the similarity of triangles SAB and SS_1S_2 :

$$l = \frac{ad}{(a-f)}$$

The distance between adjacent interference bands on the screen is

$$\Delta h = \frac{\lambda(D-b)}{l} = \frac{\lambda}{ad}(Da - Df - af)$$

The number of bands on the screen is

$$N = \frac{L}{\Delta h}$$

where $L = \frac{Dl}{b}$ is the dimension of the portion of the screen on which interference bands are observed. Therefore,

$$D = \frac{Nabf\lambda}{(adl + abN\lambda - bfN\lambda)} = 15\,\text{cm}$$

The maximum possible number of bands can be determined from the condition

$$adl + Nab\lambda - bfN\lambda = 0 \qquad (\text{here } D \to \infty).$$

Hence, $$N_{max} = \frac{adl}{(bf\lambda - ab\lambda)} = 5$$

The number of bands is finite, since the distance between them increases as the screen is moved away, and dimensions of the portion of the screen on which the interference pattern appears to grow.

Example 61

Find the distance between the neighbouring bands of an interference pattern produced by a lens with a radius of R=1 cm, described in Example 40, if this distance does not depend on the position of the screen.

At what position of the screen will the number of interference bands be maximum ?

The source sends monochromatic light with a wavelength of $\lambda = 5000\ \text{Å}$.

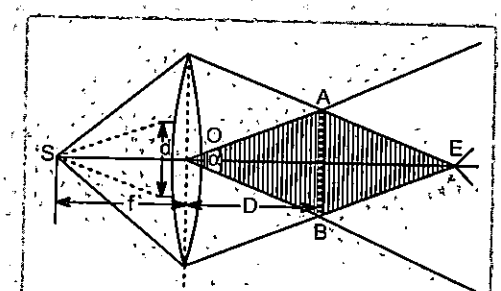


Fig. 2E.61

Solution: The distance between the interference bands will not depend on the position of the screen only if the source is arranged in the focal plane of the lens. This directly follows from the expression

$$\Delta h = \frac{\lambda}{ad}(Df - Da + af)$$

When $a = f, \Delta h = \frac{\lambda f}{d} = 10^{-2}$ cm, at any value of D.

For this case the path of the rays is shown in Fig. 2E.61. A glance at this illustration will show that the number of interference bands will be maximum, when the screen is in position AB. The distance from the screen to the lens can be found from triangle OAB, remembering that the angle $\alpha \cong \frac{d}{f}$ and $AB = R$:

$$D = \frac{R}{\alpha} = \frac{Rf}{d} = 2 \text{ metre}$$

DIFFRACTION

The light rays bend out of their straight line paths. This bending of light and other waves into the region which would be the shadow if rectilinear propagation prevailed, is called **diffraction.**

Diffraction patterns are usually classified into two categories depending on where the source and screen are placed. In **Fraunhofer diffraction** parallel rays (*i.e.*, plane waves) are used. These types of waves can be obtained by using lenses or a distant source and a distant observation point. When parallel rays are not used, the source and observation point are at finite distance relative to slit, this diffraction is called **Fresnel's diffraction.**

Figure shows parallel rays of monochromatic light that pass through a narrow slit. The rays that pass straight through are all in phase; so there will be a central bright spot on the screen. In Fig. 2.37 (b) rays make an angle θ such that a ray from the top of the slit travels exactly one wavelength more than the ray from the bottom edge and a ray passing

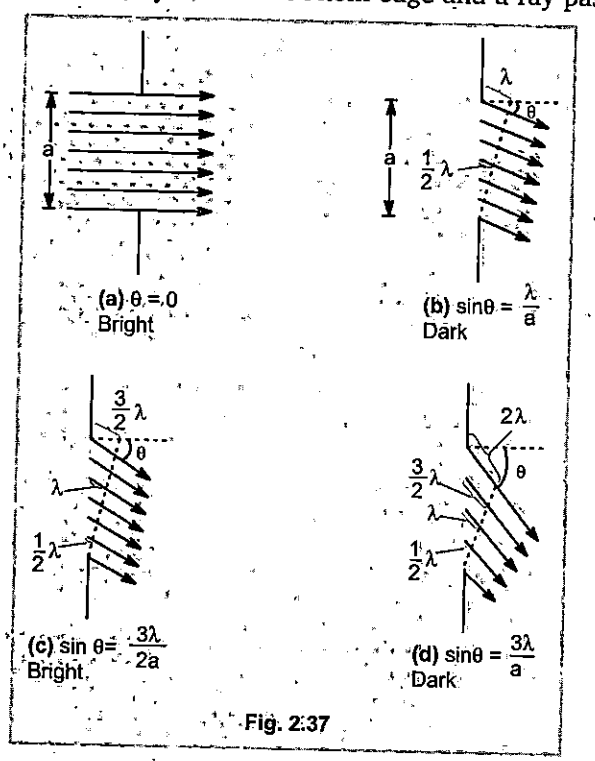


Fig. 2.37

through centre will travel one-half wavelength more than the rays from the bottom edge. Destructive interference will take place between these two rays. Similarly a ray slightly above the bottom edge will cancel a ray that is the same distance above the central one. Similarly each ray passing through the lower half of the slit will cancel the corresponding ray passing through the upper half. All the rays destructively interfere in pairs. So no light will reach the screen at this angle.

Hence $\sin\theta = \frac{\lambda}{a}$ [first minimum]

In Fig. 2.37 (c) the observation angle θ is chosen so that the ray from the top of the slit (ray C) travels a distance $3\lambda/2$ farther than the bottom ray (ray A). Once again the rays at one-third distance from the bottom will cancel in pairs with those in the middle third. But light from the top third of the slit will still reach the screen; so there will be a bright spot centred near $\sin\theta \simeq 3\lambda/2a$. Intensity at this maxima is less as compared to that at $\theta = 0°$.

In Fig. 2.37 (d) the rays from the region of the slits between A and B are half wavelength ahead of the corresponding rays from the region between B and C. These two regions of the slits therefore cancel each other's effect. Similarly the region between C and D cancels the region between D and E. Hence at this angle, there will again be a minimum of zero intensity in the diffraction pattern. In general the intensity minima occur at

$$d\sin\theta = m\lambda, \quad m = 1, 2, 3, \ldots \quad \text{[minima]}$$

- The intensity pattern consists of a bright central maxima flanked by secondary minima. The intensity of each succeeding secondary maxima decreases with distance from centre.
- As $m = 0$ corresponds to the centre of the intensity pattern, of the middle of the central maximum, this means that the width of the central maxima is twice that of secondary maxima.

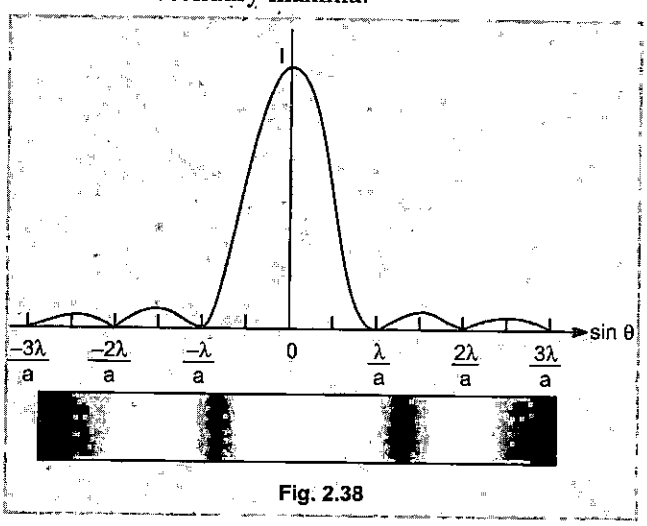


Fig. 2.38

- The secondary maxima are approximately midway between their adjacent minima. Thus the angles that locate the secondary maxima are given by

$$[d\sin\theta]_{\text{maxima}} = \frac{m\lambda + (m+1)\lambda}{2}$$

$$= \pm\left(m + \frac{1}{2}\right)\lambda$$

Example 62

Show that the intensity at any angle θ is given by $I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2$, where the angle β is the phase difference between the waves from the top and bottom edges of the slit. Hence derive the condition for intensity minima and maxima at an angle θ.

Solution: A plane wave at any time t and phase ϕ can be written as

$$y = A\sin(\omega t - \phi) \quad \ldots(1)$$

We split the slit of width d into a large number of very narrow differential slits each of the same width dS. The amplitude of wave from such an element would be $\left(\frac{A}{d}\right)dS$; the amplitude will be proportional to the width of the differential slit.

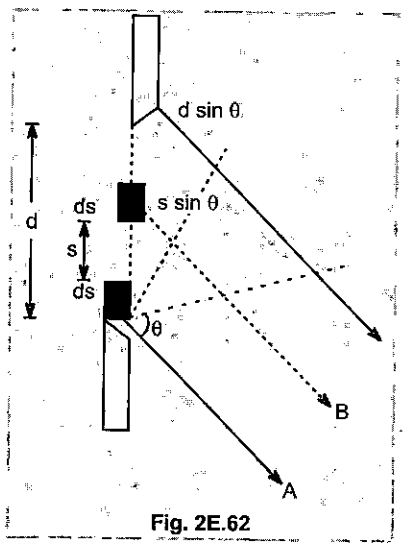


Fig. 2E.62

The wave disturbance from the bottom most portion of the slit (coming along ray A) will be represented as

$$dy_A = \left(A\frac{dS}{d}\right)\sin\omega t \quad \ldots(2)$$

A wave disturbance along the ray B will be a distance $S\sin\theta$ behind the wave which comes from A.

$$\text{The phase difference} = \frac{2\pi}{\lambda}(S\sin\theta) \quad \ldots(3)$$

Therefore the wave disturbance along ray B will have the form

$$dy = \left(A\frac{dS}{d}\right)\sin[\omega t - \phi]$$

$$= A\left(\frac{dS}{d}\right)\sin\left[\omega t - \frac{2\pi d\sin\theta}{\lambda}\right] \quad \ldots(4)$$

This is an arbitrary ray; so it represents a general ray emanating from any point of the slit.

The resultant disturbance from the slit at an angle θ can be obtained by integrating dy for all portions of the slits. The resultant disturbance is therefore

$$y\int_0^d \frac{A}{d}\sin\left(\omega t - \frac{2\pi S\sin\theta}{\lambda}\right)dS \qquad ...(5)$$

After carrying out integration over S keeping t and θ constant, S varies from 0 to width of slit d.

$$y = \frac{A\lambda/d}{2\pi\sin\theta}\left\{\cos\omega t - \cos\left[\omega t - \frac{2\pi d\sin\theta}{\lambda}\right]\right\} \qquad ...(6)$$

As $$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

it then becomes

$$-y = \frac{A\lambda/d}{\pi\sin\theta}\sin\frac{\pi d\sin\theta}{\lambda}\sin\left[\omega t - \frac{\pi d\sin\theta}{\lambda}\right] \qquad ...(7)$$

We are interested in the intensity of the wave, which is proportional to the square of amplitude.

From eqn. (7) the amplitude of the wave is

$$\frac{A\lambda/d}{\pi\sin\theta}\sin\left[\frac{\pi d\sin\theta}{\lambda}\right]$$

$$I \propto \left[\frac{(\lambda/d)}{\sin\theta}\right]^2 \sin^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$$

or $$I = I_0\left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2 \qquad ...(8)$$

where $$\beta = \frac{2\pi d\sin\theta}{\lambda} \qquad ...(9)$$

According to eqn. (8), intensity minima occurs where

$$\sin\left(\frac{\pi d\sin\theta}{\lambda}\right) = 0$$

which implies that

$$\frac{\pi d\sin\theta}{\lambda} = \pi, 2\pi, 3\pi, \ldots, m\pi \qquad ...(10)$$

or $$d\sin\theta = m\lambda \quad \text{[condition for minima]} \qquad ...(11)$$

❒ Note that m cannot be zero. Intensity for $\theta \to 0$ can be evaluated by taking limit, as $\theta \to 0$, $\beta \to 0$ and

$$\lim_{\beta\to 0}\frac{\sin(\beta/2)}{(\beta/2)} \to 1$$

So $I = I_0$, the maximum at the centre of the pattern.

Example 63

(a) Derive an expression for the intensity of the secondary maxima in terms of I_0. (b) Use your answer from part (a) to find the relative intensities for intensity I_1, I_2, I_3 at $m = 1$, $m = 2$, $m = 3$ respectively.

Solution: (a) The angles θ which locate secondary maxima are given by

$$d\sin\theta = \pm\left(m + \frac{1}{2}\right)\lambda$$

Values of β that correspond to secondary maxima are

$$\beta = \frac{\pi d\sin\theta}{\lambda} = \pm\left(m + \frac{1}{2}\right)\pi$$

Substituting β in the expression for intensity, we get

$$I = I_0\frac{\sin^2[\pm(m+1/2)\pi]}{[\pm(m+1/2)\pi]^2}$$

If we use the approximation that secondary maxima occur for integral values of m, then we have $\sin^2[\pm(m+1/2)\pi] \simeq 1$ for all m. Thus

$$I = \frac{I_0}{[(m+1/2)\pi]^2} \qquad \text{where } m = 1, 2, \ldots$$

(b) For $m = 1$, maxima

$$I_1 = \frac{I_0}{[(1+1/2)\pi]^2} = 0.045 I_0$$

For $m = 2$, maxima

$$I_2 = \frac{I_0}{[(2+1/2)\pi]^2} = 0.016 I_0$$

For $m = 3$, maxima

$$I_3 = \frac{I_0}{[(3+1/2)\pi)]^2} = 0.0083 I_0$$

Example 64

Light from a sodium lamp $\lambda = 590$ nm is diffracted by a slit of width $d = 0.30$ mm. The distance from the slit to the screen is $D = 0.87$ m. Find the width of the central maximum.

Solution: The angular positions of $m = 1$ minima are given by

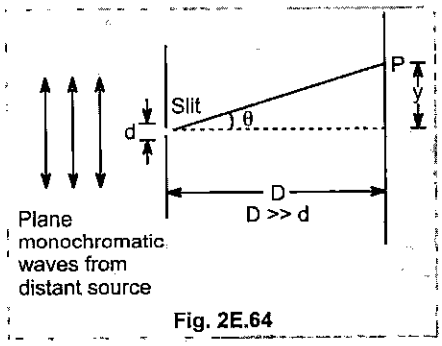


Fig. 2E.64

$$\sin\theta_1 = \pm\frac{\lambda}{d}$$

From figure, $$\sin\theta = \frac{y}{\sqrt{y^2 + D^2}}$$

If Δy is the distance between the two $m = 1$ minima, the values of y that correspond to $m = 1$ minima are $y = \pm\frac{1}{2}\Delta y$.

Therefore $$\frac{\frac{1}{2}\Delta y}{\sqrt{(1/2\,\Delta y)^2 + D^2}} = \frac{\lambda}{d}$$

$$\Delta y = \frac{2D\lambda}{\sqrt{d^2 - \lambda^2}}$$

If $d >> \lambda$, we can use the approximation

$$\sqrt{d^2 - \lambda^2} \simeq d$$

Thus $$\Delta y = \frac{2D\lambda}{d} = \frac{2(0.87)(590)}{(0.30)} = 3.4\,\text{mm}$$

Example 65

Two slits each of width a and separated by distance $d = 6a$, are illuminated by light of wavelength λ in a Young's double slit experiment. How many interference fringes appear under the central diffraction peak ?

Solution: While discussing Young's double slit experiment, we had assumed that the intensity distribution due to single slit is uniform. This assumption is valid for $a << \lambda$, which means that central diffraction peak is spread out over the entire screen. This situation is not realised in practice because infinitesimally narrow slits are impossible in practice. We had ignored the diffraction pattern due to each slit. For a far away screen the diffraction patterns due to slits overlap. Each of the two slits produces a diffraction pattern on a distant screen when illuminated at a time; the two patterns are slightly separated from one another. The expression for intensity in the double slit experiment is

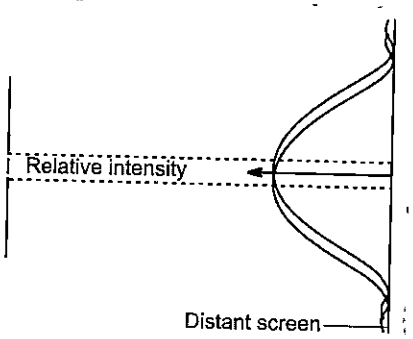


Fig. 2E.65 (a)

$$I = 4I_0 \cos^2\frac{\phi}{2}, \qquad \phi = \frac{2\pi d \sin\theta}{\lambda}$$

Now we can write a more general expression for the double slit intensity distribution. The intensity due to one slit acting alone is given by the single slit diffraction formula. We have to replace I_0 by

$$\frac{I_0\left(\sin\frac{\beta}{2}\right)^2}{\left(\frac{\beta}{2}\right)^2}$$

Hence $$I = 4I_0\left(\frac{\sin\beta/2}{(\beta/2)}\right)^2 \cos^2\left(\frac{\phi}{2}\right)$$

where $$\beta = \frac{2\pi a \sin\theta}{\lambda}$$

The factor $[\sin^2\beta/2]/\beta^2$ depends on the slit width a and is usually called *diffraction factor*. The factor $\cos^2\phi/2$ depends on slit separation and is usually called the *interference factor*. The intensity of the interference peaks is modulated by the envelope of the single slit diffraction pattern as shown in the figure.

The first minimum in the diffraction pattern occurs at

$$\sin\theta = \lambda/a$$

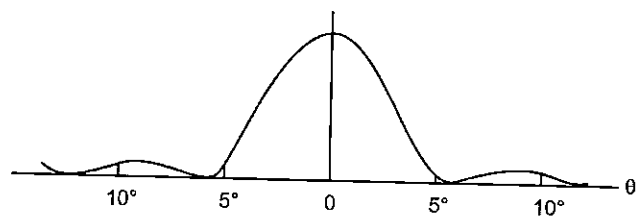
Since $d = 6a$, $$d\sin\theta = 6a\left(\frac{\lambda}{a}\right) = 6\lambda$$

As shown in Young's double slit experiment, the interference maxima occur for

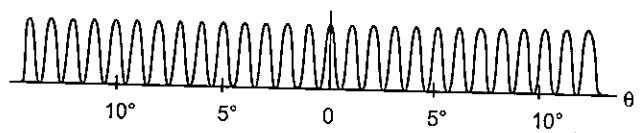
$$d\sin\theta = m\lambda, \quad m = 0, 1, 2, \ldots$$

Thus the diffraction minimum ($d\sin\theta = 6\lambda$) coincides with $m = 6$ in the interference pattern; hence $m = 6$ is not visible. Thus the central diffraction peak encloses the central interference maxima and five other maxima on either side, a total of 11. The 6th order is called the missing order.

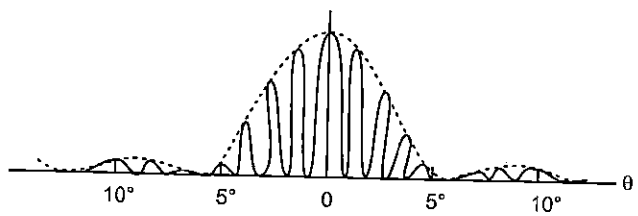
❒ The number of interference fringes in the central diffraction peak depends on the ratio d/a, independent of λ; however, the angular position on the screen depends on λ.



(a) Diffraction factor $(\sin^2\beta/2)/(\beta/2)^2$ *vs.* θ



(b) Interference factor $\cos^2\frac{\delta}{2}$ vs. θ



(c) Intensity. $I\theta$ vs.θ

Fig. 2E.65

LIMIT OF RESOLUTION

Resolution is the ability to separate and distinguish fine details of an object or to create distinction between two small objects. The **resolving power** of an optical instrument is a measure of the instrument's ability to produce separate images of two point objects.

Optical instruments usually have circular apertures, that produces a diffraction pattern similar to that of a circular slit. The expression for the angular position of the first dark ring surrounding a central maxima is

$$\sin\theta_1 = 1.22\frac{\lambda}{d}$$

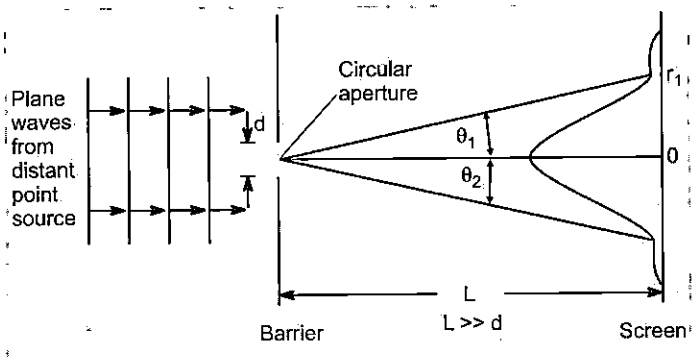


Fig. 2.39 (a)

Ordinarily the angle θ is small so that the approximations $\sin\theta \simeq \tan\theta \simeq \theta$ are valid, and we can use

$$\theta_1 \simeq 1.22\frac{\lambda}{d}$$

From figure, $\tan\theta = \frac{r_1}{D}$

$$r_1 \simeq 1.22\frac{\lambda D}{d}$$

Note that the size of the central maxima is dependent on λ/d, *i.e.*, the central maximum is more spread out for longer wavelengths and smaller apertures.

- If the aperture of an objective is large, then the limit of resolution θ is reduced, *i.e.*, relatively more closer objects will be resolved.
- Resolving power is reciprocal of resolving limit, *i.e.*,

$$\text{Resolving power} = \frac{1}{\text{Resolving limit}}$$

For a good optical instrument, resolving limit should be small and resolving power high.

- Resolving limit of normal eye is $\frac{1}{60^\circ}$ (1 minute). If two objects subtend an angle $1/60^\circ$ or more, they will be well resolved.
- If two objects separated by x at a distance D (least distance of distinct vision) are just resolved, then

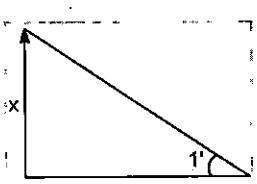


Fig. 2.39 (b)

$$\frac{x}{D} = \frac{1}{60^\circ} = \frac{1}{60}\times\frac{\pi}{180}$$

or $x = 0.00029 \times 25\,\text{cm}$

$= 0.0725\,\text{mm} \simeq 0.1\,\text{mm}$

Hence, two objects kept at D will be just resolved, by the unaided eye if they are separated by approximately one-tenth of a millimetre.

Rayleigh's Criterion

Suppose two distant stars are observed with a telescope. Each star is a point source (producing same intensity at the Earth) and its image on the film is a diffraction pattern due to the circular aperture. Both the stars produce separate diffraction patterns. No sustained interference pattern is obtained because the sources are incoherent. The resultant intensity is the sum of intensities from each diffraction pattern. Fig. 2.40 shows the diffraction patterns of two point sources when they are unresolved, just resolved and clearly resolved.

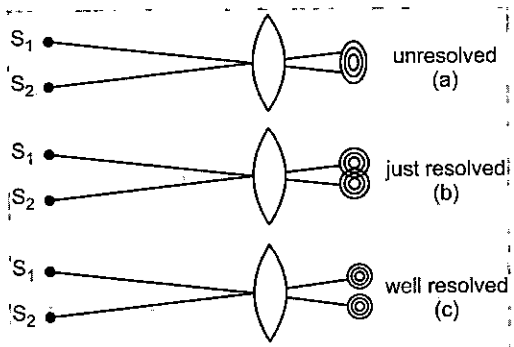


Fig. 2.40

The accepted criterion for resolution is Rayleigh's criterion.

When the central maxima of one diffraction pattern is located at the position of the first minimum of the other diffraction pattern, the two sources are said to be just resolved.

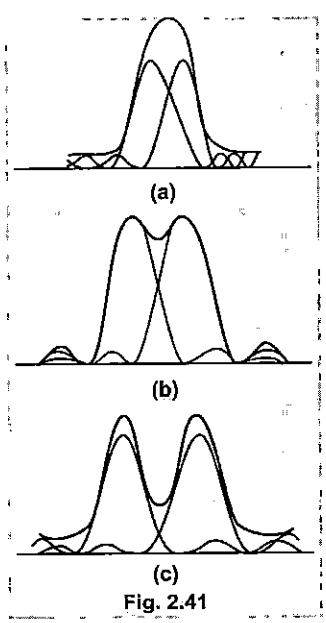


Fig. 2.41

Mathematically two point objects separated by an angle $\Delta\theta$ are resolved when $\Delta\theta > \Delta\theta_R$ where

$$\Delta\theta_R \simeq 1.22\frac{\lambda}{d}$$

The angle $\Delta\theta_R$ is the limiting angle of resolution for an optical instrument. For this criterion intensity of light at the instrument from each object should be the same.

Regardless of criterion the resolving power of an optical instrument depends on the wavelengths of the light and the diameter of the instrument's aperture.

Resolving limit of a microscope is the minimum separation between two close and resolved objects.

Reciprocal of resolving limit is resolving power.

Resolving limit

$$y = \frac{1.22\lambda}{2\sin\lambda} \simeq \frac{0.61\lambda}{\alpha}\,\text{Å}$$

where λ is the wavelength of light used and α is the semivertical angle of the cone of light as shown in Fig. 2.40.

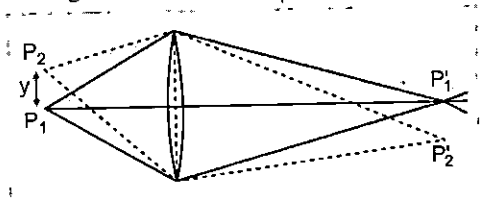


Fig. 2.42

If the medium between object and objective has refractive index μ, then

$$y = \frac{1.22\lambda}{2n\sin\alpha}\,\text{Å} \simeq \frac{0.61\lambda}{n\lambda}\,\text{Å}$$

Resolving power of a microscope is increased by using a liquid between object and objective.

Example 66

A parallel, thin, monochromatic laser beam falls on a diffraction grating at normal incidence. How does the interference pattern it produces on a viewing screen change if the grating is rotated through an angle $\phi < 90°$ around an axis, which is parallel to the lines of the grating ?

Solution: When the monochromatic laser beam falls on the diffraction grating at normal incidence, the positions θ of maxima of the interference pattern are given by

$$d\sin\theta = m\lambda, \quad (m = 0, \pm 1, \pm 2, \pm 3, \ldots),$$

where d is the grating spacing and λ is the wavelength of the laser beam. If the grating is rotated through an angle ϕ around an axis parallel to the lines of the grating, we have to modify the above equation. It is enough to consider the interference from only two slits of the grating, as shown in Fig. 2E.66.

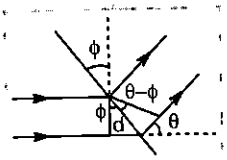


Fig. 2E.66

The optical path difference consists of two parts:

$$\Delta_1 + \Delta_2 = d\sin\phi + d\sin(\theta - \phi)$$

Thus the modified equation for the interference pattern is

$$d[\sin\phi + \sin(\theta - \phi)] = m\lambda.$$

The principal result is that the pattern becomes asymmetrical, with only the position of the zeroth-order maximum remaining unchanged. If ϕ is counted as positive, when the grating is rotated anticlockwise (and θ is counted as positive in the same sense), then the density of interference maxima becomes larger at positive angles, whilst it decreases for negative ones. If the lines of the grating are vertical, all the interference maxima lie along a horizontal straight line.

Example 67

In a certain experiment of diffraction through a single slit of width $a = 40\lambda_1$, it is observed that $\sqrt{2}$ times the geometric mean of angular width of first maxima for $\lambda_1 = 5000$ Å and that of second maxima for $\lambda_2 = 6400$ Å, is equal to the angular position of first minima in a new set-up having slit width a' and wavelength $\lambda_3 = 4000$ Å. Given that maximum order of minima in the new set-up is η and is found at angle of 90°. Determine the values of a' and η.

Solution: For minima,

$a\sin\theta = m\lambda$ where m = order of minima

Angular position $\theta = \frac{m\lambda}{a}$ $\sin\theta = \theta$ for small θ

when slit width $a = 40\lambda$ and when $\lambda = \lambda_1$:

Angular width of first maxima,

ϕ_1 = Angular position of 2nd minima – angular position of first minima

$$= \frac{2\lambda_1}{a} - \frac{\lambda_1}{a} = \frac{\lambda_1}{a}$$

When $\lambda = \lambda_2$:

Angular width of second maxima,

ϕ_2 = Difference in angular position of third and second minima

$$= \frac{3\lambda_2}{a} - \frac{2\lambda_2}{a} = \frac{\lambda_2}{a}$$

Their geometric mean $= \sqrt{\phi_1\phi_2} = \sqrt{\frac{\lambda_1\lambda_2}{a^2}}$

Now when slit width = a',

Wavelength = λ_3

No.of minima on one side of principle maxima = η

$\therefore$ Maximum order of minima = η

For maximum order of minima

$$a'\sin 90° = \eta\lambda_3$$

$\Rightarrow$ Angular position of first minima for new set-up

$$\frac{\lambda_3}{a'} = \frac{1}{\eta}$$

It is given that $\sqrt{2}\left[\frac{\lambda_1\lambda_2}{a^2}\right]^{1/2} = \frac{1}{\eta}$

$$\Rightarrow \quad \frac{\sqrt{2}[5000\times 6400]^{1/2}}{a} = \frac{1}{\eta}$$

$$\eta = (200/8) = 25$$

$$\therefore \quad a' = \eta\lambda_3 = 25\times 4000\,\text{Å} = 10\mu\text{m}$$

POLARIZATION

Let us examine waves travelling on a rope. A rope can be set into oscillation in a vertical plane as in Fig. 2.43(a), or in a horizontal plane as in Fig. 2.43(b). In either case, the wave is said to be **linearly polarized** or **plane-polarized** —that is, the oscillations are in a plane.

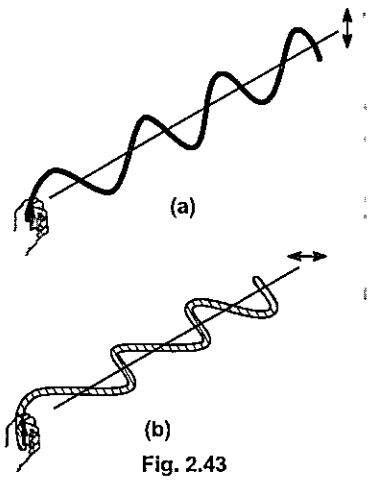


Fig. 2.43

If we now place an obstacle containing a vertical slit in the path of the wave (Fig. 2.44), a vertically polarized wave passes through, but a horizontally polarised wave will not. If a horizontal slit were used, the vertically polarized wave would be stopped. If both types of slits were used, both types of waves would be stopped. Note that polarization can exist *only* for *transverse waves*, and not for longitudinal waves such as sound.

Light is an electromagnetic wave in which the oscillating quantities are the electric and magnetic fields. In a plane wave, the directions of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and perpendicular to the direction of propagation of the wave. Light is a transverse wave, and consequently, it can be polarized in different ways.

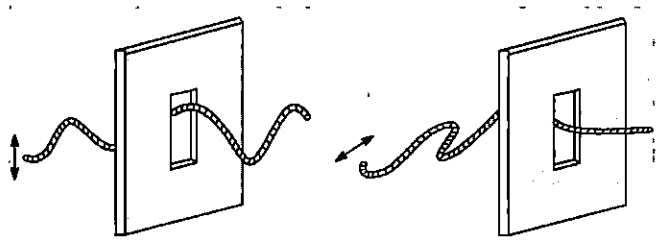


Fig. 2.44

The propagation direction of light is given by the direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$. In the figure, $\vec{\mathbf{E}}$ is directed toward $+y$ at the same position and time that $\vec{\mathbf{B}}$ is directed toward $+z$. Since $\hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}$, the direction of propagation is toward $+x$. It is customary to let the polarization direction be defined as along $\vec{\mathbf{E}}$ rather than $\vec{\mathbf{B}}$. In Fig. 2.45, the wave is polarized parallel to the y-axis because $\vec{\mathbf{E}}$ oscillates along the $\pm y$ directions. The plane that contains $\vec{\mathbf{E}}$ and the propagation direction is called the plane of polarization.

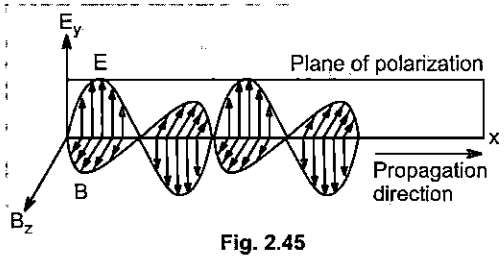


Fig. 2.45

In case of a plane-polarized wave, $\vec{\mathbf{E}}$ oscillates along a fixed line at any point. At any point along a circularly polarized wave, $\vec{\mathbf{E}}$ maintains a fixed magnitude, but its direction rotates in space with a constant angular frequency. Fig. 2.46(a) shows the time dependence of $\vec{\mathbf{E}}$ at a particular point due to a circularly polarized wave that is propagating along the $+x$ direction (out of the page). The variation of $\vec{\mathbf{E}}$

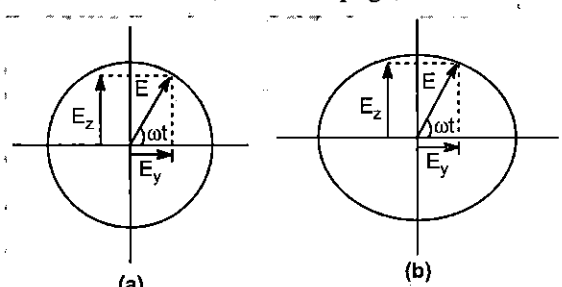


Fig. 2.46

may be represented by a vector of fixed magnitude that rotates about the x-axis with a constant angular frequency ω. In this picture, the tip of the $\vec{\mathbf{E}}$ vector traces out a circle, and the components of $\vec{\mathbf{E}}$—E_y and E_x— oscillate with the same amplitude and have a phase difference of $\lambda/2$ radian. If the tip of the $\vec{\mathbf{E}}$ vector in Fig. 2.46(a) rotates in the clockwise sense, then the light is *right*-circularly polarized, whereas if the tip rotates in the counterclockwise sense, then the light is *left*-circularly polarized.

An elliptically polarized wave [Fig 2.46(b)] is similar to a circularly polarized wave except that, at a particular point, E_y and E_x have different amplitudes. In this case the tip of the $\vec{\mathbf{E}}$ vector traces out an ellipse.

Ordinarily polarized light means *plane*-polarized light. Henceforth, we shall confine our attention to plane-polarized light.

Light emitted from an ordinary source, such as the filament of a light bulb, is *unpolarized*. Acting independently, the atoms and molecules emit wave trains of light, and the polarizations of these wave trains are unrelated. The resulting light consists of a random mixture of polarizations; it is unpolarized.

Measurement of Polarization

Light can be polarized by passing it through a *polarizer*. A polarizer is an optical device that selectively transmits light having its plane of polarization parallel to the polarizer's *transmission axis*. Light having its plane of polarization perpendicular to the transmission axis is blocked out by absorbtion or reflection. Fig. 2.47 shows a

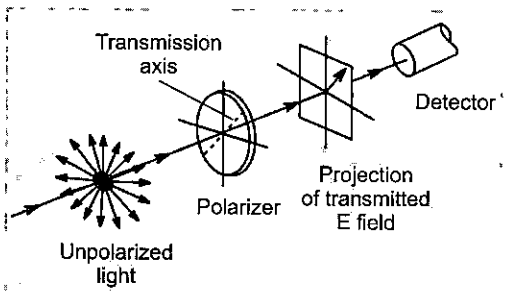


Fig. 2.47

beam of unpolarized light incident on a polarizer. The electric field of the transmitted wave at a particular point and time is parallel to the polarizer's transmission axis.

Malus's Law

Suppose we pass the polarized light from a polarizer through a second polarizer, as shown in Fig. 2.48. The

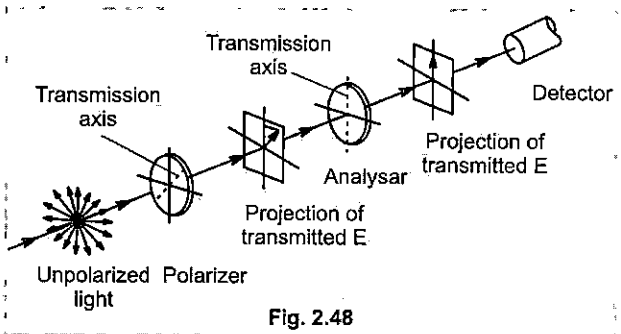


Fig. 2.48

second polarizer is often called the *analyzer* to distinguish it from the first polarizer. The azimuthal angle θ is a measure of the orientation of the polarizer's transmission axis relative to the analyzer's transmission axis. Let I_0 represent the intensity transmitted by the polarizer and incident on the analyzer. Notice that the detector can measure I_0 if the analyzer is removed temporarily.

In Fig. 2.49. we show the projection of the field components of the light as it emerges from the polarizer with amplitude E_0 and from the analyzer with amplitude $E_0 \cos\theta$. That is, the analyzer transmits only the component of the wave that is parallel to its transmission axis. Since the intensity is proportional to the square of the wave amplitude, we have that $I_0 \propto (E_0)^2$ and $I \propto (E_0 \cos\theta)^2$. The proportionality constant is the same in each case so that

$$I = I_0 \cos^2\theta$$

This is *Malus's law*. When $\theta = 0$ is substituted into Malus's law, we have $I = I_0 \cos^2 0 = I_0$. That is, the intensity transmitted by the analyzer is maximum when the transmission axes of the polarizer and analyzer are parallel. Further, when $\theta = \pi/2$ radian, $I = I_0 \cos^2(\pi/2) = 0$. That is, the intensity transmitted by the analyzer is minimum, when the transmission axes are perpendicular. Because there is no positive sense associated with a transmission axis, all possible intensity values, $0 < I < I_0$, are realized in the interval from $\theta = 0$ to $\theta = \pi/2$ radian.

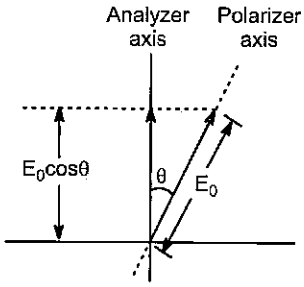


Fig. 2.49

Unpolarized light consists of light with random directions of polarization. Each of these polarization directions can be resolved into components along two mutually perpendicular directions. On average, an

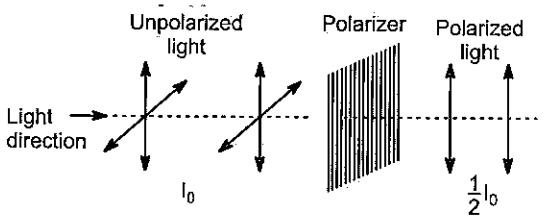


Fig. 2.50

unpolarized beam can be thought of as two plane-polarized beams of equal magnitude perpendicular to one another. When unpolarized light passes through a polarizer, one of the components is eliminated. So the intensity of the light passing through is reduced by half since half the light is eliminated,

$$I = \frac{1}{2} I_0.$$

Polarization by Reflection

When light strikes a nonmetallic surface at any angle other than perpendicular, the reflected beam is polarized preferentially in the plane parallel to the surface (Fig. 2.51). In other words, the component with polarization in the plane perpendicular to the surface is preferentially transmitted or absorbed. You can check this by

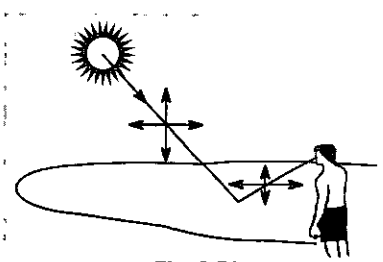


Fig. 2.51

rotating polaroid sunglasses while looking through them at a flat surface of a lake or road. Polaroid sunglasses are made with, their axes vertical to eliminate the more strongly reflected horizontal component, and thus reduce glare.

The amount of polarization in the reflected beam depends on the angle, varying from no polarization at normal incidence to 100 per cent polarization at an angle known as the **polarizing angle,** θ_p. This angle is related to the index of refraction of the two materials on either side of the boundary by the equation

$$\tan\theta_p = \frac{n_2}{n_1}, \qquad ...(1)$$

where n_1 is the index of refraction of the material in which the beam is travelling and n_2 is that of the medium beyond the reflecting boundary. If the beam is travelling in air, $n_1 = 1$, then eqn. (1) is

$$\tan\theta_p = n. \qquad ...(2)$$

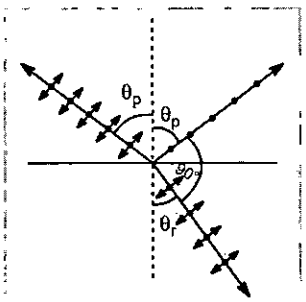


Fig. 2.52

The polarizing angle θ_p is also called **Brewster's angle,** and eqn. (2) is called *Brewster's law.* At Brewster's angle, the reflected and transmitted rays make a 90° angle to each other; *i.e.*, $\theta_p + \theta_r = 90°$ (Fig. 2.52). This can be seen by substituting eqn. (1), $n_2 = n_1 \tan\theta_p = n_1 \sin\theta_p/\cos\theta_p$, into Snell's law, $n_1 \sin\theta_p = n_2 \sin\theta_r$ and get $\cos\theta_p = \sin\theta_r$ which can only hold if $\theta_p = 90°-\theta_r$.

Degree of Polarization

In Malus's law, we assumed that both the polarizer and the analyzer were ideal. That is, all light polarized parallel to the transmission axis was transmitted, and no light polarized perpendicular to the transmission axis was transmitted. Real polarizers do not polarize light completely. The quantity used to characterize the polarization of light is the *degree of polarization P.*

Suppose the polarizer in Fig. 2.52 is not ideal, but the analyzer is effectively ideal. Let $I_{\parallel}$ represent the intensity measured by the detector, when the transmission axes are parallel ($\theta = 0$), and let $I_{\perp}$ represent the intensity when the axes are perpendicular ($\theta = \pi/2$ radian). The degree of polarization P of the light transmitted by the polarizer is defined as

$$P = \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + I_{\perp}} \qquad ...(3)$$

As an example, suppose the light incident on the analyzer is not polarized at all. Then $I_{\parallel} = I_{\perp}$ and substitution into eqn. (3) gives $P = 0$. As another example, suppose the light incident on the analyzer is completely polarized. Then $I_{\perp} = 0$ and substitution into eqn. (3) gives $P = 1$. Thus the degree of polarization ranges from a minimum of zero for unpolarized light to a maximum of 1 for completely polarized light.

Example 68

Unpolarized light passes through two polaroids; the axis of one is vertical and that of the other is at 60° to the vertical. What is the orientation and intensity of the transmitted light?

Solution: The first polaroid eliminates half the light, so the intensity is reduced by half: $I_1 = I_0/2$. The light reaching the second polarizer is vertically polarized and so is reduced in intensity to

$$I_2 = I_1(\cos 60°)^2 = \frac{1}{4}I_1.$$

Thus, $I_2 = I_0/8$. The transmitted light has an intensity one-eighth that of the original and is plane-polarized at a 60° angle to the vertical.

Example 69

When unpolarized light falls on two crossed Polaroids (axes at 90°), no light passes through. What happens if a third Polaroid, with axis at 45° to each of the other two, is placed between them ?

Solution: The first Polaroid changes the unpolarized light to plane-polarized and reduces the intensity from I_0 to $I_1 = I_0/2$. The second polarizer further reduces the intensity by $(\cos 45°)^2$,

$$I_2 = I_1(\cos 45°)^2$$
$$= \frac{1}{2}I_1 = \frac{1}{4}I_0.$$

The light leaving the second polarizer is plane polarized at 45° relative to the third polarizer, so the latter reduces the intensity to

$$I_3 = I_2(\cos 45°)^2 = \frac{1}{2}I_2$$

or

$$I_3 = \frac{1}{8}I_0.$$

Thus 1/8 of the original intensity gets transmitted. But if we do not put the 45° Polaroid in at all, zero intensity results. The 45° Polaroid must be inserted *between* the other two if transmission is to occur. Placing it before or after the other two results in zero intensity.

Subjective Problems

Interference (Elementary)

1. Consider two waves, one of frequency $\nu_1 = 5.455 \times 10^{15}$ Hz, the other with $\nu_2 = 5.450 \times 10^{15}$ Hz. If these two waves start exactly in phase at $t = 0$, (a) How long is it before they are exactly out of phase ? (b) How far has each wave travelled in this time ?

[**Ans.** (a) 1×10^{-13}s, (b) 30μm]

2. Consider two waves with frequencies ν_1 and ν_2. After travelling distance d the first wave has made one-half oscillation more than the second wave. Show that the frequencies are related by $\Delta\nu = \nu_1 - \nu_2 = v/2d$, where v is the speed of the wave.

3. Show that if two waves differ in frequency by an amount $\Delta\nu$ that is much smaller than either frequency, then the waves differ in wavelength by $\Delta\lambda = -(\Delta\nu/\nu)\lambda$.

4. Two identical waves with free-space wavelength λ_0 start in phase and travel parallel routes for a distance d. The first wave travels distance d in air ($n_0 = 1.00$) and the second wave travels distance d in a medium of refractive index n. Show that after travelling the distance d the waves have a phase difference $\Delta\phi = 2\pi[(n-1)d/\lambda_0]$.

5. Two slits separated by 0.200 mm are to be used in Young's double-slit experiment. Immediately behind the slits is a lens of focal length 0.500 m used to form the interference pattern on a screen located in the focal plane of the lens. What is the wavelength of a monochromatic light source used to illuminate the slits if adjacent maxima of the interference pattern are separated by 1.00 mm ?

[**Ans.** 400 nm]

6. A Young's double-slit experiment is observed in air. When the whole apparatus (source, slits and screen) is submerged into a fluid, the screen position originally occupied by the m^{th} bright fringe becomes the location of the m'^{th} dark fringe. In terms of m and m' m'^{th} find an expression for the index of refraction n of the fluid.

[**Ans.** $n = \left(m' - \frac{1}{2}\right)/m$]

7. Show that the total number of maxima produced by a two-slit apparatus cannot be greater than $(2D/\lambda) + 1$. Why is counting the number of maxima not a very precise way to determine λ ?

8. For a two-slit interference pattern, show that the angular half-width of each maximum—one-half the angular separation between the points on either side of the maximum at which the intensity is one-half that of the maximum—is

$$\frac{\Delta h}{R} = \Delta\theta = \frac{\lambda}{4D}.$$

9. When light of wavelength λ passes through two slits a distance d apart, what is the maximum possible number of interference fringes ?

[**Ans.** There are $\frac{2d}{\lambda} + 1$ bright fringes and $\frac{2d}{\lambda} - 1$ dark fringes]

10. One slit in a double slit is increased in size until it is 15 per cent larger than the other. (a) By what fraction is the intensity at the maxima increased? (b) What is the intensity at the former minima in terms of the intensity at the maxima ?

11. If you double the width of a single slit, the intensity of the light passing through the slit is doubled. (a) Show, however, that the intensity at the centre of the screen increases by a factor of 4. (b) Explain why this does not violate conservation of energy.

12. Two narrow parallel slits separated by 0.850 mm are illuminated by 600 nm light, and the viewing screen is 2.80 m away from the slits. (a) What is the phase difference between the two interfering waves on a screen at a point 2.50 mm from the central bright fringe ? (b) What is the ratio of the intensity at this point to the intensity at the centre of a bright fringe ?

[**Ans.** (a) 7.95 radian, (b) 0.453]

13. Monochromatic coherent light of amplitude E_0 and angular frequency ω passes through three parallel slits each separated by a distance d from its neighbour.

(a) Show that the time-averaged intensity as a function of the angle θ is

$$I(\theta) = I_{max}\left[1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]^2$$

(b) Determine the ratio of the intensities of the primary and secondary maxima.

14. Coherent light rays of wavelength λ strike a pair of slits separated by distance d at an angle of θ_1, as shown in Fig. 2.14. If an interference maximum is formed at an angle of θ_2 a great distance from the slits, show that $d(\sin\theta_2 - \sin\theta_1) = m\lambda$, where m is an integer.

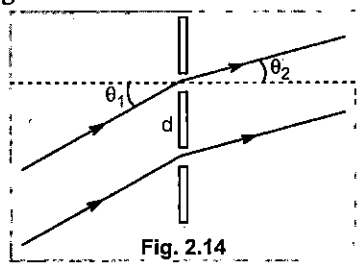


Fig. 2.14

15. Slit 1 of a double slit is wider than slit 2, so that the light from slit 1 has an amplitude 3.00 times that of the light from slit 2. Show that $I = (4I_{max}/9)(1 + 3\cos^2\phi/2)$ for this situation.

16. Determine the resultant of the two waves $E_1 = 6\sin(100\pi t)$ and $E_2 = 8\sin(100\pi t + \pi/2)$.
[**Ans.** $10\sin(100\pi t + 0.927)$]

17. Suppose that the slit openings in a Young's double-slit experiment have different sizes so that the electric fields and the intensities from each slit are different. If $E_1 = E_{01}\sin(\omega t)$ and $E_2 = E_{02}\sin(\omega t + \phi)$, show that the resultant electric field is $E = E_0\sin(\omega t + \theta)$, where

$$E_0 = \sqrt{E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos\phi}$$

and $$\sin\theta = \frac{E_{02}\sin\phi}{E_0}$$

18. When illuminated, four equally spaced parallel slits act as multiple coherent sources, each differing in phase from the adjacent one by an angle φ. Use a phasor diagram to determine the smallest value of φ for which the resultant of the four waves (assumed to be of equal amplitude) is zero.
[**Ans.** π/2]

19. Sketch a phasor diagram to illustrate the resultant of $E_1 = E_{01}\sin\omega t$ and $E_2 = E_{02}\sin(\omega t + \phi)$, where $E_{02} = 1.50E_{01}$ and $\pi/6 \le \phi \le \pi/3$. Use the sketch and the law of cosines to show that, for two coherent waves, the resultant intensity can be written in the from

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi.$$

20. Consider N coherent sources described by
$E_1 = E_0\sin(\omega t + \phi)$, $E_2 = E_0\sin(\omega t + 2\phi)$,
$E_3 = E_0\sin(\omega t + 3\phi), \ldots$, $E_N = E_0\sin(\omega t + N\phi)$.
Find the minimum value of φ for which $E_R = E_1 + E_2 + E_3 + \ldots + E_N$ is zero.
[**Ans.** $\frac{360}{N}$]

21. In a Young's double-slit experiment using light of wavelength λ, a thin piece of plexiglas having index of refraction n covers one of the slits. If the centre point on the screen is a dark spot instead of a bright spot, what is the minimum thickness of the plexiglas?
[**Ans.** $\lambda/2(n-1)$]

22. Consider the double-slit arrangement shown in Fig. 2.22, where the slit separation is d and the slit to screen distance is L. A sheet of transparent plastic having an index of refraction n and thickness t is placed over the upper slit. As a result, the central maximum of the interference pattern moves upward a distance y'. Find y'.

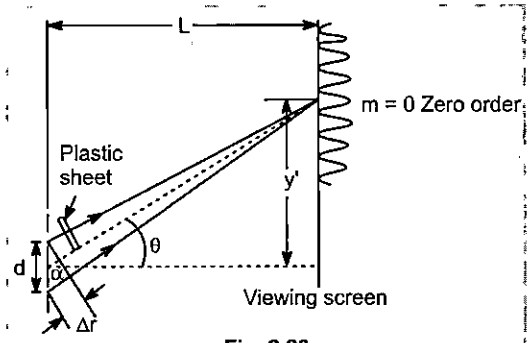


Fig. 2.22

[**Ans.** $y' = \frac{(n-1)tL}{d}$]

23. Astronomers observed a 60.0 MHz radio source both directly and by reflection from the sea. If the receiving dish is 20.0 m above sea level, what is the angle of the radio source above the horizon at first maximum ?
[**Ans.** 3.58°]

24. In a *YDSE*, let D = 120 cm and d = 0.250 cm. The slits are illuminated with coherent 600 nm light. Calculate the distance y above the central maximum for which the average intensity on the screen is 75.0% of the maximum.
[**Ans.** 48 μm]

25. An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the dominant observed colour in the reflected light and (b) the dominant colour in the transmitted light. Explain your reasoning.
[**Ans.** (a) Green, (b) Violet]

26. An air wedge is formed between two glass plates separated at one edge by a very fine wire, as shown in Fig. 2.26. When the wedge is illuminated from above by 600 nm light, 30 dark fringes are observed. Calculate the radius of the wire.

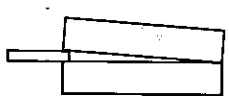


Fig. 2.26

[**Ans.** 4.35μm]

27. (a) Both sides of a uniform film that has index of refraction n and thickness d are in contact with air. For normal incidence of light, an intensity minimum is observed in the reflected light at λ_2, and an intensity maximum is observed at λ_1, where $\lambda_1 > \lambda_2$. If no intensity minima are observed between λ_1 and λ_2, show that the integer m in eqns. is given by $m = \lambda_1/2(\lambda_1 - \lambda_2)$. (b) Determine the thickness of the film if $n = 1.40, \lambda_1 = 500\,\text{nm}$ and $\lambda_2 = 370\,\text{nm}$.

28. Use phasor addition to find the resultant amplitude and phase constant when the following three harmonic functions are combined:

$E_1 = \sin(\omega t + \pi/6)$, $E_2 = 3.0\sin(\omega t + 7\pi/2)$,

$E_3 = 6.0\sin(\omega t + 4\pi/3)$.

[**Ans.** $7.99\sin(\omega t + 4.44 \text{ radian})$]

Interference from Thin Films

29. A thin film of a transparent material with $n = 1.29$ is to be placed on a glass ($n = 1.50$) surface. What is the minimum thickness for the film such that the reflection of normally incident light with $\lambda = 600\,\text{nm}$ is minimized ?

[**Ans.** 116 nm]

30. For non-normal incidence of light on a thin film show that

$$\tau_{\text{eff}} = \frac{\tau}{\cos\{\sin^{-1}[(\sin\theta)/n]\}}$$

, where n is the index of refraction of the film.

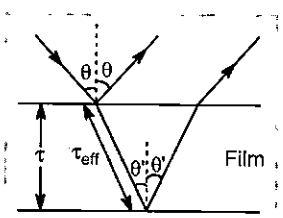


Fig. 2.30

31. A pattern similar to the double slit pattern can be produced using Lloyd's mirror (Fig. 2.31). At grazing angles almost 100 per cent of the light that falls on the glass is reflected so that the reflected beam has nearly the same amplitude at P as the beam that passes directly from slit S to P. Slit S and its virtual image S' behave as coherent sources. Develop expressions to give the positions of bright and dark fringes on the screen.

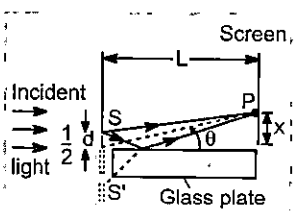


Fig. 2.31

[**Ans.** $d\sin\theta = \left(n + \frac{1}{2}\right)\lambda$ and $d\sin\theta = n\lambda$]

32. Consider a beam of light incident on a grating at an angle γ w.r.t. the normal, as shown in Fig. 2.32. Show that the interference maxima satisfy the relation

$d(\sin\gamma + \sin\theta) = \pm m\lambda \quad (m = 0, 1, 2, \ldots)$

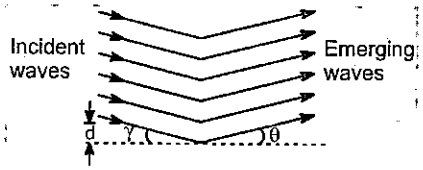


Fig. 2.32

33. In Fig. 2.33, two coherent point sources of waves, S_1 and S_2, are separated by a distance d along the x-axis. Show that the nodal lines are hyperbolas. That is, show that the coordinates of nodal lines satisfy the equation $(x/a)^2 - (y/b)^2 = 1$.

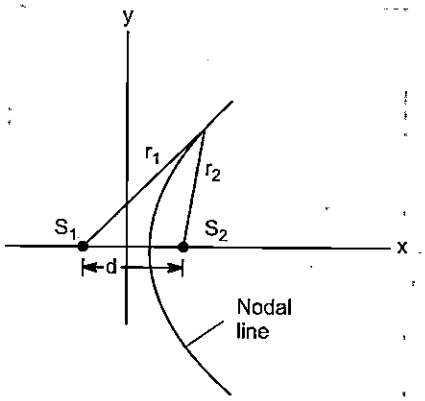


Fig. 2.33: A nodal line due to two coherent sources of waves is a hyperbola

[**Hint:** Recall that $|r_1 - r_2|$ is a constant on a nodal line.]

34. Fig. 2.34. shows a radio wave transmitter and a receiver separated by a distance d and located a distance h above the ground. The receiver can receive signals both directly from the transmitter and indirectly from signals that reflect off the ground. Assume that the ground is level between the transmitter and receiver and that a 180° phase shift occurs upon

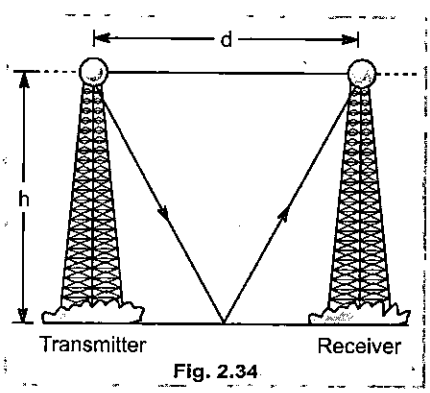


Fig. 2.34

reflection. Determine the longest wavelengths that interfere (a) constructively and (b) destructively.

[**Ans.** (a) $2(4h^2+d^2)^{1/2}-2d$, (b) $(4h^2+d^2)-d$]

35. A piece of transparent material having an index of refraction n is cut into the shape of a wedge, as shown in Fig. 2.35. The angle of the wedge is small, and monochromatic light of wavelength λ is normally incident from above. If the height of the wedge is h and the width is l, show that bright fringes occur at the positions $x = \lambda l(m+1/2)/2hn$ and that dark fringes occur at the positions $x = \lambda lm/2hn$, where $m = 0, 1, 2, \ldots$ and x is measured as shown.

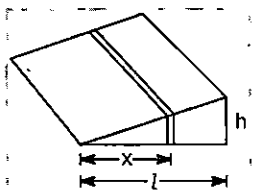


Fig. 2.35

36. The condition for constructive interference by reflection from a thin film in air assumes nearly normal incidence. Show that if the light is incident on the film at a nonzero angle ϕ_1 (relative to the normal), then the condition for constructive interference is $2nt\cos\theta_2 = (m+1/2)\lambda$, where θ_2 is the angle of refraction.

37. An air wedge is formed between two glass plates in contact along one edge and slightly separated at the opposite edge. When the plates are illuminated with monochromatic light from above, the reflected light has 85 dark fringes. Calculate the number of dark fringes that would appear if water (n = 1.33) were to replace the air between the plates.

[**Ans.** 113 dark fringes]

38. Measurements are made of the intensity distribution in a Young's interference pattern. At a particular value of y, it is found that $I/I_{max} = 0.810$, when 600 nm light is used. What wavelength of light should be used if the relative intensity at the same location is to be reduced to 64.0% ?

[**Ans.** 421 nm]

39. A useful interferometer design is the Fabry-Perot interferometer. The essential elements of this design are shown in Fig. 2.39. Show that the condition for constructive interference of the waves that reach the detector is $2d/\cos\theta = m\lambda$, where m = an integer and d is the distance between the parallel mirrors.

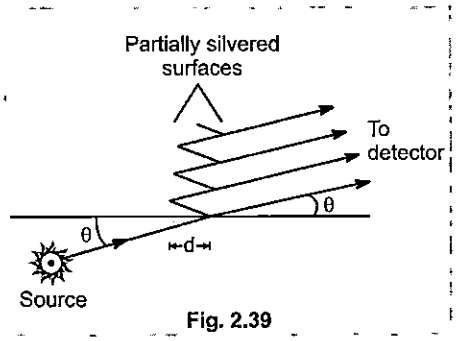


Fig. 2.39

40. In Pohl's interferometer, a thin sheet of mica is put behind a shielded light source, as in Fig. 2.40. Each side of the mica reflects light from the source, so that there are two coherent virtual sources along a line perpendicular to the screen. Show that the equation for the radius R of bright circular fringes on the screen is

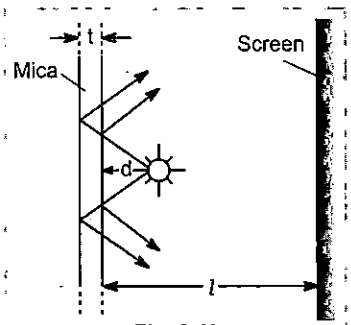


Fig. 2.40

$$\sqrt{R^2+(l+2d+2t)^2}-\sqrt{R^2+(l+2d)^2}=(n-1/2)\lambda$$

41. A two-slit interference pattern is produced by a pair of slits whose widths are equal to one-tenth, their separation. (a) Show that some of the maxima will be missing from the interference pattern. (b) What is the value of m for the first missing maximum on each side of the centre ?

42. Two optical flats are placed in near contact at one edge and are separated by a human hair at the other edge as shown in Fig. 2.42. (a) At the thin edge of the air wedge the separation of the glass surfaces is much less than the wavelength of visible light. Does this point appear bright or dark when viewed by reflected light ? (b) If filtered light from a mercury vapour source (λ = 546 nm) is used, exactly nine bright fringes are located between the edges in contact and the position of the hair. How thick is the hair ?

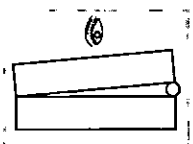


Fig. 2.42

[**Ans.** (a) Dark, (b) 2.32μm]

43. A Fresnel biprism can be used together with a point-source located distance d behind the centre line of the prism to create a two-source interference pattern. As shown in Fig. 2.43 the two images P_1' and P_2' formed by refraction at the prism act as coherent

sources. (a) Show that the separation a of the images is given approximately by $a = 2d\alpha(n-1)$.

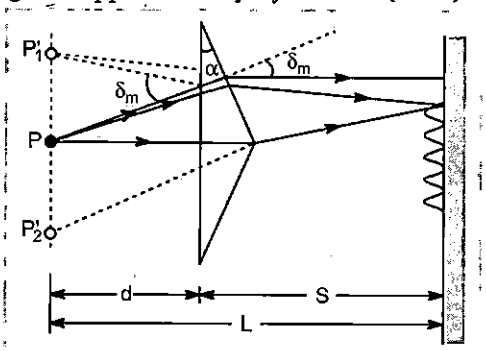


Fig. 2.43

[**Hint:** Assume that the prism angle is small ($\alpha < 1°$) and that the image is formed in the same plane as the source. Trace a ray striking the prism such that it is deviated by the angle of minimum deviation.]

(b) Calculate the fringe separation on a screen located a distance $s = 1.750$ m from the back face of a prism of refractive index $n = 1.500$ and vertex angle 1.00° if the point-source has wavelength $\lambda = 632.8$ nm and is located a distance $d = 0.750$ nm behind the back face of the prism.

[**Ans.** (b) 8.52 mm]

44. The thin glass wedge shown in Fig. 2.44 is slid slowly infront of one of the slits of a two-slit interference apparatus illuminated by sodium light ($\lambda = 589$ nm). The length of the wedge is much greater than the width of the slit, so the part of the wedge infront of the slit at any moment varies only slightly in thickness. In the process, the pattern shifts so that what was originally the tenth minimum ($m = 9$) now lies at the midline? What is the thickness of the thickest part of the wedge ? Take $n = 1.54$ for the glass.

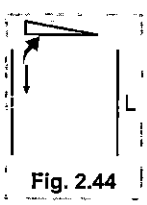


Fig. 2.44

45. In Young's double-slit experiment a thin layer of mica is placed directly behind one of the slits. The thickness of the mica is such that the optical path length of the light in the mica is 0.500λ. (a) Assuming no power is absorbed from the light passing through the mica, describe the change in the far-field diffraction pattern that occurs when the mica is inserted behind one slit. (b) If the wavelength of the light used is 632.8 nm, what is the actual thickness of the mica ? (Assume the mica is oriented so that its index of refraction is 1.552.)

[**Ans.** (a) Pattern is shifted by one-half fringe: dark to bright, light to dark, (b) 204 nm]

46. A lens with a slightly curved surface is placed on an optical flat generating a pattern of circular rings, when viewed in reflected monochromatic light. The curvature is sufficiently slight that it is not possible to tell by simple observation whether the surface is convex or concave. (a) If one pushes gently on the centre of the lens, circular fringes move toward the centre or away from the centre depending on whether the lens surface is concave or convex (Fig. 2.46). Which way do they move in the convex case? (b) An alternative technique for distinguishing concave from convex is to view the interference pattern using white light. In that case the order of the colour, from the centre out is opposite for the two cases. What is the ordere of the colours (from the centre out) for the convex case ?

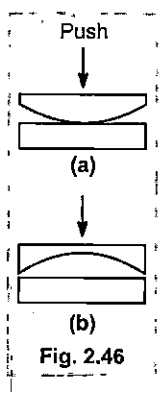


Fig. 2.46

[**Ans.** (a) Convex move out, (b) from blue to red]

47. A plano-convex lens has index of refraction n. The curved side of the lens has radius of curvature R and rests on a flat glass surface of the same index of refraction, with a film of index n_{film} between them. The lens is illuminated from above by light of wavelength λ. Show that the dark Newton's rings have radii given approximately by

$$r \cong \sqrt{m\lambda R/n_{film}}$$

where m is an integer and r is much less than R.

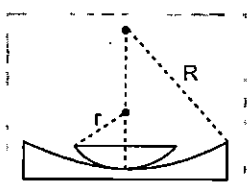


Fig. 2.47

48. The Fig. 2.48 shows a glass rod, having one cylindrical and one flat surface, in contact with a flat plate of glass. (a) What is the shape of the interference fringes ? What are the distances from the line of contact of (b) the bright fringes ? (c) the dark fringes?

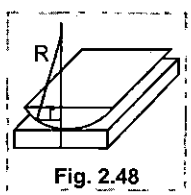


Fig. 2.48

[**Ans.** (a) The fringes run parallel to the axis of the rod. In the centre there is a wide dark fringe. The fringes get narrower the farther you move away from the centre line of contact

(b) $r = \sqrt{(n+1/2)\lambda R}$, $n = 0, 1, 2, \ldots$,

(c) $r = \sqrt{n\lambda R}$, $n = 0, 1, 2, \ldots$]

49. Two flat mirrors form an angle close to 180° (Fig. 2.49). A source of light S is placed at equal distances b from the mirrors. Find the interval between adjacent interference bands on screen MN at a distance $OA = a$ from the point of intersection of the mirrors. The length of the light

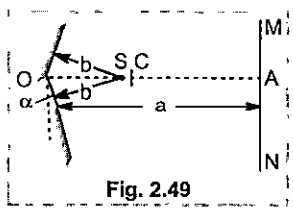


Fig. 2.49

wave is known and equal to λ. Shield C does not allow the light to pass directly from the source to the screen.

[**Ans.** $\Delta h = \dfrac{\lambda(a+b)}{2b\alpha}$]

50. Lloyd's interference experiment consisted in obtaining on a screen a pattern from source S and from its virtual image S' in mirror AO (Fig. 2.50). How will the interference pattern obtained from sources S and S' differ from the pattern obtained in Young's double slit experiment ?

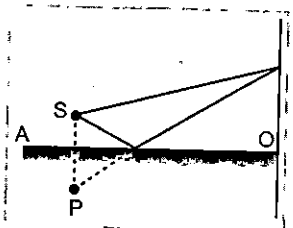


Fig. 2.50

[**Ans.** The entire pattern will be shifted by width of dark fringe.]

51. Two point sources with the same phases of oscillation are on a straight line perpendicular to a screen. The nearest source is at a distance of $D >> \lambda$ from the screen. What shape will the interference bands have on the screen ? (a) What is the distance on the screen from the perpendicular to the nearest bright band if the distance between the sources is $l = n\lambda >> \lambda$ (n is an integer)? (b) Find the radius r_k of the k^{th} bright ring if $D = l = n\lambda$, $n >> l$, and $k = n, n-1, n-2$, etc.

[**Ans.** (a) $h_{n-1} \simeq \sqrt{2D\lambda\left(\dfrac{D}{l}+1\right)}$,

(b) $r_k = \dfrac{\lambda}{2k}\sqrt{(9n^2-k^2)(n^2-k^2)}$]

52. Light from source S is incident on the Fresnel biprism shown in Fig. 2.52. The light beams refracted by the different faces of the prism partly overlap and produce an interference pattern on a screen on its section AB. (a) Find the distance between adjacent

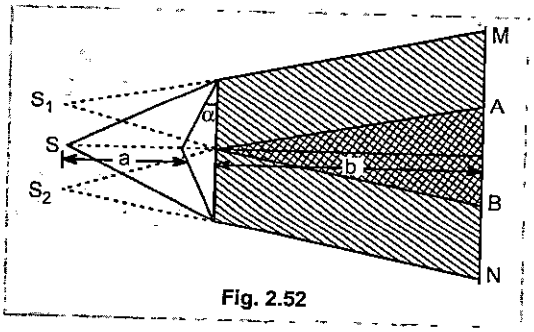


Fig. 2.52

interference bands if the distance from the source to the prism is $a = 1$ metre and from the prism to the screen $b = 4$ metre. The angle of refraction of the prism is $\alpha = 2\times10^{-3}$ radian. The glass which the prism is made of has a refraction index of $n = 1.5$. The length of the light wave $\lambda = 6000$ Å. (b) How many interference bands can be observed on a screen in an installation with the biprism described in the previous problem ?

[**Ans.** (a) $\Delta h = \dfrac{\lambda(a+b)}{2a\alpha(n-1)} = 0.15$ cm,

(b) $N = \dfrac{4ab\alpha^2(n-1)^2}{(a+b)\lambda} \simeq 5$]

53. A biprism with an angle β that appreciably differs from 180° is placed into a vessel filled with a liquid having a refraction index n_1 or serves as one of the walls of this vessel (Fig. 2.53). Calculate the angle δ of an equivalent biprism in air. The refraction index of the prism substance is n_2.

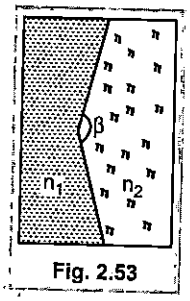


Fig. 2.53

[**Ans.** $\delta = \beta\left(\dfrac{n_2-n_1}{n_2-1}\right) + 180°\left(\dfrac{n_1-1}{n_2-1}\right)$]

54. A convergent lens with a focal length of $f = 10$ cm is cut into two halves that are, then moved apart to a distance of $d = 0.5$ mm (a double lens). Appraise the number of interference bands on a screen at a distance of $D = 60$ cm behind the lens if a point source of monochromatic light ($\lambda = 5000$ Å) is placed infront of the lens at a distance of $a = 15$ cm from it.

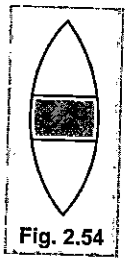


Fig. 2.54

[**Ans.** $N = 25$]

55. Why are Newton's rings formed only by the interference of rays 2 and 3 reflected from the boundaries of the air layer between the lens and the glass (Fig. 2.55), while ray 4 reflected from the flat face of the lens does not affect the nature of the interference pattern?

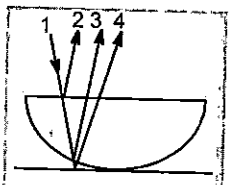


Fig. 2.55

[**Ans.** The lens is too thick, interference occurs only with thin films. The air layer between the lens and the glass is thin.]

Diffraction

56. Light of wavelength 550 nm falls on a slit that is 3.50×10^{-3} mm wide. Estimate how far from the central maximum is the first diffraction maximum fringe if the screen is 10.0 m away ?

[**Ans.** 2.4 m]

57. If a slit diffracts 550 nm light so that the diffraction maximum is 8.0 cm wide on a screen 2.50 m away, what will be the width of the diffraction maximum for light with a wavelength of 400 nm ?

[**Ans.** 5.8 cm]

58. How wide is the central diffraction peak on a screen 3.50 m behind a 0.0655 mm wide slit illuminated by 400 nm light ?

[**Ans.** 4.28 cm]

59. A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.

[**Ans.** 3.09 m]

60. A child is standing at the edge of a straight highway watching her grandparents' car driving away at 20.0 m/s. The air is perfectly clear and steady, and after 10.0 min the car's two taillights appear to merge into one. Assuming the diameter of the child's pupils is 5.00 mm, estimate the width of the car.

[**Ans.** 1.90 m if the predominant wavelength is 650 nm]

61. A diffraction pattern is formed on a screen 120 cm away from a 0.400 mm wide slit. Monochromatic 546.1 nm light is used.

Calculate the fractional intensity I/I_0 at a point on the screen 4.10 mm from the centre of the principal maximum.

[**Ans.** 0.0162]

62. The angular resolution of a radio telescope is to be 0.100° when the incident waves have a wavelength of 3.00 mm. What minimum diameter is required for the telescope's receiving dish ?

[**Ans.** 2.10 m]

63. (a) Explain why the secondary maxima in the single-slit diffraction pattern do not occur precisely at $\beta/2 = (m + 1/2)\pi$ where $m = 1, 2, 3, \ldots$ (b) Show that the secondary maxima occur when $\beta/2$ satisfies the relation $\tan(\beta/2) = \beta/2$. (c) Carefully and precisely plot the curves $y = \beta/2$ and $y = \tan\beta/2$. From their intersections, determine the values of β for the first and second secondary maxima. What is the per cent difference from $\beta/2 = (m + 1/2)\pi$?

64. Determine, approximately, the angular width at half maximum (where $I = I_0/2$) of the central diffraction peak for a single slit.

To be concrete, assume $\lambda = 550$ nm and $a = 2.60 \times 10^{-3}$ mm.

[**Hint:** Use graphical methods, or trial and error; the problem cannot be solved analytically.]

[**Ans.** 10.70°]

65. If a double-slit pattern contains exactly seven fringes in the central diffraction peak, what can you say about the slit width and separation ?

[**Ans.** $d = 4a$]

66. Two 0.010 mm wide slits are 0.030 mm apart (centre to centre). Determine (a) the spacing between interference fringes for 550 nm light on a screen 1.0 m away and (b) the distance between the two diffraction minima on either side of the central maximum of the envelope.

[**Ans.** (a) 1.8 cm, (b) 11 cm]

67. Missing orders occur for a diffraction grating when a diffraction minimum coincides with an interference maximum. Let a be the width of each slit and d the separation of slits and show (a) that if $d = 2a$, all even orders ($m = 2, 4, 6, \ldots$) are missing. (b) Show that there will be missing orders whenever

$$\frac{d}{a} = \frac{m_1}{m_2}$$

where m_1 and m_2 are integers. (c) Discuss the case $d = a$, the limit in which the space between slits becomes negligible.

68. A diffraction grating has 16000 rulings in its 2.4 cm width. Determine (a) its resolving power in first and second orders, and (b) the minimum wavelength resolution ($\Delta\lambda$) it can yield for $\lambda \approx 410$ nm.

[**Ans.** (a) 1.60×10^4; 3.20×10^4,

(b) 0.026 nm; 0.013 mm]

69. Determine a formula for the minimum difference in frequency, Δf, that a diffraction grating can resolve, when two frequencies, $f_1 \approx f_2 = f$, are incident on it.

[**Ans.** $\Delta f = f/mN$]

70. Monochromatic light falls on a transmission diffraction grating at an angle ϕ to the normal. Show that eqn. for diffraction maxima must be replaced by

$$d(\sin\phi \pm \sin\theta) = m\lambda, \; m = 0, 1, 2, \ldots$$

Explain the $\pm$ sign.

71. Two wavelengths λ and $\lambda + \Delta\lambda$ (with $\Delta\lambda \ll \lambda$) are incident on a diffraction grating. Show that the angular separation between the spectral lines in the m^{th} order spectrum is,

$$\Delta\theta = \frac{\Delta\lambda}{\sqrt{(d/m)^2 - \lambda^2}}$$

where d is the slit spacing and m is the order number.

72. Two stars, each of which produces about the same intensity at the earth, have an angular separation of 7 μ radian. If the light from the stars has an average wavelength of 600 nm, what is the minimum aperture diameter needed to resolve them ?

73. (a) How far away can a human eye distinguish two car headlights 2.0 m apart ? Consider only diffraction effects and assume an eye diameter of 5.0 mm and the wavelength of 500 nm. (b) What is the minimum angular separation an eye could resolve when viewing two stars, considering only diffraction effects ? In reality, it is about 1′ of arc. Why is it not equal to your answer in (b) ?

[**Ans.** (a) 16 km, (b) 0.42']

74. What is the highest spectral order that can be seen if a grating with 6000 lines per cm is illuminated with 633 nm laser light ? Assume normal incidence.

[**Ans.** Two orders]

75. What is the limiting angle of resolution of a telescope that has an aperture diameter of 75 mm ? The light from the objects being viewed has a wavelength of 500 nm.

[**Ans.** 8.1 μ radian]

76. Show that if the width of a slit is halved, the intensity at the centre of the screen is quartered. From this it might seem that energy is not conserved, since half as much power is passed by the smaller slit, but a quarter as much intensity shows up at the centre of the pattern. Show that the power delivered to the screen by the light in the central fringe (between the minima on either side of the central maximum) by the two slits is proportional to the width of the slit.

77. The spacing $\Delta\theta$ between two spectral lines of equal intensity is the same as their half-width $\Delta\theta_{1/2}$. Show that the ratio of the intensity at the point midway between the maxima and the intensity at the maxima is $2(2/\pi)^2 = 0.81$.

[**Hint:** $\sin(\pi/2N) \approx \pi/2N$ for large N]

78. The resolving power of an optical device whose purpose is to disperse light according to wavelength is defined as :

$$R = \frac{\lambda}{\Delta\lambda}$$

where $\Delta\lambda$ is the difference in wavelength of two spectral lines that can barely be resolved and λ is the wavelength of either line (they have nearly the same wavelength). Show that the resolving power of a grating for the m^{th} order spectrum in Nm.

79. (a) Show that the Rayleigh criterion for the resolution of the diffraction patterns due to the two sources of the same intensity passing through a single slit is $\Delta\theta_R \approx \lambda/a$, where a is the slit width (Fig. 2.79). (b) Show that this leads to an intensity at the centre of the joint pattern of two fringes of 81 per cent of the intensity at the peak on either side of the centre.

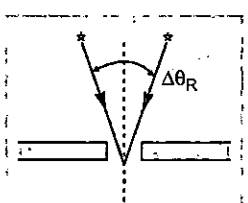


Fig. 2.79

80. Suppose that the single slit is 6.00 cm wide and infront of a microwave source operating at 7.50 GHz. (a) Calculate the angle subtended by the first minimum in the diffraction pattern. (b) What is the relative intensity I/I_{max} at $\theta = 15.0°$? (c) Consider the case when there are two such sources, separated laterally by 20.0 cm, behind the slit. What must the maximum distance between the plane of the sources and the slit be if the diffraction patterns are to be resolved ? (In this case, the approximation $\sin\theta \approx \tan\theta$ is not valid because of the relatively small value of a/λ.)

[**Ans.** (a) 41.8°, (b) 0.593, (c) 0.262 m]

81. Light of wavelength 632.8 nm illuminates a single slit, and a diffraction pattern is formed on a screen 1.00 m from the slit. Using the data in the table on the following page, plot relative intensity *vs.* distance. Choose an appropriate value for the slit width a, and on the same graph used for the experimental data, plot the theoretical expression for the relative intensity

$$\frac{I}{I_{max}} = \frac{\sin^2(\beta/2)}{(\beta/2)^2}$$

What value of a gives the best fit of theory and experiment ?

[**Ans.** $a = 99.5\mu m \mp 1\%$]

82. How much diffraction spreading does a light beam undergo ? One quantitative answer is the *full width at half maximum* of the central maximum of the Fraunhofer diffraction pattern of a single slit. You can evaluate this angle of spreading in this problem and in the next. (a) In result of problem 81 define $\beta/2 = \phi$ and show that, at the point where $I = 0.5I_{max}$, we must have $\sin\phi = \phi/\sqrt{2}$. (b) Let $y_1 = \sin\phi$ and $y_2 = \phi/\sqrt{2}$. Plot y_1 and y_2 on the same set of axes over a range from $\phi = 1$ radian to $\phi = \pi/2$ radian. Determine ϕ from the point of intersection of the two curves. (c) Then show that, if the fraction λ/a is not large, the angular full width at half maximum of the central diffraction maximum is $\Delta\theta = 0.88\lambda/a$.

83. A *pinhole camera* has a small circular aperture of diameter D. Light from distant objects passes through the aperture into an otherwise dark box, falling upon a screen located a distance L away. If D is too large, the display on the screen will be fuzzy because a bright point in the field of view will send light onto a circle of diameter slightly larger than D. On the other hand, if D is too small, diffraction will blur the display on the screen. The screen shows a reasonably sharp image if the diameter of the central disc of the diffraction pattern, is equal to D at the screen. (a) Show that for monochromatic light with plane wavefronts $L >> D$, the condition for a sharp view is fulfilled if $D^2 = 2.44\lambda L$. (b) Find the optimum pinhole diameter if 500 nm light is projected onto a screen 15.0 cm away.

[**Ans.** (b) 0.428 mm]

Polarization

84. At what angle should the axes of two polaroids be placed so as to reduce the intensity of the incident unpolarized light by an additional factor (after the first polaroid cuts cuts it in half) of (a) 25 per cent, (b) 10 per cent, (c) 1 per cent ?

[**Ans.** (a) 30°, (b) 18°, (c) 5.7°]

85. Show that if two equally intense sources of light produce light that is plane-polarized, but with their planes of polarization perpendicular to each other, then they cannot produce an interference pattern even if they are in phase at all moments.

86. Unpolarized light falls on two polarizer sheets whose axes are at right angles. (a) What fraction of the incident light intensity is transmitted ? (b) What fraction is transmitted if a third polarizer is placed between the first two so that its axis makes a 60° angle with the axis of the first polarizer ? (c) What if the third polarizer is infront of the other two ?

[**Ans.** (a) 0, (b) $0.094 I_0$, (c) no light gets transmitted]

87. What would Brewster's angle be for reflections off the surface of water for light coming from beneath the surface ? Compare to the angle for total internal reflection, and to Brewster's angle from above the surface.

[**Ans.** 36.9°, 53.1°]

88. Unpolarized light passes through five successive polaroid sheets each of whose axes makes a 45° angle with the previous one. What is the intensity of the transmitted beam ?

[**Ans.** $I_0/32$]

89. For a particular transparent medium surrounded by air, show that the critical angle for total internal reflection and the polarizing angle are related by the expression $\cot\theta_p = \sin\theta_c$.

90. How far above the horizon is the Moon, when its image reflected in calm water is completely polarized? ($n_{water} = 1.33$)

[**Ans.** 37° above the horizon]

91. In Fig. 2.91, suppose that the transmission axes of the left and right polarizing discs are perpendicular to each other. Also, let the centre disc be rotated on the common axis with an angular speed ω. Show that if unpolarized light is incident on the left disc with an intensity I_{max}, the intensity of the beam emerging from the right disc is,

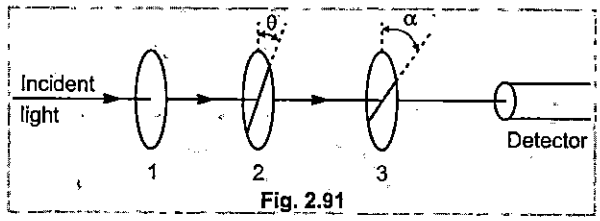


Fig. 2.91

$$I = \frac{1}{16} I_{max}(1 - \cos 4\omega t)$$

This means that the intensity of the emerging beam is modulated at a rate that is four times the rate of rotation of the centre disc.

[**Hint:** Use the trigonometric identities $\cos^2\theta = (1 + \cos 2\theta)/2$ and $\sin^2\theta = (1 - \cos 2\theta)/2$, and recall that $\theta = \omega t$.]

92. You want to rotate the plane of polarization of a polarized light beam by 45.0° with a maximum intensity reduction of 10.0%. (a) How many sheets of perfect polarizers do you need to achieve your goal ? (b) What is the angle between adjacent polarizers?

[**Ans.** (a) 6, (b) 19°]

93. The per cent polarization P of a partially polarized beam of light is defined as

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100$$

where I_{max} and I_{min} are the maximum and minimum intensities that are obtained, when the light passes through a polarizer that is slowly rotated. Such light can be considered as the sum of two unequal plane-polarized beams of intensities I_{max} and I_{min} perpendicular to each other. Show that the light transmitted by a polarizer, whose axis makes an angle ϕ to the direction in which I_{max} is obtained, has intensity

$$\frac{(1 + p\cos 2\phi)}{(1 + p)} I_{max}$$

where $p = P/100$ is the "fractional polarization".

94. Suppose we have N (ideal) polarizers in the beam in see Fig. 2.91 instead of three. The beam incident on polarizer 1 is plane polarized, and polarizer 1 has its transmission axis rotated an angle θ/N to the plane of polarization of the incident beam. Each subsequent polarizer has its axis rotated in the same sense by an angle of θ/N to the one before it, so that the last polarizer has its axis rotated by an angle θ to the plane of polarization of the beam incident on polarizer 1. Show that the intensity transmitted by the last polarizer is

$$I = I_0\left(\cos\frac{\theta}{N}\right)^{2N}$$

where I_0 is the intensity of the incident beam.

95. Suppose the angle between the transmission axes of polarizers 1 and 3 in see Fig. 2.91 is held fixed at $\pi/2$ radian while the angle θ between the axes of 1 and 2 is allowed to vary. Show that the intensity at the detector is given by

$$I = I_0\frac{(1-\cos(4\theta))}{16}$$

where I_0 is the intensity of the unpolarized beam incident on polarizer 1.

96. A partially polarized beam may be regarded as two incoherent beams with different intensities and with perpendicular planes of polarization. Let I_1 represent the intensity of the more intense beam, and let I_2 represent the intensity of the less intense beam. (a) Show that when a partially polarized beam is incident on an ideal polarizer, the transmitted intensity is

$$I = (I_1 - I_2)\cos^2\theta + I_2$$

where θ is the angle between the polarizer's transmission axis and the plane of polarization of the more intense beam. (b) Show that the intensity transmitted by the polarizer may be written as

$$I = I_1\frac{(1+P\cos(2\theta))}{(1+P)}$$

where P is the degree of polarization of the beam.

Problems

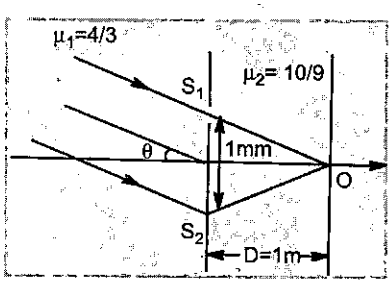
Level 1 Only One Alternative is Correct

1. In a double slit experiment, the coherent sources are spaced $2d$ apart and the screen is placed a distance D from the slits. If n^{th} bright fringe is formed on the screen exactly opposite to a slit, the value of n must be:

(a) $\frac{d^2}{2\lambda D}$ (b) $\frac{2d^2}{\lambda D}$

(c) $\frac{d^2}{\lambda D}$ (d) $\frac{d^2}{4\lambda D}$

2. In YDSE experiment shown in the figure, a parallel beam of light of wavelength ($\lambda = 0.3$ mm) in medium μ_1 is incident at an angle $\theta = 30°$ as shown ($S_1O = S_2O$). If the intensity due to each light wave at point O is I_0 then the resultant intensity at point O will be :



(a) zero (b) $2I_0[1 + \cos(40\pi/9)]$

(c) $3I_0$ (d) I_0

3. Two glass plates are touching at one end and separated by a thin wire at the other end. When a monochromatic parallel beam of wavelength 4200Å incident normally on the glass plates is reflected, an interference pattern of 30 fringes are observed. If the wavelength of light used is taken 7000Å instead of 4200Å, the number of fringes observed will be :

(a) 50 (b) 40

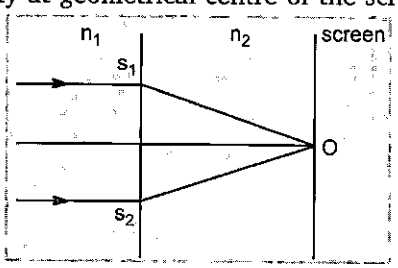
(c) 30 (d) 18

4. In the above question 3, if the free space between the plates is replaced by a transparent liquid of refractive index 1.5 and the wavelength of light remains 4200Å the number of fringes seen will be :

(a) 60 (b) 45

(c) 30 (d) 20

5. In a Young's double slit experiment the slits are illuminated by a parallel beam of light from the medium of refractive index $n_1 = 1.2$ A thin transparent film of thickness 1.2μm and refractive index $n = 1.5$ is placed infront of s_1 perpendicular to path of light. Wavelength of light measured in medium n_1 is 400nm. The refractive index of medium between plane of slits and screen is $n_2 = 1.4$. If the light coming from the film and s_2 have equal intensities I then intensity at geometrical centre of the screen is :



(a) 0 (b) $2I$

(c) $4I$ (d) none of these

6. A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge with the wedge angle of 20 second of an arc. If the refractive index of glass is 1.5, find the number of dark interference fringes per cm of the wedge length :

(a) 2 per cm (b) 5 per cm

(c) 7 per cm (d) 6.5 per cm

7. Which one of the following statements is **not** true ? The phase difference between two light waves can change :

(a) by reflection of one of the waves

(b) by the waves travelling along paths of different lengths

(c) by the waves travelling through media that have a different refractive indices but the same thickness
(d) by being transmitted through a boundary between two media with different refraction indices

8. In a Young's double slit experiment the slit separation is 0.5 mm and the screen is 0.5 m from the slit. For a monochromatic light of wavelength 500 nm the distance of 3rd maxima from the 2nd minima on the other side is :
(a) 2.75 mm (b) 2.5 mm
(c) 22.5 mm (d) 2.25 mm

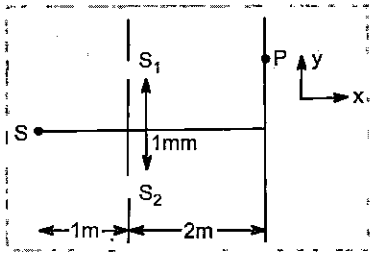
9. Two sources S_1 and S_2 separated by 2 metres vibrate according to the equations $y_1 = 0.03\sin\pi t$ and $y_2 = 0.02\sin\pi t$ where y_1 and y_2 are in metres. They send out waves of velocity 1.5 m/s. The amplitude of the resultant motion of particle collinear with S_1 and S_2 and at the middle of S_1S_2 will be :
(a) 0.05 m (b) 0.01 m
(c) 0.11 m (d) 0.06 m

10. In a standard YDSE apparatus a thin film ($\mu = 1.5$, $t = 2.1\mu m$) is placed in front of upper slit. How far above and below the centre point of the screen are two nearest maxima located ? Take $D = 1$ m, $d = 1$ mm $\lambda = 4500$Å. (Symbols have usual meaning)
(a) 1.5 mm (b) 0.6 mm
(c) 0.15 mm (d) 0.3 mm

11. In a Lloyd's mirror experiment if the mirror reflects 75% of light incident on it the ratio of intensity at interference maxima and minima will be :
(a) 75 (b) 150
(c) 194 (d) 200

12. High quality camera lenses are often coated to prevent reflection. A lens has an optical index of refraction of 1.72 and a coating with an optical index of refraction of 1.31. For near normal incidence the minimum thickness of the coating to prevent reflection for wavelength of 5.3×10^{-7} m is :
(a) 0.75μm (b) 0.2μm
(c) 0.1μm (d) 1.75μm

13. In a Young's double slit experiment set up, source S of wavelength 500 nm illuminates two slits S_1 and S_2 which act as two coherent sources. The source S



oscillates about its own position according to the equation $y = 0.5\sin\pi t$ where y is in mm and t in seconds. The minimum value of time t for which the intensity at point P on the screen exactly infront of the upper slit becomes minimum is :
(a) 1 s (b) 2 s
(c) 3 s (d) 1.5 s

14. Intensity observed in an interference pattern $I = I_0\sin^2\theta$. At $\theta = 30°$ intensity $I = 5\pm0.002$. The percentage error in angle if $I_0 = 20\frac{W}{m^2}$ is :
(a) $4\sqrt{3}\times10^{-2}\%$ (b) $\frac{4}{\pi}\times10^{-2}\%$
(c) $\frac{4\sqrt{3}}{\pi}\times10^{-2}\%$ (d) $\sqrt{3}\times10^{-2}\%$

15. A long horizontal slit is place 1 mm above a horizontal plane mirror. The interference between the light coming directly from the slit and that after reflection is seen on a screen 1 m away from the slit. If the mirror reflects only 64% of the light falling on it, the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen is :
(a) 8:1 (b) 3:1
(c) 81:1 (d) 9:1

16. Two monochromatic coherent point sources S_1 and S_2 are separated by a distance L. Each source emits light of wavelength λ, where $L >> \lambda$. The line S_1S_2 when extended meets a screen perpendicular to it at a point A.
(a) The interference fringes on the screen are circular in shape
(b) The interference fringes on the screen are straight lines perpendicular to the line S_1S_2 A
(c) The point A is an intensity maxima if $L = n\lambda/2$
(d) The point A is always an intensity maxima for any separation L

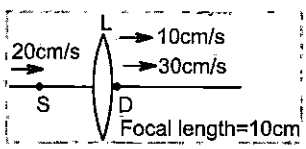
17. In YDSE, if bichromatic light having wavelength λ_1 and λ_2 is used, then the maxima due to both lights will overlap at a distance of..... from the central maxima. Take separation between slits as d and distance between screen and slits as D. Most appropriate option for the blank space is :
(a) $\left(\frac{\lambda_1+\lambda_2}{2D}\right)d$ (b) $\left(\frac{\lambda_1-\lambda_2}{D}\right)\times2d$
(c) LCM of $\frac{\lambda_1 D}{d}$ and $\frac{\lambda_2 D}{d}$ (d) HCF of $\frac{\lambda_1 D}{d}$ and $\frac{\lambda_2 D}{d}$

18. In YDSE, let A and B be two slits. Films of thicknesses t_A and t_B and refractive indices μ_A and μ_B are placed in front of A and B, respectively. If $\mu_A t_A = \mu_B t_B$, then the central maxima will :
(a) not shift
(b) shift towards A
(c) shift towards B
(d) (b) if $t_B > t_A$ and (c) if $t_B < t_A$

19. In YDSE, a laser light is used to illuminate the closely spaced slits, as a result, a clear interference pattern is obtained on screen. If we place smoke particles between the double slit and the screen, then the effects of interference will be observed :
(a) only on screen
(b) in between the space, between the screen and slits
(c) nowhere
(d) information insufficient

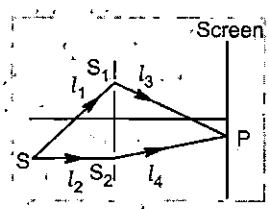
20. In YDSE of equal width slits, if intensity at the centre of screen is I_0, then intensity at a distance of $\beta/4$ from the central maxima is (β is the fringe width) :
(a) I_0 (b) $\frac{I_0}{2}$
(c) $\frac{I_0}{4}$ (d) $\frac{I_0}{3}$

21. The figure shows the initial position of a point source of light S, a detector D and a lens L. Now at $t = 0$, all the three start moving towards right with different velocities as shown



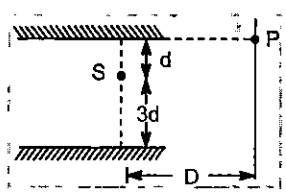
The time(s) at which the detector receives the maximum light is :
(a) 0.56s and 8.94s (b) 3.8s
(c) 8.94s and 19.62s (d) 0.56s

22. The YDSE apparatus is as shown in the figure below. The condition for point P to be a dark fringe is (λ = wavelength of light waves) :



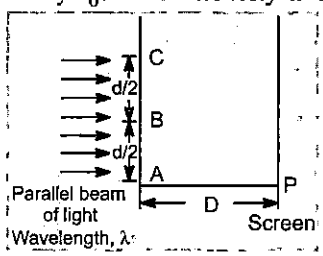
(a) $(l_1 - l_3) + (l_2 - l_4) = n\lambda$
(b) $(l_1 - l_2) + (l_3 - l_4) = n\lambda$
(c) $(l_1 + l_3) - (l_2 + l_4) = \frac{(2n-1)\lambda}{2}$
(d) $(l_1 + l_2) - (l_2 + l_3) = \frac{(2n-1)\lambda}{2}$

23. Consider the optical system shown in the figure that follows. The point source of light S is having wavelength equal to λ. The light is reaching screen only after reflection. For point P to be 2^{nd} maxima, the value of λ would be ($D >> d$ and $d >> \lambda$) :



(a) $\frac{12d^2}{D}$ (b) $\frac{6d^2}{D}$
(c) $\frac{3d^2}{D}$ (d) $\frac{24d^2}{D}$

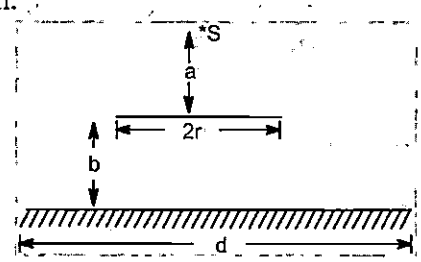
24. For the situation shown in the figure below. $BP - AP = \frac{\lambda}{3}$ and $D >> d$. The slits are of equal widths, having intensity I_0. The intensity at P would be :



(a) $4I_0$ (b) $2I_0$
(c) $3I_0$ (d) $\frac{7}{2}I_0$

25. Consider an YDSE that has different slits width, as a result, amplitudes of waves from two slits are A and $2A$, respectively. If I_0 be the maximum intensity of the interference pattern, then intensity of the pattern at a point where phase difference between waves is ϕ, is :
(a) $I_0 \cos^2 \phi$ (b) $\frac{I_0}{3}\sin^2\frac{\phi}{2}$
(c) $\frac{I_0}{9}[5 + 4\cos\phi]$ (d) $\frac{I_0}{9}[5 + 8\cos\phi]$

26. The light source S is over the center of a circular opaque plate of radius 1 m at a distance $a = 1$ m from it. The distance from the plate to the screen is $b = 0.8$ m. Find the diameter of the shadow of the plate on the screen.



(a) 1.8 m (b) 2.9 m
(c) 3.6 m (d) 5.4 m

ANSWERS

Level-1: Only One Alternative is Correct

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(b)	**5.**	(b)	**6.**	(b)	**7.**	(d)	**8.**	(d)
9.	(a)	**10.**	(c)	**11.**	(c)	**12.**	(c)	**13.**	(a)	**14.**	(c)	**15.**	(c)	**16.**	(a)
17.	(c)	**18.**	(d)	**19.**	(b)	**20.**	(b)	**21.**	(a)	**22.**	(c)	**23.**	(a)	**24**	(c)
25.	(c)	**26.**	(c)												

Solutions

Level-1: Only One Alternative is Correct

2. (d) $\Delta\phi = \frac{2\pi}{\lambda} d\sin\theta = \frac{10\pi}{3}$

$I = I_0 + 2I_0\cos[2\pi + (4\pi/3)]$

$= I_0$

3. (d) Number of fringes $= \frac{30\beta}{\beta_1} = 30 \times \frac{4200}{7000} = 18$

4. (b) Number of fringes $= 30 \times 1.5 = 45$

5. (b) $\Delta x = n_2(s_1 - t) + t \times 1.5 - n_2(s_1)$

$= t(1.5 - 1.4) = 1.2 \times 10^{-6} \times 0.1$

$= 1.2 \times 10^{-7} = 120\,\text{nm}$

$\lambda = 400\,\text{nm} \times 1.2$

$\Delta\phi = \frac{2\pi}{400 \times 1.2} \times 120 = \frac{\pi}{2}$

Alternative

Path difference at O: $(\mu_{rel} - 1)t = \left(\frac{n}{n_2} - 1\right)t$

$\therefore$ phase difference at O: $\left(\frac{n}{n_2} - 1\right)t\frac{2\pi}{\lambda_2}$ where

$n_1\lambda_1 = n_2\lambda_2$

$= \frac{\pi}{2}$ after substitution of values

6. (b) $\beta = \frac{\lambda}{2\mu\theta}$

No.of dark fringes per

$\text{cm} = \frac{1 \times 10^{-2}}{\beta} = \frac{2\mu\theta}{\lambda} \times 10^{-2} = 5$

8. (d) Fringe width $W = \frac{\lambda D}{d} = 0.5\,\text{mm}$

Distance of third maxima from central fringe $= 3W$

Distance of second maxima from central fringe $= 3W/2$ (On the other side)

Required distance $= 3W + 1.5W = 4.5W$

9. (a) $y_1 = 0.03\sin\pi t$

$y_2 = 0.02\sin\pi t$

Each wave have the same period = 2 seconds

or frequency, $n = \frac{1}{2}$. Since the velocity is 1.5 m/s, the wavelength $= v/n = \frac{1.5}{\left(\frac{1}{2}\right)} = 3\,\text{m}$

The path difference for a point R at the centre S_1 and $S_2 = 0$

Hence phase difference = 0

The resultant amplitude

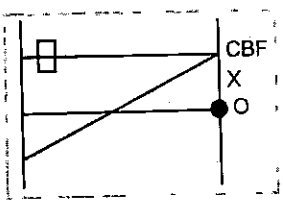
$A = (A_1^2 + A_2^2 + 2A_1A_2\cos\phi)^{1/2}$

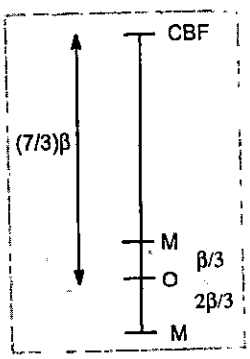
$= 0.05\,\text{m}$

10. (c) $(\mu - 1)t = \frac{xd}{D}$

$x = \frac{1}{2} \times \frac{2.1 \times 10^{-6} \times 1}{10^{-3}}$

$x = 1.05 \times 10^{-3} = 1.05\,\text{mm}$





$\beta = \frac{D\lambda}{d} = \frac{1 \times 4500 \times 10^{-10}}{10^{-3}} = 45 \times 10^{-5}$

$= 0.45 \times 10^{-3} = 0.45\,\text{mm}$

11. (c) With a narrow source and slit, the disturbance are coherent but they have slightly different amplitudes because one beam loses energy on being reflected.

The intensities of interfering disturbances can be written as a^2 and $(3/4)(a^2)$

The amplitudes are a and $\frac{a\sqrt{3}}{2}$

Maximum amplitude $= a + \frac{a\sqrt{3}}{2} = \frac{a(2+\sqrt{3})}{2}$

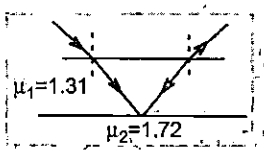
Minimum amplitude $= a - \frac{a\sqrt{3}}{2} = \frac{a(2-\sqrt{3})}{2}$

Therefore, $\frac{I_{max}}{I_{min}} = \frac{(2+\sqrt{3})^2}{(2-\sqrt{3})^2} = \left[\frac{(2+\sqrt{3})^2}{1}\right]^2$

$= (3.7321)^4 = 194$

12. (c) There is no net change in phase produced by the two reflection

$$\frac{\lambda}{2} = 2\mu_1 d\cos\theta$$



For normal incidence $\cos\theta = 1$

$$d = \frac{\lambda}{4\mu_1} = \frac{5.3\times10^{-7}}{4\times1.31} \simeq 10^{-7}\,\text{m} = 0.1\,\mu\text{m}$$

13. (a) $y' = \frac{d}{2}$, at point P exactly infront of S_1

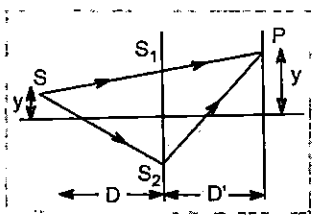
$\therefore \quad \Delta x = \frac{yd}{D} + \frac{d^2}{2D'}$

For minimum intensity $\Delta x = (2n-1)\frac{\lambda}{2}(n=1)$

Putting the value we get

$(0.5\sin\pi t)\times10^{-6} + 0.25\times10^{-6} = \frac{500}{2}\times10^{-9}$

$0.5\sin\pi t + 0.25 = \frac{0.5}{2}$



$\sin\pi t = 0 \Rightarrow \pi t = 0, \pi, 2\pi\ldots \Rightarrow t = 1\,\text{s}$

14. (c) $I = I_0\sin^2\theta$

Differentiating I with respect to θ

$\frac{dI}{d\theta} = 2I_0\sin\theta\cos\theta$

$\frac{dI}{I} = \frac{2I_0\sin\theta\cos\theta d\theta}{I_0\sin^2\theta} = 2\cot\theta d\theta$

Percentage error in angle

$= \frac{d\theta}{\theta}\times100 = \left(\frac{dI}{2I\cot\theta}\right)\frac{1}{\theta}\times100$

$= \frac{0.002}{2\times5\cot30^\circ}\times\frac{6\times100}{\pi}$

$= \frac{4}{\pi}\sqrt{3}\times10^{-2}\%$

15. (c) Intensity of direct ray $= I_0 = kA_0^2$

Intensity of reflected ray $= \frac{64}{100}I_0 = k\left(\frac{8A_0}{10}\right)^2$

$\frac{I_{max}}{I_{min}} = \frac{(A_0+0.8A_0)^2}{(A_0-0.8A_0)^2} = \left(\frac{1.8}{0.2}\right)^2 = \frac{81}{1}$

16. (a) If screen is perpendicular to the line joining the sources the fringes will be circular and central fringe will be if $S_1S_2 = n\lambda$.

17. (c) Let n_1^{th} maxima corresponding to λ_1 is overlapping with n_2^{th} maxima corresponding to λ_2. Then the required distance,

$y = \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

For least value of y

$\Rightarrow \quad y = LCM\text{ of }\frac{\lambda_1 D}{d}\text{ and }\frac{\lambda_2 D}{d}$

18. (d) $d\,\Delta x = (\mu_A - 1)t_A - (\mu_A - 1)t_B$

$= \mu_A t_A - \mu_B t_B - t_A + t_B$

$= t_B - t_A$

If $\Delta x > 0$, then fringe pattern will shift upwards.

If $\Delta x < 0$, then fringe pattern will shift downwards.

19. (b) The interference is taking place everywhere in between the slits and the screen as well as on screen. In the absence of smoke particles no path is seen in region between slits and screen

where interference is occurring but in the presence of smoke these paths are made visible by smoke.

20. (b) Let the intensity of individual waves be I, then

$$I_0 = 4I$$

$$\Rightarrow \quad I = \frac{I_0}{4}$$

$$\Rightarrow \quad \Delta x \text{ at } P = d\sin\theta$$

$$\Delta x \simeq d\tan\theta = \frac{dy}{D}$$

$$\Delta x = \frac{d}{D} \times \frac{\beta}{4}$$

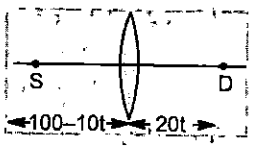
$$= \frac{d}{D} \times \frac{\lambda D}{4d} = \frac{\lambda}{4}$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \qquad [\Delta\phi = k\Delta x]$$

$$I' = I + I + 2\sqrt{I^2}\cos\frac{\pi}{2}$$

$$= 2I = \frac{I_0}{2}$$

21. (a) The detector will receive the maximum light when the image of point source of light coincides with the position of detector.



Let at any time t, image of S and detector coincides then,

for $u = -(100 - 10t)$ cm

$v = 20t$ cm

$f = 10$ cm

The detector receives maximum light.

From, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{20t} - \frac{1}{-(100-10t)} = \frac{1}{10}$$

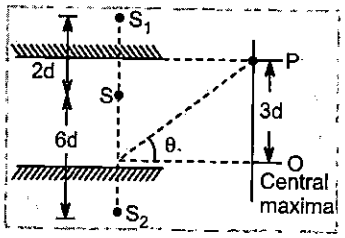
$$\Rightarrow \quad 2t^2 - 19t + 10 = 0$$

$$\Rightarrow \quad t = 0.56\,\text{s} \text{ and } 8.94\,\text{s}$$

22. (c) For dark fringe

Path difference $= (2n-1)\frac{\lambda}{2}$

23. (a)

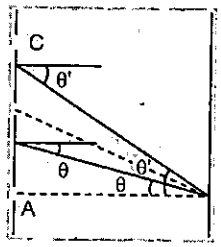


$$\text{At } P, \Delta x = \frac{(8d)\times 3d}{D}$$

For 2nd maxima,

$$\Delta x = 2\lambda \Rightarrow \frac{24d^2}{D} = 2\lambda$$

24. (c) $BP - AP = \frac{\lambda}{3}$



$$\Rightarrow \quad \frac{d}{2}\sin\theta = \frac{\lambda}{3}$$

$$\Rightarrow \quad \frac{d}{2}\tan\theta = \frac{\lambda}{3}$$

$$\Rightarrow \quad \frac{d}{2}\left[\frac{d/4}{D}\right] = \frac{\lambda}{3}$$

$$\Rightarrow \quad \frac{d^2}{8D} = \frac{\lambda}{3}$$

Path difference between waves coming from A and B is

$$\Delta x_{A/B} = \frac{\lambda}{3}$$

$$\Delta\phi_{A/B} = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

Path difference between A and C is

$$\Delta x_{A/C} = d\sin\theta' = d\tan\theta'$$

$$\Delta x_{A/C} = d \times \frac{3d/2}{D} = \frac{3d^2}{2D}$$

$$= 3\left[\frac{4\lambda}{3}\right] = 4\lambda$$

$$\Delta\phi_{A/C} = \frac{2\pi}{\lambda} \times 4\lambda = 8\pi$$

The three waves arriving at P can be written as $y < 0$,

$y < \frac{2\pi}{3}$ and $y < 8\pi$, so resultant of these three is $2y < 0$

and $y < \frac{2\pi}{3}$.

$$I_{\text{resultant}} = I_0 + 4I_0 + 2\sqrt{I_0 \times 4I_0}\cos 120° = 3I_0$$

25. (c) As amplitudes are A and $2A$, so intensities would be in the ratio 1:4, let us say I and $4I$.

$$I_{max} = I_0 = I + 4I + 2\sqrt{4I^2} = 9I$$

$$\Rightarrow \quad I = \frac{I_0}{9}$$

Intensity at any point,

$$I' = I + 4I + 2\sqrt{4I^2}\cos\phi$$

$$I' = 5I + 4I\cos\phi = \frac{I_0}{9}(5 + 4\cos\phi)$$

26. (c) $\frac{1}{1.8} = \frac{1}{d/2}$

$d = 3.6\,\text{m}$

Problems

Level 2 More than One Alternative is/are Correct

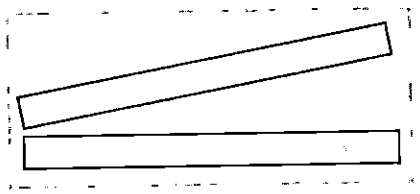
1. A thin film of thickness t and index of refraction 1.33 coats a glass with index of refraction 1.50. Which of the following thickness t will not reflect normally incident light with wavelength 640 nm in air ?
(a) 120 nm (b) 240 nm
(c) 360 nm (d) 480 nm

2. Coherent light consisting of two wavelengths $\lambda_1 = 4500$ Å and $\lambda_2 = 6000$ Å is sent through both slits of a Young's double slit apparatus :
(a) Central maxima for both wavelengths will coincide
(b) The third order bright fringe of λ_1 will coincide with the fourth order bright fringe of λ_2
(c) The third order bright fringe of λ_2 will coincide with fourth order bright fringe of λ_1
(d) The fringes of wavelength λ_1 will be wider than the fringes of wavelength λ_2

3. In Young's double slit experiment, phase difference between the waves at a point on screen having intensity less than the average intensity on screen may be :
(a) $\pi/4$ (b) $2\pi/3$
(c) π (d) $7\pi/8$

4. Two rays of light A and B with wavelength 5000 Å travel parallel to each other in air. Ray A encounters a 1 mm thick layer of glass with refractive index $n = 1.5$. Then :
(a) ray B will complete more oscillation than ray A
(b) both rays will complete same number of oscillations
(c) ray A will complete more oscillations than ray B
(d) the actual difference in number of oscillation made by two waves over the 1 mm distance is 1000

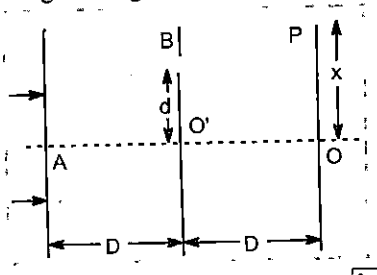
5. When monochromatic light is incident normally on a wedge-shaped thin air film, see figure, an interference pattern may be seen by reflection. Which of the following is/are correct ?



(a) Parallel fringes are observed.
(b) If water is introduced into the region between the plates, the fringe separation decreases.
(c) If the angle of the wedge is increased, the fringe separation decreases.
(d) When white light is used there will not be a completely dark fringe.

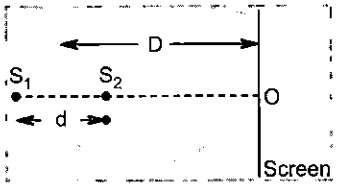
6. Imagine a Young's double slit interference experiment performed with electron waves associated with fast moving electrons coming from the electron gun. The distance between successive maximum will decrease if the :
(a) accelerating voltage in electron gun is decreased
(b) accelerating voltage is increased
(c) distance between screen and slits is increased
(d) distance between the slits is increased

7. The minimum value of d so that there is a dark fringe at O is d_{min}. For the value of d_{min}, the distance at which the next bright fringe is formed is x. Then :



(a) $d_{min} = \sqrt{\lambda D}$ (b) $d_{min} = \sqrt{\frac{\lambda D}{2}}$
(c) $x = \frac{d_{min}}{2}$ (d) $x = d_{min}$

8. Two point monochromatic and coherent sources of light of wavelength λ are each placed as shown in the figure below. The initial phase difference between the sources is zero O. $(D >> d)$. Select the correct statement (s) :



(a) If $d = \frac{7\lambda}{2}$, O will be minima

(b) If $d = \lambda$, only one maxima can be observed on screen

(c) If $d = 4.8\lambda$, then a total 10 minimas would be there on screen

(d) If $d = \frac{5\lambda}{2}$, then intensity at O would be minimum

9. A transparent slab of thickness t and refractive index μ is inserted in front of upper slit of YDSE apparatus. The wavelength of light used is λ. Assume that there is no absorption of light by the slab. Select the correct statement (s) :

(a) The intensity of dark fringes will be 0, if slits are identical

(b) The change in optical path due to insertion of plate is μt

(c) The change in optical path due to insertion of plate is $(\mu - 1)t$

(d) For marking intensity zero at the centre of screen, then thickness could be $\frac{5\lambda}{2(\mu - 1)}$

ANSWERS

1.	(b, d)	**2.**	(a, c)	**3.**	(b, c, d)	**4.**	(c, d)	**5.**	(a, b, c)	**6.**	(b, d)
7.	(b, d)	**8.**	(a, c, c, d)	**9.**	(a, c)						

Solutions

Level-2: More than One Alternative is/are Correct

1. (b, d)

$$2\mu_1 t = n\lambda$$

$$t = \frac{n\lambda}{2\mu_1} = n\frac{640\times 3}{2\times 4} = 240n$$

$$t = 240\,\text{nm},\ 480\,\text{nm},\ldots$$

2. (a, c)

Central Bright fringe position has zero path difference

(c) $\frac{3\lambda_2 D}{d} = \frac{4\lambda_1 D}{d}$

3. (b, c, d)

Let maximum intensity be I_0

$$I = I_0 \omega t \frac{\Delta\phi}{2}$$

$$I_{av} = \frac{I_0}{2}$$

$$I < I_{av} \Rightarrow \cos^2\left(\frac{\Delta\phi}{2}\right) < \frac{1}{2}$$

$$\Rightarrow \cos\left(\frac{\Delta\phi}{2}\right) < \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\pi}{2} < \Delta\phi < \frac{3\pi}{2}$$

4. (c, d)

For ray A, t_1 = time taken by the ray to travel through the glass

$$= \frac{d}{V_g} \quad [d = 1\,\text{mm and } V_g = \text{velocity of light in glass} = C/n]$$

$$= \frac{nd}{C} = \frac{1.5\times 10^{-3}}{3\times 10^8} = 5\times 10^{-12}\,\text{s}$$

For ray B, $t_2 = \frac{d}{C} = \frac{10^{-3}}{3\times 10^8} = \frac{10}{3}\times 10^{-12}\,\text{s}$

$$\Delta t = t_1 - t_2 = \left(5 - \frac{10}{3}\right)\times 10^{-12} = \frac{5}{3}\times 10^{-12}\,\text{s}$$

The frequency of the light (oscillation per second) does not change inside the glass.

$t_1 > t_2 \Rightarrow$ ray A will make more oscillations

n_1 = number of oscillations for ray $A = ft_1$

similarly, n_2 = number of oscillations for ray $B = ft_2$

$$\Delta n = n_1 - n_2 = f(t_1 - t_2) = \left(\frac{C}{\lambda}\right)(\Delta t)$$

$$= \frac{3\times 10^8}{5000\times 10^{-10}}\left(\frac{5}{3}\times 10^{-12}\right) = 10^3$$

5. (a, b, c)

$$\beta = \frac{\mu\lambda}{2\theta}$$

6. (b, d)

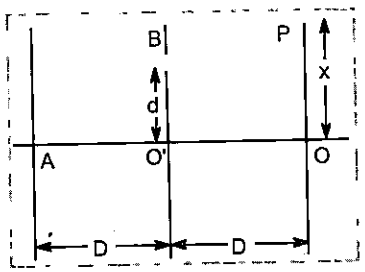
The condition for getting maxima is $d\sin\theta = m\lambda$. The wavelength of electron will be given as $\lambda = hc/v$.

The distance between successive maxima will increase if θ becomes smaller. For this either d should be increased or λ should be decreased. For the latter we must increase the voltage V.

7. (b, d)

There is a dark fringe at O if the path difference

$$\delta = ABO - AO'O = \frac{\lambda}{2}$$



$$= 2\sqrt{D^2 + d^2} - 2D = \frac{2d^2}{2D} = \frac{d^2}{D} = \frac{\lambda}{2}$$

$$\Rightarrow d_{min} = \sqrt{\frac{\lambda D}{2}}$$

The bright fringe is formed at P if the path difference

$$\delta' = AO'P - ABP = \lambda$$

$$= D + \sqrt{D^2 + x^2} - \sqrt{D^2 + d^2} - \sqrt{D^2 + (x-d)^2} = \lambda$$

$$= \frac{x^2}{2D} - \frac{d^2}{2D} - \frac{(x^2 + d^2 - 2xd)}{2D} = \lambda$$

Given $d = d_{min}$

Solving $x = d_{min} = \sqrt{\frac{\lambda D}{2}}$

8. (a, b, c, d)

Δx at $O = d$ [path difference is maximum at O]

So, if $d = \frac{7\lambda}{2}$, O will be a minima

$d = \lambda$, O will be maxima

$d = \frac{5\lambda}{2}$, O will be minima and hence intensity is minimum.

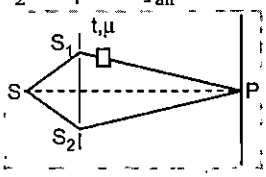
If $d = 4.8\lambda$, then total 10 minimas can be observed on screen, 5 above O and 5 below O, which correspond to

$$\Delta x = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \pm\frac{7\lambda}{2}, \pm\frac{9\lambda}{2}$$

9. (a, c) Δx at P :

$$\Delta x = (S_1P - t)_{air} + t_{medium} - S_2P_{air}$$

$$= [S_1P - S_2P + \mu t - t]_{air}$$



$$= (\mu - 1)t$$

Earlier, Δx at $P = S_1P - S_2P = 0$

So, change in optical path due to insertion of slab

$$= (\mu - 1)t$$

For intensity to be zero at P, we have

$$\Delta x = \frac{(2n-1)\lambda}{2}$$

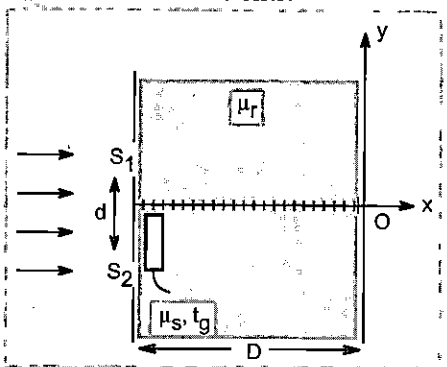
$$\Rightarrow (\mu - 1)t = t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

Problems

Level 3 Comprehension Based Problems

Passage 1

In a modified *YDSE* the region between screen and slits is immersed in a liquid whose refractive index varies with time as $\mu = \frac{5}{2} - \frac{T}{4}$ until it reaches a steady state value $\frac{5}{4}$. A glass plate of thickness 36μm and refractive index $\frac{3}{2}$ is introduced infront of one of the slits.

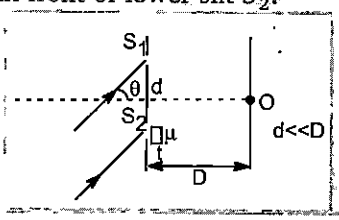


1. Find the time when central maxima is at centre of screen *O*.
(a) $T = 4$sec (b) $T = 2$sec
(c) $T = 8$sec (d) $T = 1$sec

2. What is speed of central maxima when it is at *O* ?
(a) 10^{-3} m/sec (b) 2×10^{-3} m/sec
(c) 3×10^{-3} m/sec (d) 4×10^{-3} m/sec

Passage 2

A monochromatic beam of light falls on Young's double slit experiment apparatus as shown in figure. A thin sheet of glass is inserted in front of lower slit S_2.



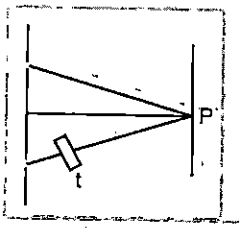
1. The central bright fringe can be obtained :
(a) at *O* only
(b) at *O* or below *O* only
(c) at *O* or above *O* only
(d) anywhere on the screen

2. If central bright fringe is obtained on screen at *O*.
(a) $(\mu-1)t = d\sin\theta$ (b) $(\mu-1)t = d\cos\theta$
(c) $(\mu-1)t + d\sin\theta = 0$ (d) $\frac{t}{\mu-1} = \frac{d}{\sin\theta}$

3. The phase difference between the waves interfering at fifth minima is :
(a) 5π (b) 9π
(c) 10π (d) 11π

Passage 3

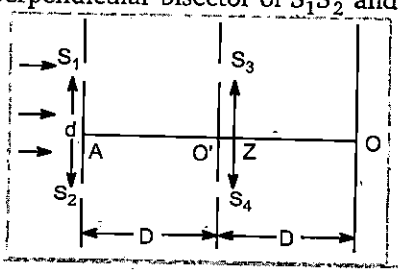
The figure shows a double-slit experiment in which coherent monochromatic light of wavelength λ from a distant source is incident upon the two slits, each of width w ($w >> \lambda$) and the interference pattern is viewed on a distant screen. A thin piece of glass of thickness t and refractive index n is placed between one of the slit and the screen; perpendicular to the light path. Intensity at P for $t = 0$ is given is I_0.



1. The intensity at a point P as a function of t is :
   (a) $I_0 \sin^2\left(\frac{\pi}{\lambda}(n-1)t\right)$ (b) $I_0 \cos^2\left(\frac{\pi}{\lambda}(n-1)t\right)$
   (c) $I_0 \cos^2\left(\frac{\pi}{2\lambda}(n-1)t\right)$ (d) $I_0 \sin^2\left(\frac{\pi}{2\lambda}(n-1)t\right)$
2. The intensity at P is minimum if the value of t is :
   (a) λ/n (b) $\lambda/\{2(n-1)\}$
   (c) $\lambda/(n-1)$ (d) $2\lambda/(n-1)$
3. If the width w of one of the slits is increased to $2w$, the maximum intensity at P for t_0 is :
   (a) $(3/2)I_0$ (b) $(9/2)I_0$
   (c) $(9/4)I_0$ (d) $\left(\frac{3}{4}+\frac{1}{\sqrt{2}}\right)I_0$

Passage 4

In the arrangement shown in the figure, slits S_3 and S_4 are having a variable separation Z. Point O on the screen is at the common perpendicular bisector of S_1S_2 and S_3S_4.

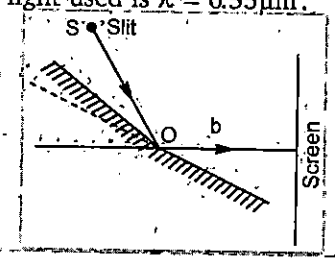


1. When $Z = \frac{\lambda D}{2d}$ the intensity measured at O is I_0. The intensity at O when $Z = \frac{2\lambda D}{d}$ is :
   (a) I_0 (b) $2I_0$
   (c) $3I_0$ (d) $4I_0$
2. The minimum value of Z for which the intensity at O is zero, is :
   (a) $\frac{3\lambda D}{2d}$ (b) $\frac{\lambda D}{2d}$
   (c) $\frac{\lambda D}{3d}$ (d) $\frac{\lambda D}{d}$
3. If a hole is made at O' on $AO'O$ and the slit S_4 is closed, then the ratio of the maximum to minimum observed on screen at O if $O'S_3$ is equal to $\frac{\lambda D}{4d}$, is :
   (a) 1 (b) infinity
   (c) 34 (d) 4

Consider the arrangement shown in figure. The distance D is large compared to the separation between the slits.

Passage 5

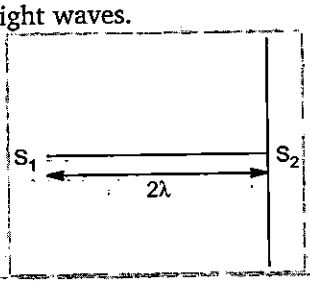
The experimental arrangement with Fresnel's double mirror at an inclination of 12' is shown in figure. The distances from the intersection of the mirrors to the slit and the screen are ascectively equal to 10 cm and 130 cm. The wavelength of light used is $\lambda = 0.55\mu m$.



1. The width of the fringe in the interference pattern is :
   (a) 1.1 mm (b) 2.2 mm
   (c) 1.5 mm (d) 3 mm
2. The number of possible fringes is :
   (a) 8 (b) 6
   (c) 9 (d) 7
3. Shift of the interference pattern when the slit is displaced by 1 mm along the arc of a circle of radius $r = 0.1$ m with centre at 0 is :
   (a) 0.2 cm (b) 0.13 cm
   (c) 0.3 cm (d) 0.15 cm
4. The fringes on the screen will be sharp when the width of the slit is kept at a maximum of :
   (a) 0.55 mm (b) 30μm
   (c) 20μm (d) 15μm

Passage 6

Two similar and coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between the sources is 2λ. A person initially at S_2 starts walking along a straight line which is passing through S_2 and perpendicular to S_1S_2. During his journey he finds variation in intensity of light which is obviously because of interference of light waves.



1. The smallest distance from S_2 where the person finds minimum intensity is :

(a) $\frac{\lambda}{3}$ (b) $\frac{7\lambda}{12}$

(c) $\frac{15\lambda}{4}$ (d) 2λ

2. At some place during his walk the person finds intensity of light is equal to intensity due to one of the sources, the path difference between the waves arriving at that place is :

(a) $\frac{\lambda}{3}$ (b) $\frac{7\lambda}{12}$

(c) $\frac{15\lambda}{4}$ (d) 2λ

3. A source slit of adjustable width is used to illuminate two point sources 0.1 mm apart in an interference experiment. The wavelength of light used is 6000 Å. Initially the slit is closed and then it is gradually opened. The width at which the fringes on a screen $D = 1$ m apart will first disappear is :

(a) 1 m rad (b) 2 m rad

(c) 3 m rad (d) 4 m rad

Passage 7

The YDSE apparatus is modified by placing an isotropic transparent plate of high melting point in front of one of the slits. The refractive index of the plate is $\mu_r = 1.5$ at room temperature and its thickness is $t = 2\mu m$. The refractive index of plate will increase when temperature increases and temperature coefficient of refractive index of the plate (*i.e.*, the fractional change in refractive index per unit rise in temperature) is $2 \times 10^{-6}/°C$. The incident light is having wavelength $\lambda = 600$ Å. The separation between the slits is $d = 0.2$ cm, and separation between the slit and the screen is 2 m. Assume that slits are of equal intensity.

Based on the above information, answer the following questions :

1. Before insertion of plate, mid-point of screen is location of central maxima, then this point after the insertion of slab is :

(a) a point of central maxima

(b) a point of maxima but not the central maxima

(c) a point of minima

(d) neither a point of maxima nor of minima

2. If the plate is heated so that it temperature rises by 10°C, then how many fringes will cross a particular point on the screen ? (Neglect the thermal expansion of plate) :

(a) 1000 (b) 10^{-4}

(c) 5000 (d) $\frac{10000}{3}$

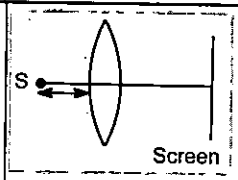
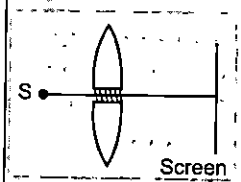
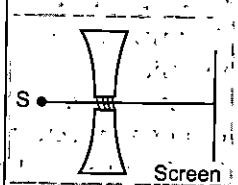
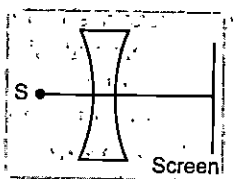
3. If intensity of unconverted slit is I_0 and that of covered slit is $\frac{I_0}{2}$, then intensity at mid-point of the screen, is :

(a) $I_0\left[\frac{3}{2} - \frac{1}{\sqrt{2}}\right]$ (b) $I_0 \times \frac{3}{2}$

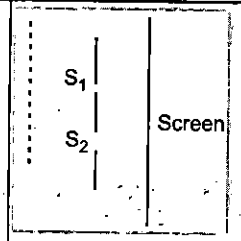
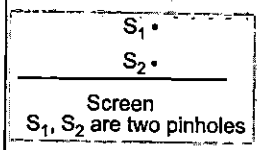
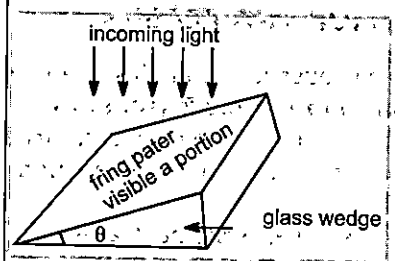
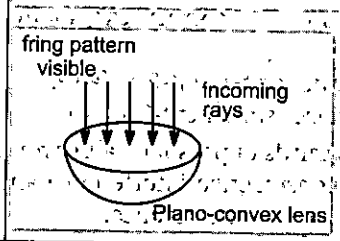
(c) $4I_0$ (d) $I_0\left[1 - \frac{1}{\sqrt{2}}\right]$

MATCHING TYPE PROBLEMS

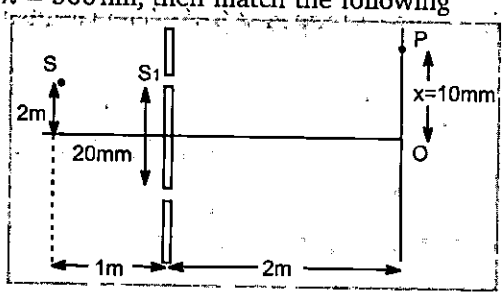
1. Light from source $S(|u|<|f|)$ falls on lens and screen is placed on the other side. The lens is formed by cutting it along principal axis into two equal parts and are joined as indicated in column II.

Column-I		Column-II	
(a)	Plane of image move toward screen if $\|f\|$ is increased	(p)	 Small portion of each part near pole is removed. The remaining parts are joined.
(b)	Images formed will be virtual	(q)	 The two parts are separated slightly. The gap is filled by opaque material.
(c)	Separation between images increase if $\|u\|$ decreases	(r)	 The two parts are separated slightly. The gap is filled by opaque material.
(d)	Interference pattern can be obtained if screen is suitably positioned.	(s)	 Small portion of each part near pole is removed. The remaining parts are joined.

2. Match the column

	Column-I (Arrangement to see fringe pattern)		Column-II (Shape of fringe)
(a)	 S_1, S_2 are two pinholes	(p)	Hyperbolic
(b)		(q)	Circuler
(c)		(r)	Elliptical
(d)		(s)	Linear

3. In Young's double slit experiment, the point source S is placed slightly off the central axis as shown in figure. If $\lambda = 500$ nm, then match the following



	Column-I		Column-II
(a)	Nature and order of interference at the point P, $OP = 10$ mm	(p)	Bright fringe of order 80
(b)	Nature and order of interference point O	(q)	Bright fringe of order 262
(c)	If a transparent paper (refractive index $\mu = 1.45$) of thickness $t = 0.02$ mm is pasted on S_1 *i.e.* one of the slits, the nature and order of interference at P	(r)	Bright fringe of order 62
(d)	After inserting the transparent paper in front of slits S_1, the nature and order of interference at O	(s)	Bright fringe of order 280

ASSERTION AND REASON

Direction: Each question contains statement 1 **(assertion) and statement 2 (reason)**

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

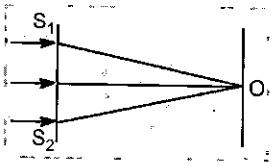
(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

1. Statement-1: Two thirds of possible values for phase difference result in constructive interference and one third results in destructive interference.

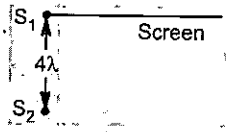
Statement-2: Interference is called constructive if the amplitude of the superposed waves exceeds that of one individual wave.

2. Statement-1: In YDSE, as shown in figure, central bright fringe is formed at O. If a liquid is filled between plane of slits and screen, the central bright fringe is shifted in upward direction.



Statement-2: If path difference at O increases y-coordinate of central bright fringe will change.

3. Statement-1: For the situation shown below, two identical coherent light sources produce interference pattern on the screen. The intensity of minima-nearest to S_1 is not exactly zero



Statement-2: Minimum intensity is zero, when interfering waves have same intensity at the location of superposition.

4. Statement-1: In YDSE, if separation between slits is less than wavelength of light, then no interference pattern could be observed.

Statement-2: For interference pattern to be observed, light sources have to be coherent.

5. Statement 1: In Young's double slit experiment the two slits are at distance d apart. Interference pattern is observed on a screen at distance D from the slits. At a point on the screen when it is directly opposite to one of the slits, a dark fringe is observed. Then the wavelength of wave is proportional to square of distance between the two slits.

Statement 2: For a dark fringe intensity is zero.

6. Statement 1: In interference experiment, the fringe system will shift, if a glass plate is interposed in the path of the one of the interfering beams and the shift will be on the side of the glass plate.

Statement 2: The velocity of light in glass is smaller than in air.

ANSWERS

Level-3: Comprehension Based Problems

Passage-1: **1.** (a) **2.** (c)

Passage-2: **1.** (d) **2.** (a) **3.** (d)

Passage-3: **1.** (b) **2.** (b) **3.** (d)

Passage-4: **1.** (b) **2.** (d) **3.** (c)

Passage-5: **1.** (a) **2.** (d) **3.** (b) **4.** (a)

Passage-6: **1.** (b) **2.** (a) **3.** (c)

Passage-7: **1.** (b) **2.** (d) **3.** (d)

Matching Type Problems

1. (a)-p, q; (b)-p, q, r, s; (c)-r, s; (d)-p

3. (a)-s; (b)-p; (c)-q; (d)-r

Assertion and Reason

1.	(a)	**2.**	(d)	**3.**	(c)	**4.**	(b)	**5.**	(c)	**6.**	(a)

Solutions

Level 3 Comprehension Based Problems

Level-3: Comprehension Based Problems

Passage-1

2. (c) $\Delta x = \dfrac{\mu_l yd}{D} + (\mu_g - \mu_l)t$

for central maxima

$\Delta x = 0$

$$y = -\left(\frac{\mu_g - \mu_l}{\mu_e}\right)\frac{tD}{d} = \frac{(4-T)+D}{(10-T)d}$$

$y = 0$; for $T = 4\,\text{sec}$

$$(h)v = \frac{dy}{dt} = \frac{6Dt}{(10-T)^2 d}$$

central maxima is 0 at $T = 4\,\text{sec}$

$$v = \frac{6Dt}{36d} = 3\times 10^{-3}\ \text{m/sec}$$

Passage-2

1. (d) $\Delta x = d\sin\theta - (\mu - 1)t = 0$ for C. maxima.

2. (a) If C max. is at O $d\sin\theta = (\mu - 1)t$

3. (d) For 5^{th} minima $\phi = 11\pi$

Passage-3

1. (b) A = path difference between two waves arriving at

$P = (n-1)t$

ϕ = phase difference between two waves arriving at

$$P = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}(n-1)t$$

I = Intensity at the central point

We know, $I \propto 4a^2\cos^2(\phi/2)$

and $I_0 \propto 4a^2$

$\therefore\quad I/I_0 = \cos^2(\phi/2)$

$$\therefore\quad I = I_0\cos^2\left(\frac{\pi}{\lambda}(n-1)t\right)$$

2. (b) Intensity at P will be minimum if

$$\frac{\phi}{2} = \left(\frac{2k+1}{2}\right)\pi \qquad \text{where } k = 0, \pm 1, \pm 2....$$

i.e. $$\frac{\pi}{\lambda}(n-1) = \frac{(2k+1)}{2}\pi$$

$$\therefore\quad t = \frac{(2k+1)}{2(n-1)}\lambda$$

for $k = 0$, $t = \dfrac{\lambda}{2(n-1)}$

3. (d) As the width of one of the slits is increased to $2w$, the amplitude due to slit become $2a$.

$$I \propto (a_1^2 + a_2^2 + 2a_1a_2\cos\phi)$$

$$\propto a^2 + (2a)^2 + (2a)(2a)\cos\phi$$

$$\propto 5a^2 + 4a^2\cos\phi$$

$$\propto a^2(5 + 4\cos\phi) = a^2\left[5 + 4\cos\left(\frac{2\pi(n-1)t}{\lambda}\right)\right]$$

$$\therefore\quad \frac{I}{I_0} = \frac{1}{4}\left[5 + 4\cos\left\{\frac{2\pi(n-1)t}{\lambda}\right\}\right]$$

for $t = 0$, the intensity at P is maximum

$$\therefore\quad I_{\max} = I_0\left(\frac{9}{4}\right)$$

Passage-4

1. (b) Intensity at O is proportional to intensity at S_3 and at S_4

$I = k\cos^2\delta/2$, where k constant, δ-phase difference

$\delta = \frac{2\pi}{\beta}\times\frac{Z}{2} = \frac{\pi Z}{\beta}$, where β is fringe width, $\beta = \frac{\lambda D}{d}$

when $Z = \lambda D/2d$

$\delta = \frac{\pi d}{\lambda D}\times\frac{\lambda D}{2d} = \frac{\pi}{2}$

$I = I_0 = k\cos^2(\pi/4)$

$k = 2I_0$

when $Z' = \frac{2\lambda D}{d}$

Phase difference $\delta' = \frac{2\pi}{\beta}\frac{Z'}{2} = \frac{2\pi d}{\lambda d}\times\frac{2\lambda D}{2d} = 2\pi$

Required intensity at O is

$I' = k\cos^2\delta/2 = 2I_0\cos^2(\pi) = 2I_0$

2. (d) Intensity at O is zero if $\frac{\delta}{2} = \frac{\pi}{2}$

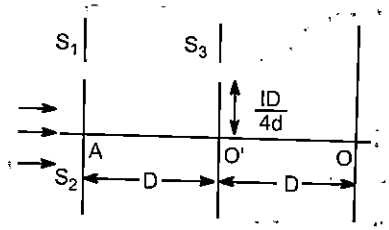
i.e., $\delta = \frac{2\pi}{\beta}\frac{Z}{2} = \pi \Rightarrow Z = \beta = \frac{\lambda D}{d}$

3. (c) Intensity at $S_3 = (A_0^2 + A_0^2 + 2A_0^2\cos\phi)$

where $\phi = \frac{2\pi}{\beta}\frac{\lambda D}{4d} = \frac{2\pi d}{\lambda D}\times\frac{\lambda D}{4d} = \frac{\pi}{2}$

$I_{S_3} = A_0^2(2 + 2\cos\pi/2) = 2A_0^2\times 2\cos^2\frac{\pi}{4} = \frac{4}{2}A_0^2$

$= 2A_0^2$



Amplitude of wave at S_3, $A_{S_3} = \sqrt{2}A_0$

Maximum intensity at $O' = (A_0 + \sqrt{2}A_0)^2$

Maximum intensity at $O = (\sqrt{2}A_0 - A_0)^2$

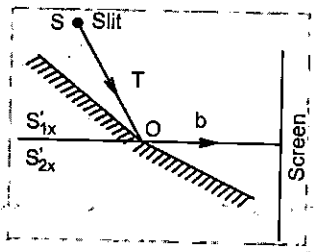
$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = (\sqrt{2}+1)^4 = 34$

Passage-5

1. (a) When a ray turns through a reflected ray turns through 2α.

Hence $d = 2\alpha r$

$D = b + r$



$\beta = \frac{\lambda D}{d} = \frac{\lambda(b+r)}{2\alpha r} = \frac{\lambda\left(1+\frac{b}{r}\right)}{2\alpha}$

$= 0.55\times10^{-6}\frac{\left(1+\frac{1.3}{0.1}\right)}{2\times12}\times\frac{180\times60}{\pi} = 1.1\,\text{mm}$

2. (d) The spread on the screen whose interference fringes are formed is $2\alpha b$.

$\therefore$ N (No. of fringes formed)

$= \frac{4\times1.3\times(12\pi\times180\times60)^2}{0.55\times10^{-6}\left(1+\frac{1.3}{0.1}\right)} = 8.22$

Since n must be an odd number lower than 8.22

$N = 7$

3. (b) The angle through the ray turns is $\frac{\delta l}{r}$. The reflected ray turns through the same angle. The linear distance through which the ray moves on the area $= \left(\frac{\delta l}{r}\right)b$. The central fringe and hence all other fringes will shift by the same distance.

$\therefore$ Required shift $= \frac{\delta l}{r}\cdot b$

$= \frac{1\times10^{-3}}{0.1}\times1.3 = 0.013\,\text{m} = 1.3\,\text{cm}$

4. (a) The fringes are still sharp when $d \le \frac{\beta}{2}$

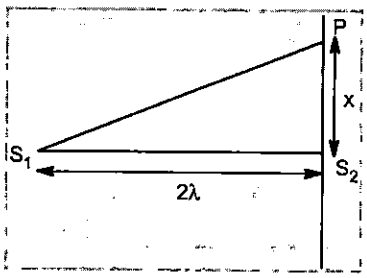
$\delta_{max} = \frac{1.1}{2}\,\text{mm} = 0.55\,\text{mm}$

Passage-6

1. (b) Let the person finds minimum intensity at point P at distance x from S_2.

Path difference

$$\sqrt{(2\lambda)^2 + x^2} - x = (2n-1)\frac{\lambda}{2} \quad ...(1)$$



To have minimum intensity at smallest distance from S_2.

$$\sqrt{(2\lambda)^2 + x^2} - x = 3\frac{\lambda}{2} \quad ...(2)$$

$$\Rightarrow \quad x = \frac{7\lambda}{12}$$

2. (a) Maximum possible path difference = distance between the sources = 2λ

Let intensity of light from each source is I, then $I = I + I + 2\sqrt{II}\cos\phi$

$$\cos\phi = -\frac{1}{2} \Rightarrow \phi = \frac{2\pi}{3} \text{ path diff.} = \frac{\lambda}{3}$$

3. (c) The required width $= \frac{\lambda}{2d}$

$$= \frac{6000\times10^{-10}}{0.2\times10^{-3}} = 3\times10^{-3} = 3\,\text{m rad}$$

Passage-7

1. (b) When temperature of the plate is increased, its refractive index increases and, as a result, the fringes cross at a particular point. Here the number of fringes crossing through a point is due to change in refractive index of the plate.

Shift in the location of a particular fringe due to changes in refractive index is, $y = \frac{(\mu_2 - \mu_1)tD}{d}$

So, the number of crossed fringes,

$$n = \frac{(\mu_2 - \mu_1)t}{\lambda}$$

$$= \frac{1.5\times2\times10^{-6}\times10\times2\times10^{-6}}{6\times10^{-7}}$$

$$= 10^{-4}$$

2. (d) At O, $\Delta x = (\mu_0 - 1)\,t = 10\times10^{-7}\,\text{m}$

$$\Delta\phi = \frac{2\pi}{\lambda}\times\Delta x = \frac{2\pi\times10\times10^{-7}}{6\times10^{-7}} = \frac{10\pi}{3}$$

So, intensity at O is

$$I = I_0 + \frac{I_0}{2} + 2\sqrt{\frac{I_0\times I_0}{2}}\cos\left(\frac{10\pi}{3}\right)$$

$$= \frac{3I_0}{2} + \sqrt{2}I_0\times(-0.5) = I_0\left[\frac{3}{2} - \frac{1}{\sqrt{2}}\right]$$

3. (d) For satellite to be geostationary it has to revolve in an equatorial plane and its sense of rotation must be same as that of earth's rotation about its own axis.

In addition to this, the satellite must have a time period of 24 h.

For the satellite to revolve around the earth, the plane of the orbit must contain the centre of earth.

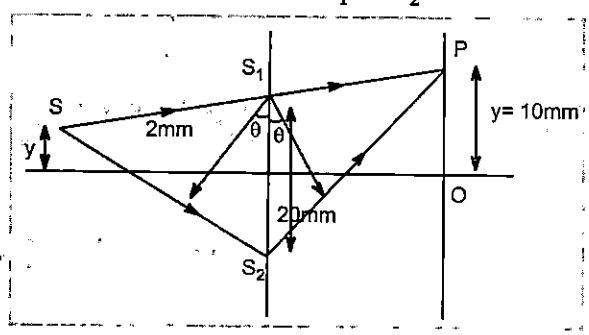
Matching Type Problems

3. The optical path difference between the two waves arriving at P is

$$\delta = (SS_2 + S_2P) - (SS_1 + S_1P)$$

$$= (SS_2 - SS_1) + (S_2P - S_1P)$$

$$= d\sin\theta_0 + d\sin\theta = \frac{dy_0}{D_1} + \frac{dy}{D_2}$$



$d = 20\,\text{mm}$ $y_0 = 2\,\text{mm}$, $D_2 = 2m$,

$D_1 = 1\text{m}$, $y = 10\,\text{mm}$

$$\delta = \frac{20\times2}{1000} + \frac{20\times10}{2000} = 0.14\,\text{mm}$$

For a bright fringe, $\delta = \frac{dy_0}{D_1} = 0.04\,\text{mm}$

$$n' = \frac{\delta'}{\lambda} = \frac{0.04}{0.5\times10^{-3}} = 80$$

Due to transparent paper, the change in optical path

$$= (\mu - 1)t = (1.45 - 1)(0.02)\,\text{mm} = 0.009\,\text{mm}$$

$$\delta^H = 0.14\,\text{mm} - 0.009\,\text{mm} = 0.131\,\text{mm}$$

$$n = \frac{0.131}{0.5\times10^{-3}} = 262$$

Due to transparent paper, the path difference at O

$$\delta^H = \delta^1 - (\mu - 1)t = (0.04 - 0.009)\,\text{mm} = 0.0131\,\text{m}$$

$$n = \frac{0.131}{0.5 \times 10^{-3}} = 62$$

Assertion and Reason

1. (a)

$$\left|\cos\left(\frac{\Delta\phi}{2}\right)\right| > \frac{1}{2}$$

$$-\frac{2\pi}{3} < \Delta\phi < \frac{2\pi}{3}$$

3. (c)

At the location of minima, two waves have different intensities and hence minimum intensity is not exactly zero.

4. (b)

$\Delta x = d\sin\theta$

$\Delta x_{max} < d$ if $d < \lambda$, then $\Delta x_{max} < \lambda$, so maxima can be present and interference pattern cannot be observed. Statement 2 is true but doesn't explain Statement 1.

5. (c)

Here $S_1P = D$ and $S_2P = \sqrt{D^2 + d^2} = D\left[1 + \frac{d^2}{2D^2}\right]$

$$\therefore \quad \Delta x = S_2P - S_1P = \frac{d^2}{2D} = \frac{\lambda}{2}(2n - 1)$$

6. (a)

The fringe system will shift on the side of the glass plate. The shift establishes the fact the velocity of light in glass is smaller than that in air.

Previous Year Problems

Only One Alternative is Correct

1. An electric bulb illuminates a plane surface. The intensity of illumination on the surface at a point 2 m away from the bulb is 5×10^{-4} phot (lumen/sq.cm). The line joining the bulb to the point makes an angle 60° with the normal to the surface. The intensity of the bulb in candela is : **(IIT 1980; 1M)**

(a) $40\sqrt{3}$ (b) 40

(c) 20 (d) 40×10^{-4}

2. When a ray of light enters a glass slab from air : **(IIT 1980; 1M)**

(a) its wavelength decreases

(b) its wavelength increases

(c) its frequency increases

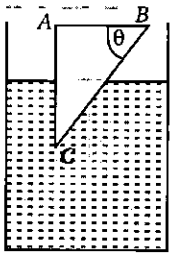
(d) neither its wavelength nor its frequency changes

3. In Young's double slit experiment, the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is: **(IIT 1981; 2M)**

(a) uncharged (b) halved

(c) doubled (d) quadrupled

4. A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam incident normally on the face AB is totally reflected to reach on the face BC if :



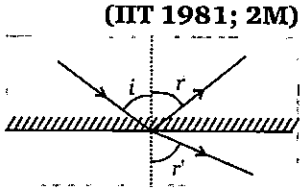
(a) $\sin\theta \geq \frac{8}{9}$

(b) $\frac{2}{3} < \sin\theta < \frac{8}{9}$

(c) $\sin\theta \leq \frac{2}{3}$

(d) none of these **(IIT 1981; 2M)**

5. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i (see fig.). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. The critical angle is : **(IIT 1983; 1M)**



(a) $\sin^{-1}(\tan r)$ (b) $\sin^{-1}(\tan i)$

(c) $\sin^{-1}(\tan r')$ (d) $\tan^{-1}(\sin i)$.

6. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are : **(IIT 1988; 1M)**

(a) $5I$ and I (b) $5I$ and $3I$

(c) $9I$ and I (d) $9I$ and $3I$

7. A beam of light of wavelength 600 nm, from a distant source, falls on a single slit 1 mm wide and a resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of central bright fringe is : **(IIT 1994; 1M)**

(a) 1.2 cm (b) 1.2 mm

(c) 2.4 cm (d) 2.4 mm

8. Spherical aberration, in a thin lens, can be redued by : **(IIT 1994; 1M)**

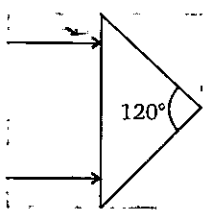
(a) using a monochromatic light

(b) using a doublet combination

(c) using a circular annular mask over the lens

(d) increasing the size of the lens

9. An isosceles prism of angle 120° has a refractive index 1.44.Two parallel monochromatic rays enter the prism, parallel to each other in air, as shown. The rays which emerge from the opposite faces :



(a) are parallel to each other

(b) are diverging

(c) make an angle $2[\sin^{-1}(0.72) - 30°]$ with each other

(d) make an angle $2\sin^{-1}(0.72)$ with each other **(IIT 1995; 2M)**

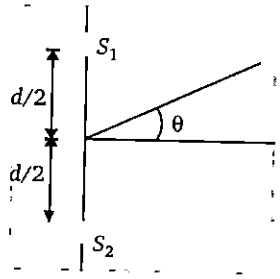
10. The focal lengths of the objective and the eye piece of a compound microscope are 2.0 cm and 3.0 cm, respectively. The distance between the objective and the eyepiece is 15.0 cm. The final image formed by the eyepiece is at infinity. The two lenses are thin. The distance, in cm, of the object and the image produced

by the objective, measured from the objective lens, are respectively : **(IIT 1995; 2M)**

(a) 2.4 and 12.0 (b) 2.4 and 15.0

(c) 2.0 and 12.0 (d) 2.0 and 3.0

11. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance $d = 150.0$ m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by : **(IIT 1995; 2M)**



(a) $I(\theta) = I_0/2$ for $\theta = 30°$

(b) $I(\theta) = I_0/4$, for $\theta = 90°$

(c) $I(\theta) = I_0$, for $\theta = 0°$

(d) $I(\theta)$ is constant for all values of θ

12. Consider Fraunhofer diffraction pattern obtained with a single slit illuminated at normal incidence. At the angular position of the first diffraction minimum, the phase difference (in radian) between the wavelets from the opposite edges of the slit is: **(IIT 1995; 2M)**

(a) $\pi/4$ (b) $\pi/2$ (c) 2π (d) π

13. An eye specialist prescribes spectacles having combination of convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination, in dioptre, is : **(IIT 1997; 1M)**

(a) +1.5 (b) −1.5 (c) +6.67 (d) −6.67

14. Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal, **(IIT 1999; 2M)**

(a) that the central maximum in narrower

(b) more number of fringes

(c) less number of fringes

(d) no diffraction pattern

15. A concave lens of glass, refractive index 1.5, has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a : **(IIT 1999; 2M)**

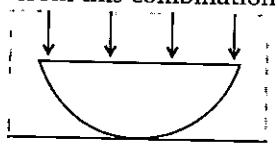
(a) convergent lens of focal length $3.5R$

(b) convergent lens of focal length $3.0R$

(c) divergent lens of focal length $3.5R$

(d) divergent lens of focal length $3.0R$

16. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate as shown in figure. The observed interference fringes from this combination shall be :



(IIT 1999; 2M)

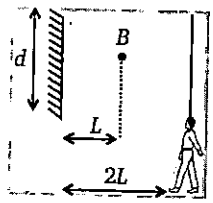
(a) straight

(b) circular

(c) equally spaced

(d) having fringe spacing which increase as we go outwards.

17. A point source of light B is placed at a distance L in front of the center of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in figure. The greatest distance over which he can see the image of the light source in the mirror is: **(IIT 2000; 2M)**



(a) $d/2$ (b) d (c) $2d$ (d) $3d$

18. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled with : **(IIT 2000; 2M)**

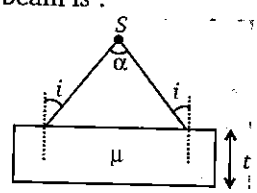
(a) air and placed in air

(b) air and immersed in L_1

(c) L_1 and immersed in L_2

(d) L_2 and immersed in L_1

19. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index μ, then the divergence angle of the emergent beam is :



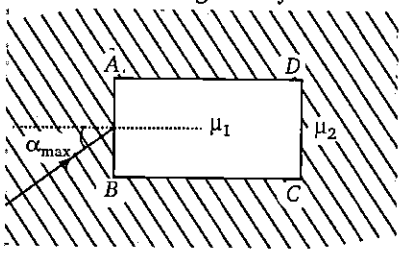
(IIT 2000; 2M)

(a) zero (b) α

(c) $\sin^{-1}\left(\frac{1}{\mu}\right)$ (d) $2\sin^{-1}\left(\frac{1}{\mu}\right)$

20. A rectangular glass slab $ABCD$ of refractive index μ_1 is immerged in water of refractive index μ_2 ($\mu_1 > \mu_2$). A ray of light is incident at the surface AB of the slab as shown in the figure. The maximum value of the angle

of α_{max} such that the ray comes out only from the other surface *CD* is given by :



(IIT 2000; 2M)

(a) $\sin^{-1}\left[\frac{\mu_1}{\mu_2}\cos\left(\sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)\right)\right]$

(b) $\sin^{-1}\left[\mu_1\cos\left(\sin^{-1}\left(\frac{1}{\mu_2}\right)\right)\right]$

(c) $\sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$

(d) $\sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$

21. In a compound microscope, the intermediate image is: **(IIT 2000; 2M)**

(a) virtual, erect and magnified
(b) real, erect and magnified
(c) real, inverted and magnified
(d) virtual, erect and reduced

22. In a double slit experiment instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern : **(IIT 2000; 2M)**

(a) the intensities of both the maxima and the minima increase
(b) the intensity of the maxima increases and the minima has zero intensity
(c) the intensity of the maxima decreases and that of the minima increases
(d) the intensity of the maxima decreases and the minima has zero intensity

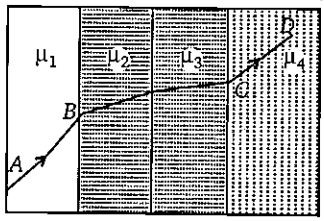
23. In Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by : **(IIT 2001; 1M)**

(a) 12 (b) 18 (c) 24 (d) 30

24. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B . Then the difference between the resultant intensities at A and B is : **(IIT 2001; 2M)**

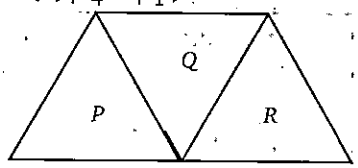
(a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$

25. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray *CD* is parallel to the incident ray *AB*, we must have : **(IIT 2001; 2M)**



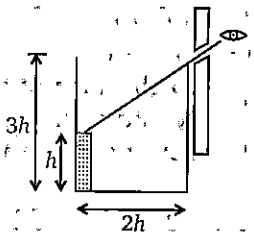
(a) $\mu_1 = \mu_2$ (b) $\mu_2 = \mu_3$
(c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$

26. A given ray of light suffers minimum deviation in an equilateral prism *P*. Additional prism *Q* and *R* of identical shape and of the same material as *P* are now added as shown in the figure. The ray will now suffer : **(IIT 2001; 2M)**



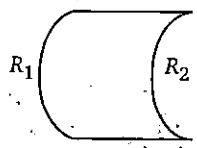
(a) greater deviation
(b) no deviation
(c) same deviation as before
(d) total internal reflection

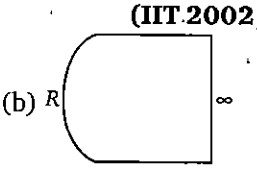
27. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is : **(IIT 2002; 2M)**

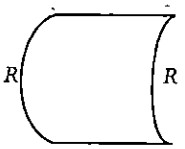


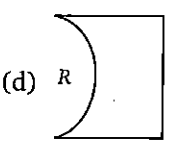
(a) $\frac{5}{2}$ (b) $\sqrt{\frac{5}{2}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\frac{3}{2}$

28. which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams. **(IIT 2002; 2M)**

(a) 

(b) 

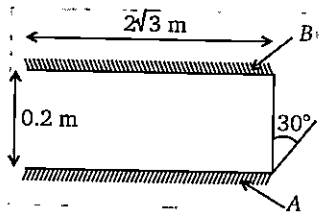
(c) 

(d) 

29. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interferring beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is : **(IIT 2002; 2M)**

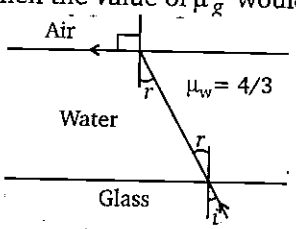
(a) 2λ (b) $2\lambda/3$ (c) $\lambda/3$ (d) λ

30. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is : **(IIT 2002; 2M)**



(a) 28 (b) 30 (c) 32 (d) 34

31. A ray of light is incident at the glass-water interface at an angle i. It emerges finally parallel to the surface of water. Then the value of μ_g would be :



(IIT 2003; 2M)

(a) $(4/3)\sin i$ (b) $1/\sin i$

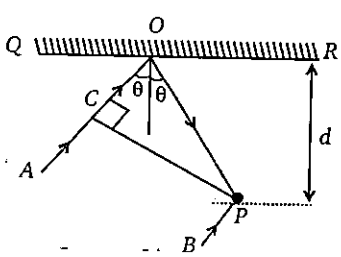
(c) 4/3 (d) 1

32. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image. **(IIT 2003; 2M)**

(a) 1.25 cm (b) 2.5 cm

(c) 1.05 cm (d) 2 cm

33. In the given diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition in θ for constructive interference at P between the ray BP and reflected ray OP : **(IIT 2003; 2M)**



(a) $\cos\theta = 3\lambda/2d$ (b) $\cos\theta = \lambda/4d$

(c) $\sec\theta - \cos\theta = \lambda/d$ (d) $\sec\theta - \cos\theta = 4\lambda/d$

34. A beam of white light is incident on glass air interface from glass to air such that green light just suffers total internal reflection. The colours of the light which will come out to air are : **(IIT 2004; 2M)**

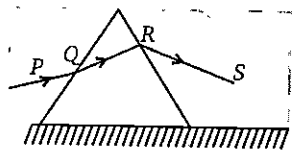
(a) violet, indigo, blue

(b) all colours except green

(c) yellow, orange, red

(d) white light

35. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation :



(IIT 2004; 2M)

(a) PQ is horizontal

(b) QR is horizontal

(c) RS is horizontal

(d) any one will be horizontal

36. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is : **(IIT 2004; 2M)**

(a) 6 cm (b) 4 cm

(c) 12 cm (d) 9 cm

37. A source emits sound of frequency 600 Hz inside water. The frequency heard in air will be equal to (velocity of sound in water = 1500 m/s, velocity of sound in air = 300 m/s) **(IIT 2004; 2M)**

(a) 3000 Hz (b) 120 Hz

(c) 600 Hz (d) 6000 Hz

38. Monocromatic light of wavelength 400 nm and 560 nm are incident simultaneously and normally on double slit apparatus whose slit separation is 0.12 mm and screen distance is 1m. Distance between areas of total darkness will be : **(IIT 2004; 2M)**

(a) 4 mm (b) 5.6 mm

(c) 14 mm (d) 28 mm

39. A convex lens is in contact with concave lens. The magnitude of the ratio of their focal lengths is 2/3. Their equivalent focal length is 30 cm. What are their individual focal lengths? **(IIT 2005; 2M)**

(a) −15, 10 (b) −10, 15

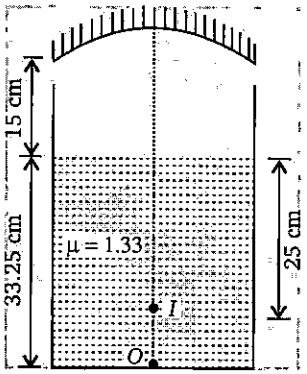
(c) 75, 50 (d) −75, 50

40. In Young's double slit experiment intensity at a point is (1/4) of the maximum intensity. Angular position of this point is : **(IIT 2005; 2M)**

(a) $\sin^{-1}(\lambda/d)$ (b) $\sin^{-1}(\lambda/2d)$

(c) $\sin^{-1}(\lambda/3d)$ (d) $\sin^{-1}(\lambda/4d)$

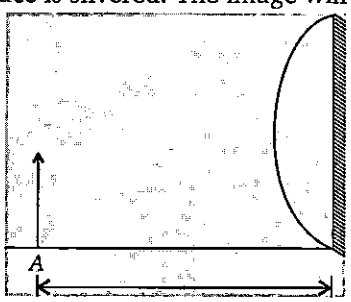
41. A container is filled with water ($\mu = 1.33$) upto a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. Focal length of the mirror is :



(IIT 2005; 2M)

(a) 15 cm (b) 20 cm
(c) −18.31 cm (d) 10 cm

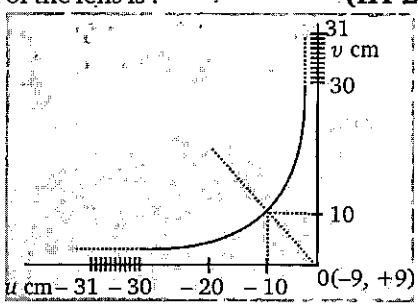
42. Focal length of the plano-convex lens is 15 cm. A small object is placed at A as shown in the figure. The plane surface is silvered. The image will form at :



(IIT 2006; 3 M)

(a) 60 cm to the left of lens
(b) 12 cm to the left of lens
(c) 60 cm to the right of lens
(d) 30 cm to the left of lens

43. The graph shows relationship object distance and image distance for an equiconvex lens. Then, focal length of the lens is : **(IIT 2006; 3 M)**



(a) 0.50 ± 0.05 cm (b) 0.50 ± 0.10 cm
(c) 5.00 ± 0.05 cm (d) 5.00 ± 0.10 cm

44. Rays of light from sun fall on a biconvex lens of focal length f and the circular image of sun of radius r is formed on the focal plane of the lens. Then : **(IIT 2006; 3M)**

(a) area of image is πr^2 and area is directly proportional to f
(b) area of image is πr^2 and area is directly proportional to f^2
(c) intensity of image increases if f is increased
(d) if lower half of the lens is covered with black paper area will become half

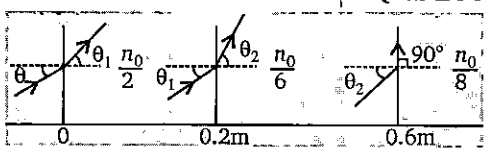
45. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be : **(IIT JEE 2007; 3M)**

(a) only a reflected ray and no refracted ray
(b) only a refracted ray and no reflected ray
(c) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

46. In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then, **(IIT 2007; 3M)**

(a) $x < f$ (b) $f < x < 2f$
(c) $x = 2f$ (d) $x > 2f$

47. A light beam is travelling from region I to region IV (Refer figure). The refractive index in regions I, II, III and IV are n_0, $\frac{n_0}{2}$, $\frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering region IV is : **(IIT 2008; 3M)**



(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$
(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

48. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be : **(IIT 2008; 3M)**

(a) 30° for both the colours
(b) greater for the violet colour
(c) greater for the red colour
(d) equal but not 30° for both the colours

49. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of

water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take $g = 10\,m/s^2$]: **(IIT 2009; 3M)**

(a) 9 m/s (b) 12 m/s
(c) 16 m/s (d) 21.33 m/s

50. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is : **(IIT 2010; 3M)**

(a) virtual and at a distance of 16 cm from the mirror
(b) real and at a distance of 16 cm from the mirror
(c) virtual and at a distance of 20 cm from the mirror
(d) real and at a distance of 20 cm from the mirror

More than One Alternatives is/are Correct

1. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of the combination is : **(IIT 1982; 3M)**

(a) −1.5 dioptre (b) −6.5 dioptre
(c) +6.5 dioptre (d) +6.67 dioptre

2. In Young's double slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that : **(IIT 1982; 3M)**

(a) the intensities at the screen due to the two slits are 5 units and 4 units respectively
(b) the intensities at the screen due to the two slits are 4 units and 1 units respectively
(c) the amplitude ratio is 3
(d) the amplitude ratio is 2

3. White light is used to illuminate the two slits in Young's double slit experiment. The separation betwen the slits is b and the screen is at a distance $d(> b)$ from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are : **(IIT 1984; 2M)**

(a) $\lambda = \frac{b^2}{d}$ (b) $\lambda = \frac{2b^2}{d}$ (c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

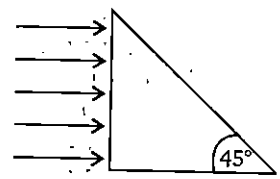
4. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen : **(IIT 1986;2M)**

(a) half the image will disappear
(b) complete image will be formed
(c) intensity of the image will increase
(d) intensity of the image will decrease

5. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to : **(IIT 1988; 2M)**

(a) $b\left(\frac{u-f}{f}\right)^{1/2}$ (b) $b\left(\frac{f}{u-f}\right)^{1/2}$
(c) $b\left(\frac{u-f}{f}\right)$ (d) $b\left(\frac{f}{u-f}\right)^{2}$

6. A beam of light consisting of red, green and blue colours is incident on a right angled prism, as shown in figure. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will : **(IIT 1989; 2M)**



(a) separate part of the red colour from the green and blue colours
(b) separate part of the blue colour from the red and green colours
(c) separate all the three colours from one another
(d) not separate even partially any colour from the other two colours

7. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of the eyepiece are : **(IIT 1989; 2M)**

(a) $f_o = 45$ cm and $f_e = -9$ cm
(b) $f_o = 50$ cm and $f_e = 10$ cm
(c) $f_o = 7.2$ cm and $f_e = 5$ cm
(d) $f_o = 30$ cm and $f_e = 6$ cm

8. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is : **(IIT 1990; 2M)**

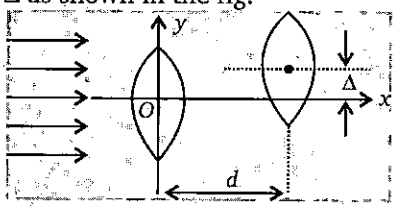
(a) 5.33° (b) 4° (c) 3° (d) 2.6°

9. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm. **(IIT 1992; 2M)**

(a) The distance between the objective and the eyepiece is 16.02 m
(b) The angular magnification of the planet is − 800
(c) The image of the planet is inverted
(d) The objective is larger than the eyepiece

10. Two thin convex lenses of focal lengths f_1 and f_2 are seperated by a horizontal distance d (where $d < f_1$,

$d < f_2$) and their centres are displaced by a vertical separation Δ as shown in the fig.



Taking the origin of coordinates O, at the centres of the first lens, the x and y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by : **(IIT 1993; 2M)**

(a) $x = \dfrac{f_1 f_2}{f_1 + f_2}$, $y = \Delta$

(b) $x = \dfrac{f_1(f_2 + d)}{f_1 + f_2 - d}$, $y = \dfrac{\Delta}{f_1 + f_2}$

(c) $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = \dfrac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(d) $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = 0$

11. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by appropriately placing : **(IIT 1995; 2M)**

(a) a concave mirror of suitable focal length
(b) a convex mirror of suitable focal length
(c) a convex lens of focal length less than 0.25 m
(d) a concave lens of suitable focal length

12. Which of the following form(s) a virtual and erect image for all positions of the object? **(IIT 1996; 2M)**

(a) Convex lens (b) Concave lens
(c) Convex mirror (d) Concave mirror

13. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration: **(IIT 1998; 2M)**

(a) is absent
(b) is smaller if the curved surface of the lens faces the object
(c) is smaller if the plane surface of the lens faces the object
(d) is the same whichever side of the lens faces the object

14. A concave mirror is placed on a horizontal table, with its directed vetically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be : **(IIT 1998; 2M)**

(a) real, and will remain at C
(b) real, and located at a point between C and ∞
(c) virtual, and located at a point between C and O
(d) real, and located at a point between C and O

15. A parallel monchromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is : **(IIT 1998; 2M)**

(a) 0 (b) $\pi/2$ (c) π (d) 2π

16. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total inernal reflection. If μ is the refractive index of the medium with respect to air, select the possible value(s) of μ from the following : **(IIT 1998; 2M)**

(a) 1.3 (b) 1.4 (c) 1.5 (d) 1.6

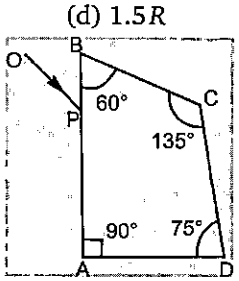
17. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s). **(IIT 1998; 2M)**

(a) If $d = \lambda$, the screen will contain only one maximum
(b) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
(c) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
(d) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

18. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O, and $PO = OQ$. The distance PO is equal to : **(IIT 1998; 2M)**

(a) $5R$ (b $3R$ (c) $2R$ (d) $1.5R$

19. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following statement(s) is (are) correct? **(IIT 2010; 3M)**



(a) The ray gets totally internally reflected at face CD.
(b) The ray comes out through face AD.

(c) The angle between the incident ray and the emergent ray is 90°.

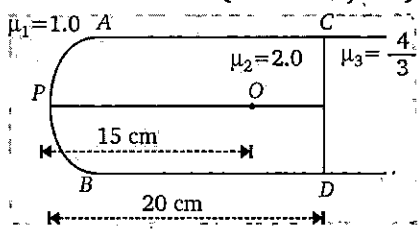
(d) The angle between the incident ray and the emergent ray is 120°.

True/False

1. The setting sun appears higher in the sky than it really is. **(IIT 1980)**
2. The intensity of ligth at a distance r from the axis of a long cylindrical source is inversely proportional to r. **(IIT 1981; 2M)**
3. A convex lens of focal length 1 metre and a concave lens of focal length 0.25 metre are kept 0.75 metre apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. **(IIT 1983; 2M)**
4. A beam of white ligth passing through a hollow prism gives no spectrum. **(IIT 1983; 2M)**
5. The two slits in a Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed on the screen. **(IIT 1984; 3M)**
6. In Young's double slit experiment, performed with a source of white light, only black and white fringes are observed. **(IIT 1987; 2M)**
7. A parallel beam of white light falls on a combination of a concave and a convex lens, both of the same material. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the other side of the lens system, one sees coloured patterns with violet colour at the outer edge. **(IIT 1988; 2M)**

Fill in the Blanks

1. A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1.5. In the medium the velocity of the light wave is ..... and its wavelength is ..... **(IIT 1983; 2M)**
2. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then d is equal to ..... cm. **(IIT 1985; 2M)**
3. A monochromatic beam of light of wavelength 6000 Å in vacuum enters a medium of refractive index 1.5. In the medium, its wavelength is ..... its frequency is ..... **(IIT 1985; 2M)**
4. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same set-up the two slits are sources of equal amplitude A and wavelength λ, but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is ..... **(IIT 1986; 2M)**
5. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index 4/3, its focal will become .....cm. **(IIT 1987; 2M)**
6. A point source emits waves equally in all directions in a non-absorbing medium. Two points P and Q are at a distance of 9 metre and 25 metre respectively from the source. The ratio of amplitudes of the waves at P and Q is ..... **(IIT 1989; 2M)**
7. A slab of a material of refractive index 2 shown in fig. has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is air and on the right of CD is water with refractive indices as given in the figure. An object Q is placed at a distance of 15 cm from the pole P as shown. The distance of the final image of O from P, as viewed from the left is ..... **(IIT 1991; 2M)**



8. A thin rod of length $\frac{f}{3}$ is placed along the optic axis of a concave mirror of focal length f such that its image, which is real and elongated, just touches the rod. The magnification is ..... **(IIT 1991; 1M)**
9. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism, with the base of the prism is ..... **(IIT 1992; 1M)**
10. The resolving power of electron microscope is higher than that of an optical microscope because the wavelength of electrons is ..... than the wavelength of visible light. **(IIT 1992; 1M)**
11. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is ..... degree. **(IIT 1997; 2M)**
12. Two thin lenses, when in contact, produce a combination of power +10 dioptre. When they are 0.25 m apart, the power reduces to +6 dioptre. The

focal length of the lenses are ..... m and .....m. **(IIT 1997; 2M)**

13. A slit of width d is placed in front of a lens of focal length 0.5 m and is illuminated normally with light of wavelength 5.89×10^{-7} m. The first diffraction minima on either side of the central diffraction maximum are separated by 2×10^{-3} m. The width d of the slit is ..... m. **(IIT 1997; 1M)**

Assertion and Reason

Instructions: *Each question contains statement-1 (assertion) and statement-2 (reason). Of these statements, mark correct choice if:*

(a) Statements-1 and 2 are true and statement-2 is a correct explanation for statement-1

(b) Statements-1 and 2 are true and statement-2 is not a correct explanation for statement-1

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true.

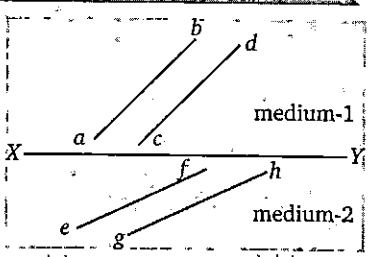
1. Statement-1 : The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

Statement-2 : Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

(IIT 2007; 3M)

Comprehension Based Problems

The figure shows a surface XY separating two transparent media, medium-1 and medium-2. The lines ab and cd represent wavefronts of a light wave travelling in medium-1 and incident on XY. The lines ef and gh represent in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



(IIT 2007; 3 × 4 M)

1. Light travels as a :
(a) parallel beam in each medium
(b) convergent beam in each medium
(c) divergent beam in each medium
(d) divergent beam in one medium and convergent beam in the other medium.

2. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$.
(a) ϕ_c can not be equal to ϕ_d
(b) ϕ_d can be equal to ϕ_e
(c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
(d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

3. Speed of light is :
(a) the same in medium-1 and medium-2
(b) larger in medium-1 than in medium-2
(c) larger in medium-2 than in medium-1
(d) different at b and d

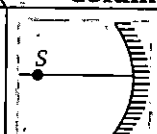
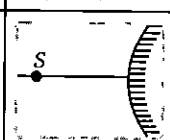
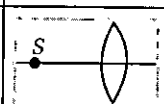
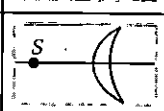
Matching Type Problems

1. A simple telescope used to view distant objects has eyepiece and objective lenses of focal lengths f_e and f_o respectively. Then **(IIT 2006; 6M)**

	Column I		Column II
(a)	Intensity of light received by lens	(p)	Radius of aperature
(b)	Angular magnification	(q)	Dispersion of lens
(c)	Length of telescope	(r)	Focal length of objective lens and eyepiece lens
(d)	Sharpness of image	(s)	Spherical aberration

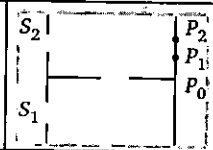
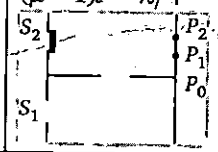
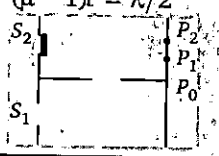
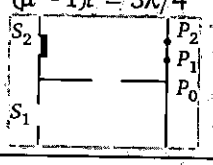
2. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in column II. Match all the properties of images from Column II with the appropriate components given in Column I.

(IIT 2008; 8M)

	Column I		Column II
(a)		(p)	Real image
(b)		(q)	Virtual image
(c)		(r)	Magnified image
(d)		(s)	Image at infinity

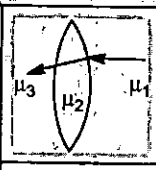
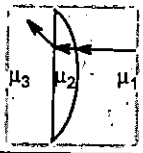
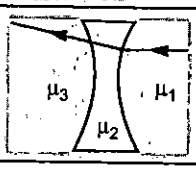
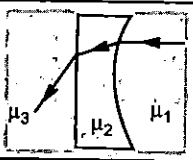
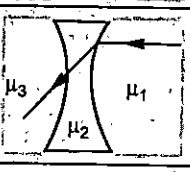
3. Column-I shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0, S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$ where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column-I** with the statement (s) in **Column-II** valid for that situation.

(IIT 2009; 8M)

	Column-I		Column-II
(a)		(p)	$\delta(P_0) = 0$
(b)	$(\mu - 1)t = \lambda/4$ 	(q)	$\delta(P_1) = 0$
(c)	$(\mu - 1)t = \lambda/2$ 	(r)	$I(P_1) = 0$
(d)	$(\mu - 1)t = 3\lambda/4$ 	(s)	$I(P_0) > I(P_1)$
		(t)	$I(P_2) > I(P_1)$

4. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column-II. A ray traversing these media is also shown in the figure. In Column-I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in Column-II.

(IIT 2010; 6M)

	Column-I		Column-II
(a)	$\mu_1 < \mu_2$	(p)	
(b)	$\mu_1 > \mu_2$	(q)	
(c)	$\mu_2 = \mu_3$	(r)	
(d)	$\mu_2 > \mu_3$	(s)	
		(t)	

Integer Answer Type Problems

1. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm infront of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is: **(IIT 2010; 4M)**

2. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour?

(IIT 2010; 4M)

3. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

(IIT 2010; 4M)

ANSWERS

Only One Alternative is Correct

1.	(b)	**2.**	(a)	**3.**	(d)	**4.**	(a)	**5.**	(a)	**6.**	(c)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(a)
11.	(c)	**12.**	(c)	**13.**	(b)	**14.**	(d)	**15.**	(a)	**16.**	(a)	**17.**	(d)	**18.**	(d)	**19.**	(b)	**20.**	(a)
21.	(c)	**22.**	(a)	**23.**	(b)	**24.**	(b)	**25.**	(d)	**26.**	(c)	**27.**	(b)	**28.**	(c)	**29.**	(a)	**30.**	(b)
31.	(b)	**32.**	(b)	**33.**	(b)	**34.**	(c)	**35.**	(b)	**36.**	(a)	**37.**	(c)	**38.**	(d)	**39.**	(a)	**40.**	(c)
41.	(c)	**42.**	(b)	**43.**	(c)	**44.**	(b)	**45.**	(c)	**46.**	(b)	**47.**	(b)	**48.**	(a)	**49.**	(c)	**50.**	(b)

More than One Alternatives is/are Correct

1. (a)	**2.** (b, d)	**3.** (a, c)	**4.** (b, d)	**5.** (d)
6. (a)	**7.** (d)	**8.** (c)	**9.** (a, b, c, d)	**10.** (c)
11. (a, c)	**12.** (b, c)	**13.** (b)	**14.** (d)	**15.** (d)
16. (c, d)	**17.** (a, b)	**18.** (a)	**19.** (a, b, c)	

True/False

1. True	**2.** True	**3.** False	**4.** True	**5.** True
6. False	**7.** True			

Fill in the Blanks

1. 2×10^{8} m/s, 4×10^{-7} m	**2.** $d = 15$	**3.** 4000Å, 5×10^{14} Hz	
4. 2	**5.** 60	**6.** 25/9	**7.** 30 cm to right of P
8. 1.5	**9.** zero	**10.** Smaller	**11.** 15°
12. 0.125, 0.5	**13.** 2.945×10^{-4}		

Assertion and Reason

1. (c)

Comprehension Based Problems

1. (a) **2.** (c) **3.** (b)

Matching Type Problems

1. a → p; b → r; c → r; d → p, q, s

2. a → p, q, r, s; b → q; c → p, q, r, s; d → p, q, r, s

3. a → p, s; b → q; c → t; d → r, s, t

4. a→p, r; b→q, s, t; c→ p, r,t; d→q, s

Integer Answer Type Problems

1. 6 **2.** 3 **3.** 6

Solutions

Only One Alternative is Correct

1. (b) The intensity of illumination at a surface is given by

$$E = \frac{I\cos\theta}{r^2}$$

$$E = 5\times10^{-4}\ \text{lumen/cm}^2$$

$$= 5\times10^{-4}\times10^{4}\ \text{lumen/m}^2$$

or $$I = \frac{Er^2}{\cos\theta} = \frac{5\times(2)^2}{\cos 60^\circ} = 40\ \text{candela}$$

2. (a) Wavelength $(\lambda) = \dfrac{\text{velocity}\ (v)}{\text{frequency}\ (f)}$

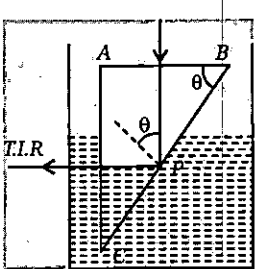
When the ray of light travels from air to glass, (f) remains unchanged while velocity (v) decreases.
Hence wavelength λ should decreases.

3. (d) In double slit experiment, fringe width $\beta = \dfrac{\lambda D}{d}$

$$\therefore \quad \frac{\beta_2}{\beta_1} = \left(\frac{D_2}{D_1}\right)\left(\frac{d_1}{d_2}\right) = (2)(2) = 4 \text{ or } \beta_2 = 4\beta_1$$

Fringe width will become four times.

4. (a) Total internal reflection occurs when the ray of light travels from denser medium to rarer medium and the angle of incidence is greater than critical angle.
At P, glass is denser and water is rarer.



$$^{g}\mu_{w} = \frac{^{a}\mu_{w}}{^{a}\mu_{g}} = \frac{4/3}{3/2} = \frac{8}{9}$$

$\therefore \quad \sin C = \dfrac{8}{9}$ where C = critical angle.

$\therefore \quad \theta = C$ or $\sin\theta = 8/9$ for critical angle.

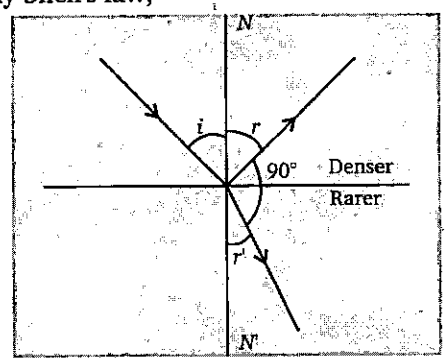
For total internal reflection, $\sin\theta > \dfrac{8}{9}$

5. (a) $r + r' + 90^\circ = 180^\circ$

$\therefore \quad r' = 90^\circ - r$ .... (1)

$i = r'$ ... (2)

Apply Snell's law,



$$\mu_D \sin i = \mu_R \sin r'$$

or $$\mu_D \sin r = \mu_R \sin(90^\circ - r)$$

or $$\frac{\mu_R}{\mu_D} = \tan r$$

or $$\sin\theta_C = \tan r$$

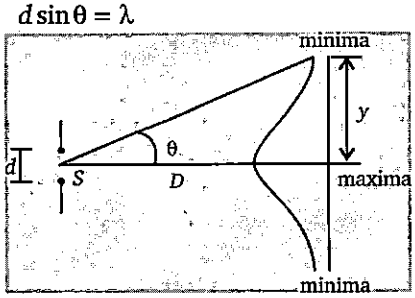
or $$\theta_C = \sin^{-1}(\tan r)$$

6. (c) $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{4I} + \sqrt{I})^2 = 9I$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{4I} - \sqrt{I})^2 = I$

7. (d) For first dark fringe, in diffraction pattern,

$d\sin\theta = \lambda$



or $$d\left(\frac{y}{D}\right) = \lambda$$

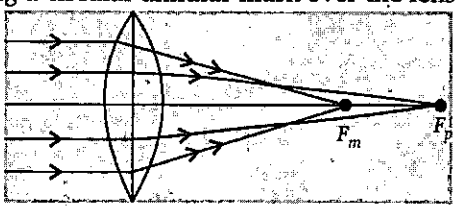
or $y = \frac{D\lambda}{d}$

or Distance $= \frac{2D\lambda}{d}$

or Distance $= \frac{2\times(2)(600\times10^{-9})}{1\times10^{-3}}$ m

$= 2.4\times10^{-3}$ m $= 2.4$ mm

8. (c) Spherical aberration is caused because the marginal rays and the paraxial rays incident on a lens are brought to focus at different points. To reduce spherical aberration, the marginal rays are blocked by using a circular annular mask over the lens.



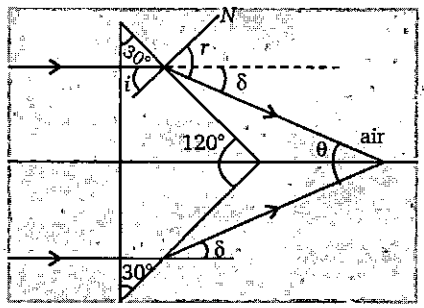
9. (c) $\angle i = 30°$, by geometry

$$\frac{\sin i}{\sin r} = \frac{\sin 30°}{\sin r}$$

$$\frac{1}{1.44} = \frac{\sin 30°}{\sin r}$$

or $\sin r = 0.72$

$\delta = \angle r - \angle i$



$\therefore$ $\delta = \sin^{-1}(0.72) - 30°$

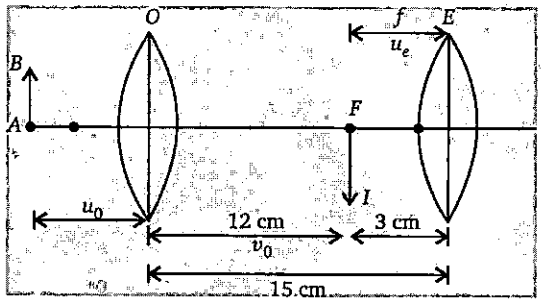
or $(2\delta) = 2[\sin^{-1}(0.72) - 30°]$

or $\theta = 2[\sin^{-1}(0.72) - 30°]$

θ is the angle at which the emergent rays from the opposite faces meet each other.

10. (a) The eyepiece forms the final image at infinity. Its object should therefore lie at its focus.

F denotes focus of eyepiece. I denotes image formed by the objective lens which serves as object for eye piece. It should be at 3 cm from eyepiece.



$\therefore$ v_0 for objective lens $= 15 - 3 = 12$ cm ... (1)

$\therefore$ $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

or

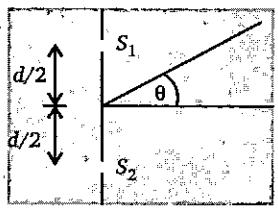
$$\frac{1}{12} - \frac{1}{u_0} = \frac{1}{2} \Rightarrow \frac{1}{u_0} = \frac{1}{12} - \frac{1}{2} = -\frac{5}{12}$$

or $u_0 = -2.4$ cm

From objectives lens, $u_0 = 2.4$ cm (to left)

$v_0 = 12$ cm (to right)

11. (c) The intensity $I(\theta)$ is measured as a function of θ. I_0 denotes the maximum intensity. θ varies from 0° to 90°.



$\because$ $I(\theta) = I_0 \cos^2\left(\frac{\delta}{2}\right)$

When $\delta = \frac{2\pi}{\lambda} \times \frac{dy}{D} = \frac{2\pi d \tan\theta}{\lambda}$

$\therefore$ $I(\theta) = I_0 \cos^2\left(\frac{\pi d \tan\theta}{\lambda}\right)$

where $\lambda = \frac{3\times10^8}{10^6} = 300$ m

or $I(\theta) = I_0 \cos^2\left(\frac{\pi\times150\times\tan\theta}{300}\right)$

or $I(\theta) = I_0 \cos^2\left(\frac{\pi}{2}\tan\theta\right)$

(a) When $\theta = 30°$ $I(\theta) = I_0 \cos^2\left(\frac{\pi\times1}{2\sqrt{3}}\right)$

Option (a) is not correct.

(b) When $\theta = 90°$, $I(\theta) = I_0 \cos^2(\infty)$

Option (b) is not correct.

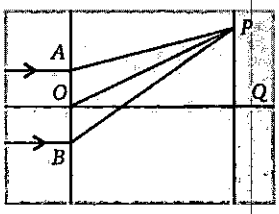
(c) When $\theta = 0$, $I(\theta) = I_0 \cos^2(0°)$

$\therefore$ $I(\theta) = I_0$. The option (c) is correct.

(d) $I(\theta)$ varies with value of θ. Option (d) is not correct.

Hence only option (c) is correct.

12. (c) The figure depicts Fraunhofer diffraction pattern.



For P to be first minimum, the phase difference between OA or OB is π.

$\therefore$ Phase difference from opposite edges $= 2\pi$ radian

13. (b) For convex lens, power $P_1 = \dfrac{100}{40} = 2.5\,\text{D}$

For concave lens, power $P_2 = -\dfrac{100}{25} = -4.0\,\text{D}$

$\therefore$ Power of combination $= P_1 + P_2 = 2.5 - 4.0$
$= -1.5\,\text{D}$

14. (d) X-rays have very small wavelength of the order of 1Å. Light rays have wavelength of the order of 6000 Å. For diffraction pattern to be produced by incident waves, the dimension of slit should be comparable to the wavelength of waves. No diffraction pattern will be observed with X-rays which replace the yellow light.

15. (a) When glass lens is immersed in a medium, its refractive index is ${}^{m}\mu_{g}$.

$${}^{m}\mu_{g} = \frac{{}^{a}\mu_{g}}{{}^{a}\mu_{m}} = \frac{1.50}{1.75} = \frac{6}{7}$$

$\therefore$ By lens maker's formula $\dfrac{1}{f} = ({}^{m}\mu_{g} - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

or $\dfrac{1}{f} = \left(\dfrac{6}{7} - 1\right)\left(-\dfrac{1}{R} - \dfrac{1}{R}\right)$ or $\dfrac{1}{f} = \left(-\dfrac{1}{7}\right)\left(-\dfrac{2}{R}\right)$

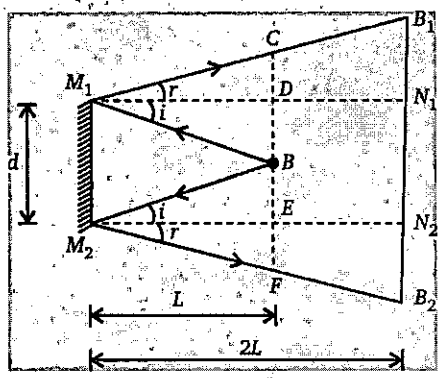
or $$f = \frac{7R}{2} = 3.5R$$

Positive focal length shows that it behaves like a convergent lens of focal length 3.5 R.

Note. When a glass lens is immersed in a denser medium, the nature of the lens is reversed. A convex lens behaves as a divergent lens and a concave lens behaves as a convergent lens. Glass lens (${}^{a}\mu_{g} = 1.5$) behaves like this pattern when dipped in carbon disulfide (${}^{a}\mu_{g} = 1.5$)

16. (a) Locus of equal path difference consists in the lines running parallel to the axis of the cylinder. Hence fringes obtained are straight.

17. (d) The ray diagram denotes the passage of light.



B_1 is the image of source B formed by reflection at M_1. Similarly B_2 is the image of B formed by reflection at M_2.

$N_1N_2 = M_1M_2 = d$
($\because M_1N_1$ and M_2N_2 are normals)

$DB = DC = d/2$

ΔM_1CD and $\Delta M_1B_1N_1$ are similar

$\therefore$ $\dfrac{M_1D}{M_1N_1} = \dfrac{CD}{B_1N_1}$

or $B_1N_1 = (CD)\left(\dfrac{M_1N_1}{M_1D}\right)$

or $B_1N_1 = \dfrac{d/2}{1}\left(\dfrac{2L}{L}\right) = d$

Similarly $B_2N_2 = d$

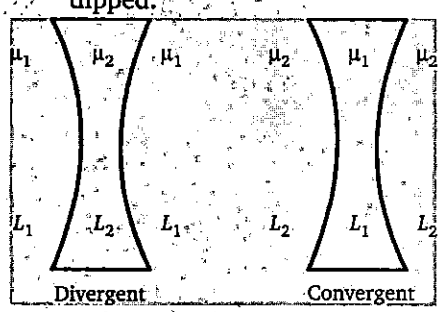
$\therefore$ $B_1B_2 = B_1N_1 + N_1N_2 + N_2B_2$

$B_1B_2 = (d + d + d) = 3d$

18. (d) According to lens maker's formula

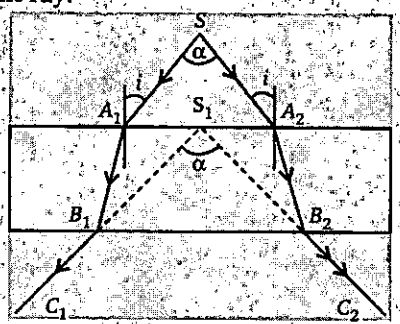
$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where μ_L = refractive index of material of lens
μ_m = refractive index of medium in which lens is dipped.



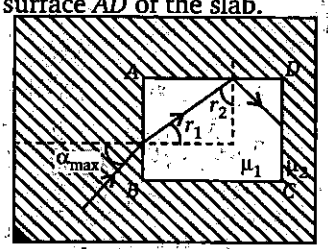
If $\mu_L > \mu_m$, the lens maintains its nature.
A concave lens behaves as a divergent lens and a convex lens behaves as a convergent lens.
If $\mu_L < \mu_m$, the nature of lens is reversed.
Since $\mu_2 > \mu_1 > 1$, the concave lens will maintain its divergent nature if it is filled with L_2 and immersed in L_1.

19. (b) When a ray is incident on a glass slab, after refraction through slab, the emergent ray is parallel to incident ray.



Emergent ray B_1C_1 is parallel to SA_1.
Emergent ray B_2C_2 is parallel to SA_2.
$\therefore$ Angle between SA_1 and SA_2 is equal to angle between B_1C_1 and B_2C_2 drawn backwards.
$\therefore$ Angle $= \alpha$.

20. (a) Rays come out only from the surface CD when the light is incident at the surface AB of the slab.
It means that rays after refraction from AB undergo total internal reflection at surface AD of the slab.



$\therefore \quad r_1 + r_2 = 90°$

$\therefore \quad r_1 = 90° - r_2$

$(r_1)_{max} = 90° - (r_2)_{min}$

$(r_2)_{min}$ = critical angle θ_c

$$\sin(\theta_c) = \frac{\mu_2}{\mu_1}$$

$$(r_1)_{max} = 90° - \theta_c$$

For refraction at AB, apply snell's law.

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{\sin(\alpha_{max})}{\sin(r_1)_{max}} = \frac{\sin(\alpha_{max})}{\sin(90-\theta_c)}$$

$$= \frac{\sin(\alpha_{max})}{\cos(\theta_c)}$$

$$\text{or} \quad \sin(\alpha_{max}) = \frac{\mu_1}{\mu_2}\cos(\theta_c)$$

$$\text{or} \quad \alpha_{max} = \sin^{-1}\left[\frac{\mu_1}{\mu_2}\cos(\theta_c)\right]$$

Put θ_c from eq. (1),

$$\text{or} \quad \alpha_{max} = \sin^{-1}\left[\frac{\mu_1}{\mu_2}\cos\left(\sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)\right)\right]$$

21. (c) The intermediate image in a compound microscope is real, inverted and magnified.

22. (a) When slits are of equal width,

$$I_{max} \propto (\sqrt{I} + \sqrt{I})^2 \propto 4I = 4KI \text{ (say)} \quad \ldots (1)$$

$$I_{max} \propto (\sqrt{I} + \sqrt{I})^2 \propto \text{zero} = \text{zero} \quad \ldots (2)$$

In second case,

$$\frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{W}{2W} = \frac{1}{2}, \quad \therefore I'_{max} = K(\sqrt{I_1} + \sqrt{I_2})^2$$

$$I'_{max} = K(\sqrt{I_1} + \sqrt{2I_1})^2 = KI_1(1+\sqrt{2})^2$$

$$= KI_1 \times (2.414)^2 = 5.8KI_1 = 5.8KI \quad \ldots (3)$$

$$I'_{min} = K(\sqrt{I_1} - \sqrt{I_2})^2 = KI_1(1-\sqrt{2})^2$$

$$= KI(-0.414)^2 = 0.17KI \quad \ldots (4)$$

$I'_{max} > I_{max}$ from eq. (3) and (1)

$I'_{min} > I_{min}$ from eq. (4) and (2)

$\therefore$ Intensities of both, the maxima and the minima, increase.

23 (b) Fringe width $\beta = \frac{\lambda D}{d}$

12 fringes of λ_1, occupy a segment of screen.
Let N fringes of λ_2 occupy the same segment.

$$\therefore \quad N\left(\frac{\lambda_2 D}{d}\right) = 12\left(\frac{\lambda_1 D}{d}\right) \text{ or } N = \frac{12\times 600\times 10^{-9}}{400\times 10^{-9}}$$

or $\quad N = 18$

24. (b) $I(\phi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I_1 = I, I_2 = 4I,$

$\therefore$ At A when $\phi = \pi/2, I_A = I + 4I = 5I \quad \ldots (1)$

Again at B when $\phi = \pi$,

$I_B = I + 4I + 2\sqrt{I \times 4I}\cos\pi$ or $I_B = 5I - 4I$

or $I_B = I$

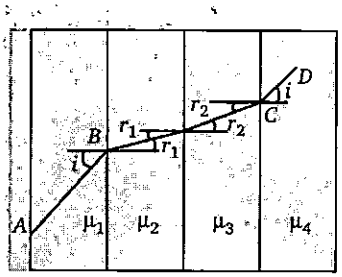
$\therefore \quad I_A - I_B = 5I - I$ or $I_A - I_B = 4I$

25. (d) Apply Snell's law

$$^1\mu_2 = \frac{\sin i}{\sin r_1}$$

$$^2\mu_3 = \frac{\sin r_1}{\sin r_2}$$

$$^3\mu_4 = \frac{\sin r_2}{\sin i}$$

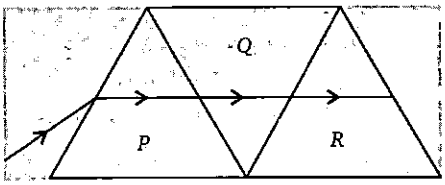


$$^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_4 = 1$$

$$\frac{\mu_2}{\mu_1} \times \frac{\mu_3}{\mu_2} \times \frac{\mu_4}{\mu_3} = 1$$

or $\quad \mu_4 = \mu_1$

26. (c) No refraction will occur when ray travels from P to Q or Q to R because P, Q, R are made of same material. They are all identical prisms.



Deviation suffered in prism P remains as such.

27. (b) When liquid is filled in the beaker up to a height $2h$, the image of the lower end A of the rod AB is formed at B so that it is visible to the eye.

$\because \quad DE = EB = 2h$

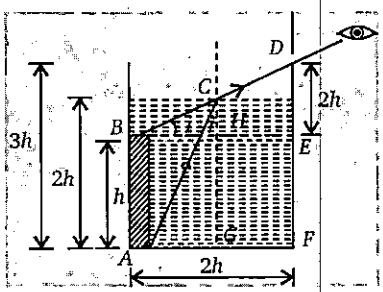
$\therefore \quad \angle i = 45°$ ... (1)

$\sin r = \frac{AG}{AC} = \frac{h}{h\sqrt{5}}$ $[\because AC^2 = (GC)^2 + (GA)^2]$

$\therefore \quad \mu = \frac{\sin i}{\sin r}$

or $\mu = \frac{\sin 45°}{1/\sqrt{5}}$

or $\mu = \sqrt{\frac{5}{2}}$



28. (c) In figure option (c), both the curved surfaces have same R on the same side. Hence no dispersion is exhibited.

For a lens, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For no dispersion $d\left(\frac{1}{f}\right) = 0$

$\therefore \quad 0 = (d\mu)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ or $R_1 = R_2$

29. (a) Path difference due to slab $= (\mu - 1)t$

or $(\mu - 1)t = n\lambda$

For minimum thickness t of plate, n should be minimum *i.e.* $n = 1$

$\therefore \quad (\mu - 1)t = \lambda$

or $t = \frac{\lambda}{\mu - 1}$ or $t = \frac{\lambda}{1.5 - 1}$ or $t = 2\lambda$

30. (b) Let N = no. of reflections

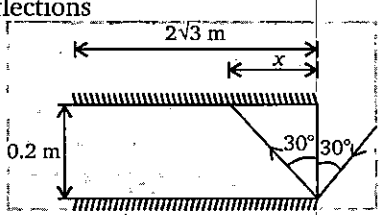
$\therefore N \cdot x = 2\sqrt{3}$

or $N = \frac{2\sqrt{3}}{x}$

or $N = \frac{2\sqrt{3}}{0.2 \tan 30°}$

$= \frac{6}{0.2}$

or $N = 30$



31. (b) Apply Snell's law,

At glass- water surface,

$^g\mu_w = \frac{\sin i}{\sin r}$

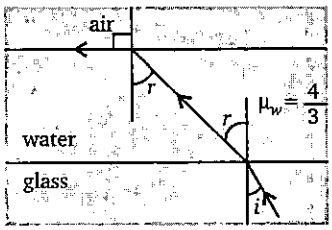
At water - air surface

$^w\mu_a = \frac{\sin r}{\sin 90°}$

$\because \quad {}^g\mu_w \times {}^w\mu_a \times {}^a\mu_g = 1$

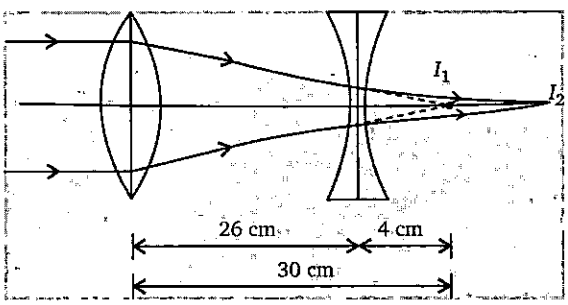
$\therefore \quad \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin 90°} \times {}^a\mu_g = 1$ or $\sin i \times {}^a\mu_g = 1$

or $^a\mu_g = \frac{1}{\sin i}$



32. (b) Convex lens forms the image at I_1. I_2 is at the second focus of convex lens. Size of $I_1 = 2$ cm.

I_1 acts as virtual object for concave lens. Concave lens forms the image of I_1 at I_2.



Lens formula : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For concave lens,

$\frac{1}{v} - \frac{1}{4} = -\frac{1}{20}$ or $\frac{1}{v} = \frac{1}{20} + \frac{1}{4} = \frac{4}{20} = \frac{1}{5}$

or $v = 5$ cm = Distance of I_2 from concave lens.

$\therefore$ Magnification $= \frac{v}{u} = \frac{\text{size of image}}{\text{size of object}} = \frac{5}{4}$

or $\frac{\text{size of image}}{2} = 1.25$

or size of image due to concave lens = 2.5 cm.

33. (b) In ΔOPM,

$\frac{PM}{OP} = \cos\theta$

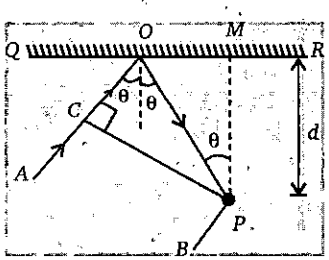
or $OP = \frac{d}{\cos\theta}$

In ΔCOP,

$\cos 2\theta = \frac{OC}{OP}$

or $OC = (OP)\cos 2\theta$ or $OC = \frac{d\cos 2\theta}{\cos\theta}$

Path difference $= CO + OP + \frac{\lambda}{2}$



$$= \frac{d\cos 2\theta}{\cos\theta} + \frac{d}{\cos\theta} + \frac{\lambda}{2}$$

$$= \frac{d(2\cos^2\theta - 1)}{\cos\theta} + \frac{d}{\cos\theta} + \frac{\lambda}{2}$$

$$= 2d\cos\theta + \frac{\lambda}{2}$$

For constructive interference at P, between BP and OP,

Path difference $= n\lambda$

or $2d\cos\theta + \frac{\lambda}{2} = n\lambda$ or $2d\cos\theta = \left(n - \frac{1}{2}\right)\lambda$

or $\cos\theta = \frac{(2n-1)\lambda}{4d}$, for $n = 1$, $\cos\theta = \frac{\lambda}{4d}$

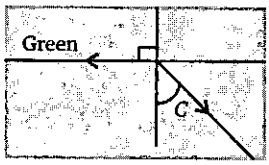
34. (c) λ increases in the order V, I, B, G, Y, O, R.

Y, O, R have λ greater than that of G.

V, I, B have λ less than that of G.

$$\mu = \frac{1}{\sin C}$$

Also $\mu \propto \frac{1}{\lambda}$

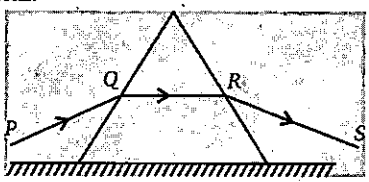


$\therefore$ $\sin C \propto \lambda$

$\therefore$ C is greater for Y, O, R

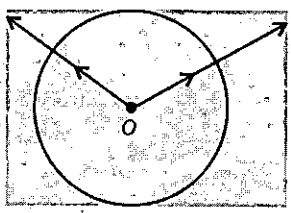
$\therefore$ Y, O, R will come out to air.

35. (b) In equilateral prism, the refracted ray QR runs parallel to base. Base is horizontal. Hence QR is horizontal.



36. (a) A Point object is at the centre of a glass sphere of radius 6 cm.

The rays from the object fall normally on the surface of the sphere and emerge undeviated. When drawn backwards, they meet at O.



The image will be formed at the centre O itself.

$\therefore$ Distance of virtual image from surface = 6 cm.

37. (c) Frequency of sound = 600 Hz inside water.

Frequency of a wave depends on the source and it does not change during refraction when wave travels from one medium to another.

$\therefore$ Frequency heard in air = 600 Hz.

38. (d) At the area of total darkness, in double slit apparatus, minima will occur for both the wavelengths which are incident simultaneously and normally.

$$\therefore \frac{(2n+1)\lambda_1}{2} = \frac{(2m+1)\lambda_2}{2} \text{ or } \frac{2n+1}{2m+1} = \frac{\lambda_2}{\lambda_1}$$

or $\frac{2n+1}{2m+1} = \frac{560}{400} = \frac{7}{5}$ or $10n = 14m + 2$

By inspection, the two solutions are

(i) if $m_1 = 2$, $n_1 = 3$

(ii) if $m_2 = 7$, $n_2 = 10$

$\therefore$ Distance between areas correspond to these points,

$$\therefore \text{Distance } \Delta S = \frac{D\lambda_1}{d}\left[\frac{(2n_2+1) - (2n_1+1)}{2}\right]$$

Put $n_2 = 10$ and $n_1 = 3$,

$$\Delta S = \frac{1 \times (400 \times 10^{-9})}{0.1 \times 10^{-3}}\left[\frac{21-7}{2}\right]$$

or $\Delta S = 4 \times 7 \times 10^{-3}$ m or $\Delta S = 28$ mm

39. (a) Let focal length of convex lens $= f_1 = f$ (suppose)

f_2 = focal length of concave lens $= \frac{-3f}{2}$

Equivalent focal length = 30 cm

$$\therefore \frac{1}{30} = \frac{1}{f} - \frac{2}{3f}\left(\because \frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}\right) \text{or } \frac{1}{30} = \frac{1}{3f}$$

or $f = 10$ cm = Focal length of convex lens.

$\therefore$ Focal length of concave lens $= \frac{-3}{2} \times 10 = -15$ cm

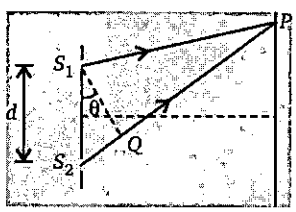
$\therefore$ Focal lengths are – 15 cm (concave lens) and 10 cm (convex lens).

40. (c) In Young's double slit experiment, intensity at any point P is given by $I = I_{max}\cos^2(\phi/2)$.

where ϕ denotes the phase difference at P.

$$\because \quad I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\therefore \quad \frac{I_{max}}{4} = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$



or $\cos\left(\frac{\phi}{2}\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

or $\frac{\phi}{2} = \frac{\pi}{3}$ or $\phi = \frac{2\pi}{3}$ or $\frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{3}$

or $\Delta x = \lambda/3$

Path difference $\Delta x = S_2Q = d\sin\theta$

or $d\sin\theta = \lambda/3$

or $\theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$

41. (c) O= Object, I = Image of object.

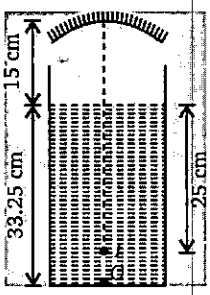
$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$\therefore$ Apparent depth $= \dfrac{\text{Real depth}}{\mu}$

From mirror :

Object distance $= u$

$$u = 15 + \frac{33.25}{1.33} = 15 + 25 = 40\,\text{cm}$$

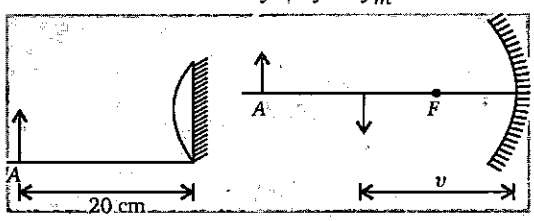


$$v = 15 + \frac{25}{1.33} = 15 + 18.8 = 33.8\,\text{cm}$$

$$\therefore \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{-33.8} + \frac{1}{-40} = \frac{1}{f}$$

or $f = -18.31\,\text{cm}$

42 (b) Focal length of equivalent mirror when a lens is silvered at a surface is $\dfrac{1}{f} = \dfrac{2}{f} + \dfrac{1}{f_m}$



Here the mirror is a plane (surface) mirror.

$\therefore \quad f_m = \infty$

$$\therefore \quad \frac{1}{f} = \frac{2}{15} + \frac{1}{\infty} = \frac{2}{15} \text{ or } f = \frac{15}{2}\,\text{cm}$$

Positive focal length means a converging mirror.

For mirror : As per sign convention

$u = -20\,\text{cm}, f = -\dfrac{15}{2}\,\text{cm}$

$$\because \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} + \frac{1}{-20} = \frac{1}{-15/2}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{20} - \frac{2}{15} = \frac{3-8}{60} = \frac{-5}{60} = -\frac{1}{12}$$

or $v = -12\,\text{cm}$

Image is formed at 12 cm to the left of lens.

Note. There are three stages of formation of final image

(i) Lens forms the image I_1 of object at A.

(ii) I_1 acts as virtual object for mirrored surface.

(iii) I_2, the image formed by mirror acts as virtual object for lens. The lens forms the final image.

43. (c) Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

or $$\frac{1}{f} = \frac{1}{10} - \frac{1}{-10}$$

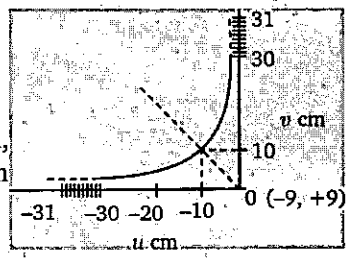
or $$\frac{1}{f} = \frac{2}{10} = \frac{1}{5}$$

or $f = +5\,\text{cm}$

$\Delta u = 0.1$, from graph,

$\Delta v = 0.1$, from graph

$$\because \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$



For maximum error in f,

errors are added.

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} \quad \text{or} \quad \frac{\Delta f}{(5)^2} = \frac{0.1}{(10)^2} + \frac{0.1}{(10)^2}$$

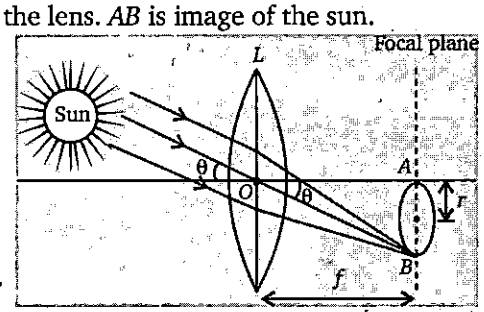
or $$\Delta f = \frac{25 \times 2 \times 0.1}{100} \quad \text{or} \quad \Delta f = 0.05$$

$\therefore$ Focal length of lens $= f \pm \Delta f = 5.00 \pm 0.05\,\text{cm}$.

Note. As the graph between v and u indicates, it is the curve of a convex lens where $u = -10\,\text{cm}, v = 10\,\text{cm}, f = 5\,\text{cm}$. Object is placed at $2f$ and real, inverted and equal size image is formed at $2f$, on the other side of the lens.

44. (b) Rays of light from sun are incident upon a biconvex lens. Sun is at infinity and incident rays form a parallel beam.

The beam is brought to convergence in the focal plane of the lens. AB is image of the sun.



Let r = radius of real image of the sun. Sun is above principal axis of lens.

The real and inverted image of diminished size is formed below the principal axis in the focal plane of the lens as shown in figure.

Area of circular image of sun $= \pi r^2$ ... (1)

In the ΔOAB, θ is inclination of sun and is constant.

$$\tan\theta = \frac{r}{f} \Rightarrow r = f\tan\theta$$

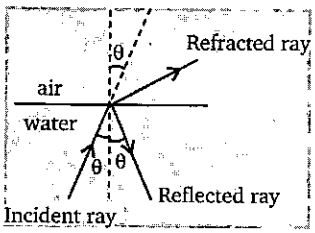
or Area of image $= \pi (f\tan\theta)^2$

or Area of image $= (\pi \tan^2\theta) f^2$ ...(2)

From eq. (1) and (2), area of image $= \pi r^2$ and area is proportional to f^2.

45. (c) The ray is incident on the interface of a rarer medium (air) from a denser medium (water), and the angle of incidence is less than the critical angle. The way will be partly refracted and partly reflected. Also,

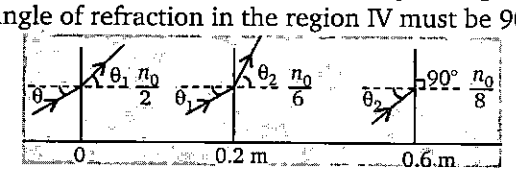
the Snell's law $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$ tells that the angle of refraction will be more than the angle of incidence, on entering the air the ray will bend away from the normal. See the fig.



The angle between the reflected and the refracted ray is less than 180°–2θ.

46. (b) On shifting the eye towards left, the image will appear to the right of the object pin if the image distance is more than the object distance and the image is real. This happens when the object lies between the focus and the centre of curvature $f < x < 2f$.

47. (b) As the beam just misses entering the region IV, the angle of refraction in the region IV must be 90°.

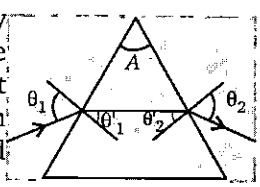


Application of Snell's law successively at different interfaces gives

$$n_0 \sin\theta = \frac{n_0}{2}\sin\theta_1 = \frac{n_0}{6}\sin\theta_2 = \frac{n_0}{8}\sin 90°$$

$$\Rightarrow \sin\theta = \frac{1}{8} \text{ or } \theta = \sin^{-1}\frac{1}{8}$$

48. (a) When the deviation of a ray through a prism is the minimum, it deviates equally at both the refracting surfaces. In this condition $\theta_1 = \theta_2$ and $\theta_1' = \theta_2'$ see in figure.



Now $\theta_1' + \theta_2' = A$, so $\theta_1' = \theta_2'$

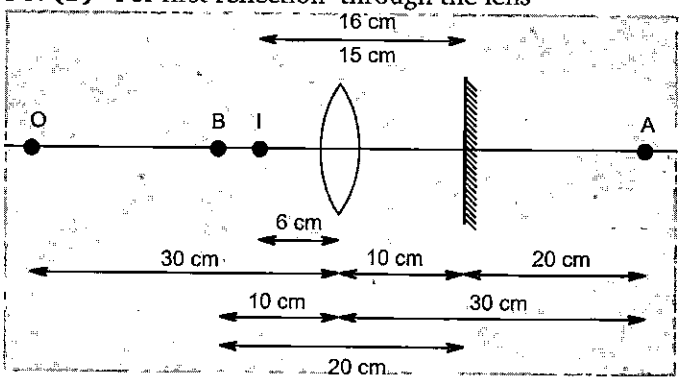
$$= \frac{A}{2} = \frac{60°}{2} = 30°.$$

Since both the beams are set for minimum deviation, the angle of refraction will be 30° for both the colours.

49. (c) $x' = \dfrac{x}{n_{rel}}$,

$$v' = \frac{v}{n_{rel}} = \frac{\sqrt{2\times 10\times(20-12.8)}}{1}\times\frac{4}{3} = 16\,\text{m/s}.$$

50. (b) For first reflection through the lens



$$\frac{1}{V} - \frac{1}{-30} = \frac{1}{+15}$$

$$\Rightarrow \quad V = +30\text{cm} \qquad \text{(Real image at } A\text{)}$$

Image A is the virtual object for plane mirror. Its image is at B which acts as virtual object for the lens

$$\frac{1}{V} - \frac{1}{+10} = \frac{1}{+15}$$

$$\Rightarrow \quad V = +6\text{: Real image.}$$

Distance from mirror is 16 cm.

More than One Alternatives is/are Correct

1. (a) Power of convex lens $= \dfrac{100}{40} = 2.5\text{D}$

Power of concave lens $= \dfrac{-100}{25} = -4.0\text{D}$

$\therefore$ Power of combination $= P_1 + P_2$ or $P = 2.5 - 4.0$

or $P = -1.5$ dioptre.

2. (b,d) $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$

or $\dfrac{9}{1} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$

or $3(\sqrt{I_1}-\sqrt{I_2}) = \sqrt{I_1}+\sqrt{I_2}$ or $2\cdot\sqrt{I_1} = 4\cdot\sqrt{I_1}$

or $\dfrac{I_1}{I_2} = \dfrac{4}{1}$

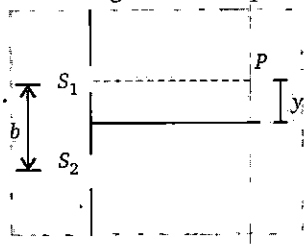
Option (b) is correct.

Again $\dfrac{I_1}{I_2} = \dfrac{4}{1}$

or $\dfrac{a^2}{b^2} = \dfrac{4}{1}$ $\quad [\because I \propto (\text{amplitude})^2]$ or $\dfrac{a}{b} = \dfrac{2}{1}$

Option (d) is correct.

3. (a, c) Those wavelengths are missing for which path difference Δx = odd $\left(\frac{\lambda}{2}\right)$



P is directly in front of slit S_1.

$\because \quad y = (2n-1)\frac{\lambda}{2}\frac{D}{d}$

or $\quad \frac{b}{2} = \frac{(2n-1)\lambda}{2b}$

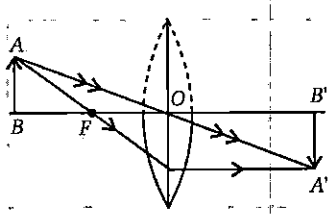
$\left[\because y = \frac{b}{2}, d = b, D = d\right]$

or $\quad \lambda = \frac{b^2}{(2n-1)d}$ where $n = 1, 2, 3....$

$\therefore$ For $n = 1, \lambda_1 = \frac{b^2}{d}$; For $n = 2, \lambda_2 = \frac{b^2}{3d}$

For $n = 3, \quad \lambda_3 = \frac{b^2}{5d}$

4. (b, d) Image formed will be complete when upper half of lens is blocked. Intensity of the image will decrease as the incident rays from upper half are cut off.



5. (d) The object is short. It lies along axis of concave mirror. Differentiate for small variations of v and u.

For a mirror, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \therefore \quad -\frac{dv}{v^2}-\frac{du}{u^2}=0$

or $\quad \frac{dv}{du} = -\frac{v^2}{u^2}$ .... (1)

Again, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{u}{v}+\frac{u}{u}=\frac{u}{f}$

or $\quad \frac{u}{v}=\frac{u-f}{f}$ or $\frac{v}{u}=\frac{f}{u-f}$ ... (2)

From eq. (1) and (2), we get

$$\frac{dv}{du} = -\left(\frac{f}{u-f}\right)^2$$

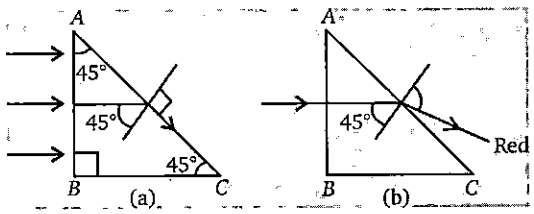
or $\quad dv = -(du)\left(\frac{f}{u-f}\right)^2$ where $(-du) = b$ = Object size

or $\quad dv = b\left(\frac{f}{u-f}\right)^2$

6. (a) Let C = critical angle. Consider $C = 45°$

$\therefore \quad \mu = \frac{1}{\sin C} = \frac{1}{\sin 45°} = 1.414$

Rays fall normally on face AB.



If angle of incidence is greater than C, the ray will suffer total internal reflection. Thus if μ is greater than 1.414, the colour will suffer total internal reflection.

For red colour, $\mu_R = 1.39$. It is less than 1.414.

For green and blue colours, $\mu_G = 1.44$.

Red colour will emerge out of prism. Blue and green colours undergo total internal reflection at the face AC of the prism.

7. (d) In an astronomical telescope when the object and final image are at infinity, M and L are given as shown :

Angular magnification $M = f_o/f_e$

Separation between lenses $L = f_o + f_e$

$\therefore \quad \frac{f_o}{f_e} = 5 \quad$ or $\quad f_o = 5f_e$

$f_o + f_e = 36$ or $5f_e + f_e = 36$

or $\quad f_e = 6\,\text{cm}$

$\therefore \quad f_o = 5f_e$ or $f_o = 30\,\text{cm}$

Hence $f_o = 30\,\text{cm}, \quad f_e = 6\,\text{m}$

8. (c) Net deviation due to two prisms is zero.

$\therefore \quad \delta_1 + \delta_2 = 0$

$A_1(\mu_1 - 1) + A_2(\mu_2 - 1) = 0$

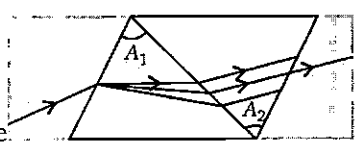
A_1, μ_1 and δ_1 are for prism P_1.

A_2, μ_2 and δ_2 are for prism P_2.

$\therefore \quad A_2 = \frac{-A_1(\mu_1 - 1)}{(\mu_2 - 1)} = \frac{-4°(1.54-1)}{(1.72-1)}$

$A_2 = \frac{-4° \times 0.54}{0.72}$

$A_2 = -3°$



The two prisms are combined such that the position of one prism is inverted w.r.t. other prism.This is the indication of negative sign.

9. (a,b,c,d) Tube length of telescope = $f_o + f_e$

Angular magnification = f_o/f_e

(a) Distance between O and E lens = $f_o + f_e$

$\therefore$ Distance = 16 + 0.02 or Distance = 16.02 m

(b) Angular magnification = f_o/f_e

$\therefore \quad M = \frac{-16}{0.02} = -800$

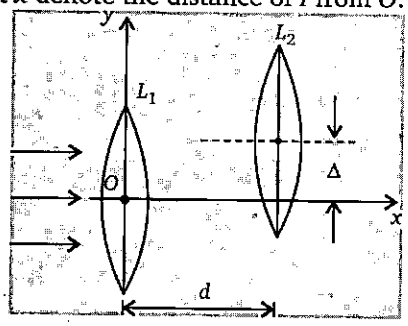
Negative sign shows that final image is inverted.

(c) The image formed by astronomical telescope is inverted w.r.t. planet.

(d) Objective lens is larger than the eyepiece in terms of aperture and focal length.

10. **(c)** A parallel beam of rays coming from the left is incident on the first lens of focal length f_1. The lens forms the image (I_1) at its focus. This image (I_1) serves as virtual object for second lens. The second lens forms the final image I at distance v_2 from the second lens. I is the focal point of the lens system.

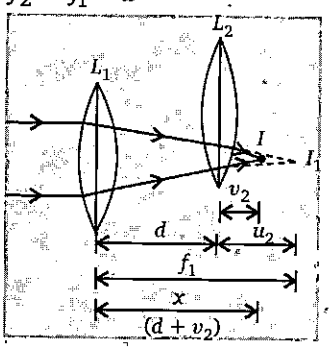
(i) Let x denote the distance of I from O.



For lens L_2,

$$\frac{I}{v_2}-\frac{1}{u_2}=\frac{1}{f_2} \text{ or } \frac{I}{v_2}-\frac{1}{(f_1-d)}=\frac{1}{f_2}$$

or $$\frac{1}{v_2}=\frac{1}{f_2}+\frac{1}{f_1-d}$$



or $$v_2=\frac{f_2(f_1-d)}{f_1+f_2-d} \quad \text{.... (1)}$$

$\therefore$ $$x=d+v_2$$

or $$x=d+\frac{f_2(f_1-d)}{f_1+f_2-d}$$

or $$x=\frac{f_1f_2+d(f_1-d)}{f_1+f_2-d} \quad \text{..... (2)}$$

(ii) y-coordinate of the focal point of lens system :

For lens L_2,

Magnification $=\frac{v_2}{u_2}$ $\therefore$ $m=\frac{\frac{f_2(f_1-d)}{f_1+f_2-d}}{(f_1-d)}$

or $m=\frac{f_2}{f_1+f_2-d}$ or $\frac{h_2}{\Delta}=\frac{f_2}{f_1+f_2-d}$

or $h_2=\frac{\Delta f_2}{f_1+f_2-d}$ $\therefore$ $y=\Delta-h_2$

or $y=\Delta-\frac{\Delta f_2}{f_1+f_2-d}$ or $y=\frac{\Delta(f_1-d)}{f_1+f_2-d}$

11. **(a,c)** An image obtained on screen is a real image. Convex mirror and concave lens do not form real image of an object. Options (b) and (d) do not hold good.

Options (a) and (c) hold good.

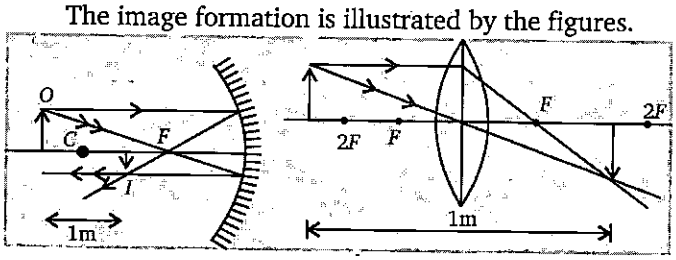
For concave mirror, the object should be beyond C to form a real diminished image between F and C of mirror.

For convex lens, minimum distance between an object and its real image should be $4f$.

$\therefore$ Minimum $(d) = 4f$ or $1M = 4f$, or $f = 0.25\,m$

Convex lens of focal length less than 0.25 m can be used when object is placed beyond $(2f)$ and image is formed between $2f$ and f on other side of the lens.

The image formation is illustrated by the figures.



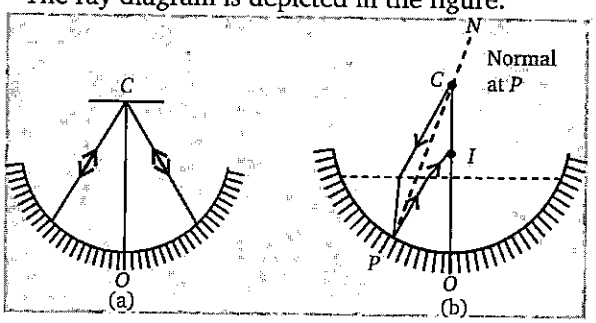
Options (a) and (c) are correct.

12. **(b, c)** Concave lens and convex mirror are diverging in nature. For all positions of objects, they form virtual and erect image.

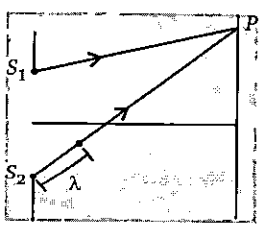
13. **(b)** Spherical aberration is smaller if the curved surface of the lens faces the object. The total deviation is shared between the curved and the plane surfaces.

14. **(d)** The image formed will be real and located at a point between C and O.

The ray diagram is depicted in the figure.



15. **(d)** The path difference of rays coming from the top of upper zone *i.e.* the upper edge of slit and the bottom of lower zone *i.e.* the lower edge of the slit is λ.



λ corresponds to a phase difference of 2π.

Hence the phase difference between the rays coming from the two edges of slit is 2π.

16. (c,d) For total internal reflection to occur,

$i >$ critical angle C

or $\sin i > \sin C$

or $\sin 45° > \frac{1}{\mu}$ or $\frac{1}{\sqrt{2}} > \frac{1}{\mu}$ or $\mu > \sqrt{2}$

or $\mu > 1.414$

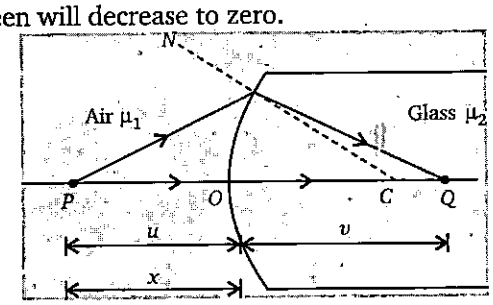
Possible value of μ can be (c) 1.5 or (d) 1.6.

17. (a,b) If $d = \lambda$, then maximum path difference (path difference is given by $d\sin\theta$) will be less than λ. So there will be only central maximum on the screen, because in the equation $d\sin\theta = n\lambda$, n can take only one value.

If $\lambda < d < 2\lambda$, then the maximum path difference will be less than 2λ. So there will be two more maximum on screen in addition to the central maximum.

Intensity of the dark fringes become zero when intensities at the two slits are equal. Initial intensity at both the slits are unequal so there will some brightness at dark fringe. Hence when intensity of both slits is made same the intensity at dark fringe on screen will decrease to zero.

18. (a)



When a ray of light travels from μ_1 to μ_2 after refraction at a single curved surface.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

As per sign convention, $u = -x, v = +x, R$ is +ve,

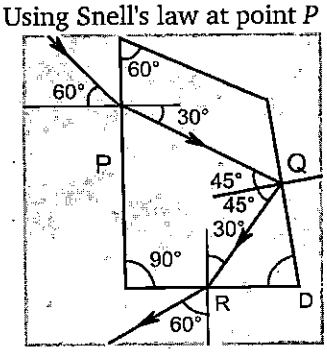
$\mu_1 = 1, \mu_2 = 1.5$

$$\therefore \quad \frac{1.5}{x} - \frac{1}{-x} = \frac{1.5-1}{R} \quad \text{or,} \quad \frac{1.5}{x} + \frac{1}{x} = \frac{0.5}{R}$$

or $\frac{2.5}{x} = \frac{0.5}{R}$ or $x = 5R$

$\therefore$ Distance $PO = 5R$

19. (a, b, c) Using Snell's law at point P

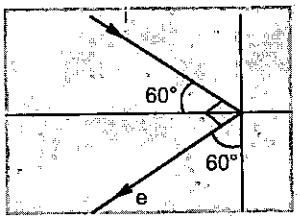


$$\frac{\sin 60°}{\sin r} = \sqrt{3} \quad \Rightarrow \quad r = 30°$$

From geometry, angle of incidence at Q is 45°

At Q, $\sqrt{3}\sin(45°) = \frac{\sqrt{3}}{\sqrt{2}} > 1$

Thus total internal reflection takes place.



At point R,

angle of incidence is 30°

By symmetry $R = 60°$

Net deviation is 90° (second diagram)

True/False

1. True

It appears higher due to atmospheric refraction.

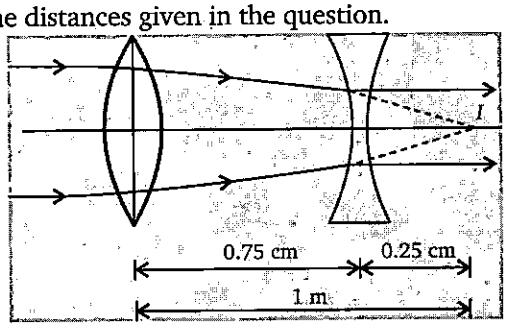
2. True

It is a line source of power P and length l.

Intensity $I = \frac{P}{S} = \frac{P}{2\pi rl}$ or $I \propto \frac{1}{r}$

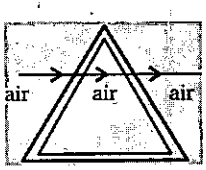
3. False

Parallel rays incident on convex lens meet at its second focus after refraction from lens. Let I be the image. If happens to lie at the first focus of concave lens, as per the distances given in the question.



$\therefore$ Final image will be at infinity. The statement is false.

4. True



The statement is true. Light travels from air to air in case of hollow prism. No refraction and no dispersion therefore occur.

The glass slabs forming the prism are very thin and permit the rays to pass undeviated. Hence a hollow prism gives no spectrum.

5. True

For sustained interference pattern, the two sources should be coherent. Two different lamps cannot act as coherent sources. Hence no interference pattern will be observed on the screen.

6. False

With source of white light, Young's double slit experiment gives coloured fringes on either side of the central white fringe.

7. True

Violet colour is deviated more than red colour when white light passes through a lens or a prism: $F_V < F_R$, for a lens.

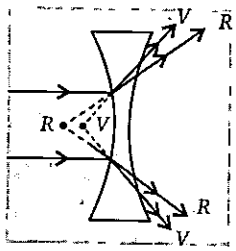
Equivalent focal length $= F$

$$\therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = -\frac{1}{15} + \frac{1}{30} = -\frac{1}{30}$$

or $F = -30$ cm.

The combination behaves like a concave lens.

Hence it is true that one sees coloured patterns with violet colour at the outer edge.



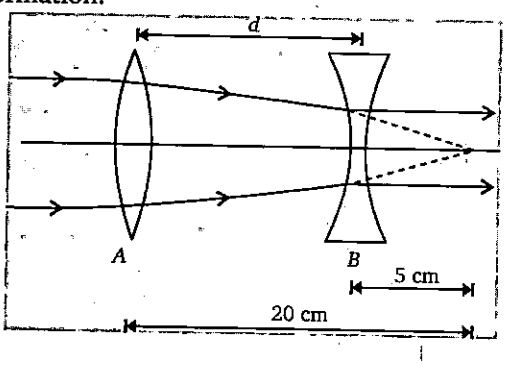
Fill in the Blanks

1. Velocity $v = \frac{c}{\mu}$

or $$v = \frac{3\times10^8}{1.5} = 2.0\times10^8 \text{ m/s}$$

$$\lambda = \frac{\text{velocity}}{\text{frequency}} = \frac{2\times10^8}{5\times10^{14}} = 4.0\times10^{-7} \text{ m}$$

2. The second focus of the convex lens should coincide with the first focus of concave lens for the given image formation.



$d = 20 - 5 = 15$ cm.

3. For medium, $\lambda_m = \frac{\lambda_{air}}{\mu}$ or $\lambda_m = \frac{6000}{1.5} = 4000$Å

Frequency $f = \frac{\text{velocity in air}}{\lambda \text{ in air}}$

$$\therefore \quad f = \frac{3.0\times10^8}{6\times10^{-7}} = 5\times10^{14} \text{Hz}$$

Frequency of light does not change during refraction.

4. When sources are coherent, $I_m = 4I_0$

When sources are incoherent, $I = I_0 + I_0 = 2I_0$

$$\therefore \quad \text{Ratio} = \frac{I_m}{I} = \frac{4I_0}{2I_0} = 2$$

5. According to lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When lens is immersed in a medium, ${}^m\mu_g$ must be taken for μ

$${}^a\mu_g = \frac{{}^a\mu_g}{{}^a\mu_m} = \frac{1.5}{4/3} = \frac{4.5}{4} = 1.125$$

$$\therefore \quad \frac{1}{f_a} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_m} = ({}^m\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $f_a({}^a\mu_g - 1) = f_m({}^m\mu_g - 1)$

or $15(1.5 - 1) = f_m(1.125 - 1)$

or $$f_m = \frac{15\times0.5}{0.125} = 60 \text{ cm} \quad \therefore \quad f_m = 60 \text{ cm}$$

6. In case of point source, at distances r_1 and r_2,

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

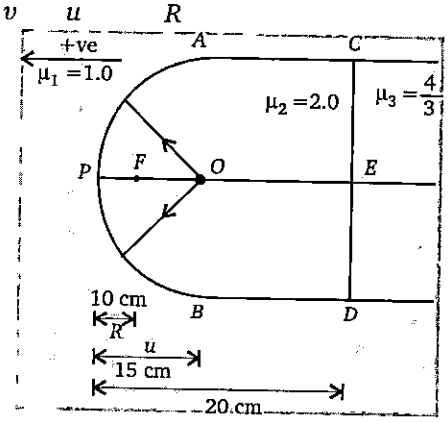
For amplitudes A_1 and A_2,

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{r_2}{r_1} = \frac{25}{9}, \quad \therefore \quad \text{Ratio of amplitudes} = \frac{25}{9}$$

7. When light travels from μ_2 medium to μ_1 medium at a single spherical surface, the formula used is

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$



Direction of light is in positive direction.

$\therefore \quad \frac{1.0}{v} - \frac{2.0}{-15} = \frac{1.0-2.0}{-10}$

F is centre of curvature of APB.

or $\quad \frac{1}{v} = \frac{1}{10} - \frac{2}{15} = \frac{3-4}{30} = \frac{-1}{30}$ or $v = -30\,\text{cm}$

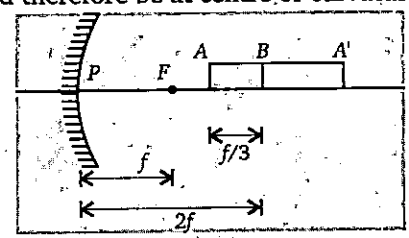
$\therefore$ The distance of the final image of O from P, as viewed from the left, is 30 cm to right of P. The image formed will be virtual.

Note. When the light travels from medium μ_1 to medium μ_2 at a single spherical surface, the formula used is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Distances measured in the direction of light are positive.

8. The question deals with lateral magnification when rod is placed along optic axis of concave mirror. Image of A is formed at A'. Image of B is formed at B itself. B should therefore be at centre of curvature of mirror.



AB lies between f and $2f$ for real and elongated image.

Object distance for $A = PA$.

$\therefore \quad u = PB - AB = 2f - \frac{f}{3} = \frac{5f}{3}$

Formula for mirror : $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\therefore \frac{1}{v} + \frac{1}{-5f/3} = \frac{1}{-f}$ or $\frac{1}{v} = -\frac{1}{f} + \frac{3}{5f} = \frac{-2}{5f}$

or $\quad v = -\frac{5}{2}f = -2.5f \quad \therefore PA' = 2.5f$

or $\quad BA' = PA' - PB = 2.5f - 2f$

or $\quad BA' = 0.5f$

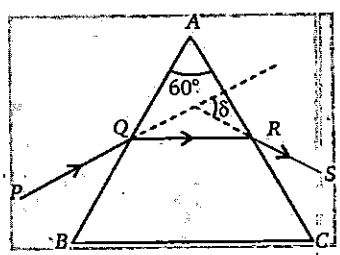
$\therefore$ Magnification $= \frac{BA'}{BA}$ or $M = \frac{0.5f}{f/3}$ or $M = 1.5$

9. For a prism,

$$\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin(A/2)}$$

or $\quad \sqrt{2} = \frac{\sin\left(\frac{60°+\delta}{2}\right)}{\sin(60°/2)}$

or $\sqrt{2} \times \frac{1}{2} = \sin\left(\frac{60°+\delta}{2}\right)$



or $\quad \sin 45° = \sin\left(\frac{60°+\delta}{2}\right)$ or $45° = \frac{60°+\delta}{2}$

or $\quad \delta = 30°$

According to questions also $\delta = 30°$. It means that the ray undergoes minimum deviation. This is possible when QR is parallel to base BC. Angle between BC and QR = zero.

10. The resolving power of a microscope is inversely proportional to the wavelength used.

The resolving power of an electron microscope is higher than that of an optical microscope because the de Broglie wavelength associated with electron is much smaller than the wavelength of visible light.

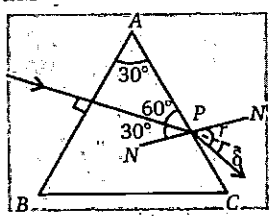
11. Apply Snell's law of refraction at P

$$\frac{\sin 30°}{\sin r} = \frac{1}{\sqrt{2}}$$

or

$$\sin r = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}} = \sin 45°$$

or $\quad r = 45°$



$\therefore \quad \delta = r - 30° = 45° - 30° = 15°$

$\therefore \quad$ Deviation of ray = 15°.

12. When the lenses are in contact,

$P = P_1 + P_2$ or $P_1 + P_2 = 10$ .... (1)

When lenses have d separation.

$P_1 + P_2 - dP_1P_2 = P$ or $P_1 + P_2 - \frac{P_1P_2}{4} = 6$

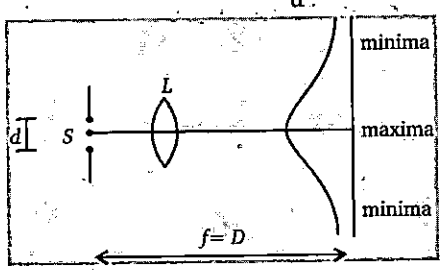
or $\quad 10 - \frac{P_1P_2}{4} = 6$ or $P_1P_2 = 16$ .... (2)

From eq. (1) and (2), we get

$P_1 = 8\,\text{D}; P_2 = 2\,\text{D} \quad \therefore \quad f_1 = \frac{1}{8} = 0.125\,\text{m}$

$f_2 = \frac{1}{2} = 0.5\,\text{m}$

13. Width of central maxima $= \frac{2\lambda D}{d}$, during diffraction.



or $\quad 2 \times 10^{-3} = \frac{2 \times (5.89 \times 10^{-7}) \times 0.5}{d}$

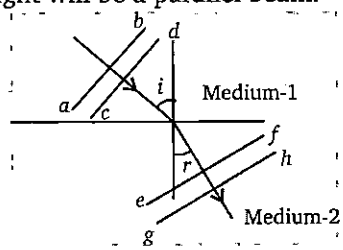
or $\quad d = 2.945 \times 10^{-4}\,\text{m}$

Assertion and Reason

1. (c) The formula is derived under paraxial approximation. Only those rays are considered which make very small angle with the principal axis. This requires a small aperture. Statement-1 is true. Statement-2 is false, the laws of reflection are valid whenever the light is reflected. Nature of surface does not matter.

Comprehension Based Problems

1. (a) The wavefronts, in both the media, are parallel, the light will be a parallel beam.



2. (c) Points c and d are on the same wavefronts, $\phi_c = \phi_d$. Also $\phi_e = \phi_f$ because points e and f are on the same wavefronts.
Then clearly,
$\phi_d - \phi_f = \phi_c - \phi_e$.

3. (b) The ray diagram suggests that on entering medium-2, the ray bends towards normal, medium-2 is denser than medium-1.

Matching Type Problems

1. (a) → (p)
Greater the radius of aperature, greater will be the amount of light entering the telescope.
(b) → (r)
For a telescope, angular magnification $\frac{f_o}{f_e}$
(c) → (r)
For a telescope, tube length $= \frac{f_o}{f_e}$
(d) → (r)
For a telescope, tube length $= f_o + f_e$
(d) → (p), (q) and (s)
Sharpness of image depends on radius of aperture, dispersion of lens and spherical aberration.

2. (a) As per the sign convention f is to be taken as negative and μ is also negative.

$$v = \frac{uf}{u-f} = \frac{f}{1-f/u} = \frac{u}{u/f-1} \text{ and } m = -\frac{v}{u}$$

Values of v may be positive, negative or infinity, also it can have values less than or greater than u.
(a) → (p), (q), (r), (s)
(b) Focal length f is positive. So v will be positive and less than u.
(b) → (q)

(c) $v = \frac{f}{1+f/u} = \frac{u}{u/f+1}$ Here $u < 0$, and $f > 0$

v may be positive, negative or infinity. v may be greater than or less than u.
(c) → (p), (q), (r), (s)
(d) The lens is a concavo-convex lens, a converging lens, with positive focal length.
This case is same as in case (c) for the nature of image.
So (d) → (p), (q), (r), (s)

3. (a) $I(P_1) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\frac{2\pi}{4}$

$$= I_0 + I_0 + 2I_0 \cdot \frac{1}{\sqrt{2}} = (2+\sqrt{2})I_0$$

$$I(P_2) = I_1 + I_2 + 2I_0 \cdot \frac{1}{\sqrt{2}} = 2\sqrt{I_1 I_2} \cdot \cos\frac{2\pi}{3}$$

$$= I_0 + I_0 + 2I_0 \cdot \left(-\frac{1}{\sqrt{2}}\right) = I_0$$

$\therefore \quad I(P_1) > I(P_2)$
(a) → (p), (s)

(b) $\delta(P_0) = (\mu-1)t \cdot \frac{2\pi}{\lambda} = \frac{\lambda}{4} \cdot \frac{2\pi}{\lambda} = \frac{\pi}{2}$

$$\delta(P_1) = \left[(\mu-1)t - \frac{\lambda}{4}\right] \cdot \frac{2\pi}{\lambda} = 0$$

$$\delta(P_2) = \left[(\mu-1)t - \frac{\lambda}{3}\right] \cdot \frac{2\pi}{\lambda} = \frac{-\pi}{6}$$

$$\delta(P_0) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos\delta(P_0)$$

$$= I_0 + I_0 + 2I_0 \cdot \cos\frac{\pi}{2} = 2I_0$$

$$I(P_0) = 4I_0$$

$$I(P_2) = I_0 + I_0 + 2I_0 \cos\left(-\frac{\pi}{6}\right)$$

$$= (2+\sqrt{3})I_0$$

(b) → (g)

(c) $\delta(P_0) = (\mu-1)t \cdot \frac{2\pi}{\lambda} = \frac{\lambda}{2} \cdot \frac{2\pi}{\lambda} = \pi$

$$\delta(P_1) = \left[(\mu-1)t \cdot \frac{2\pi}{\lambda} - \frac{\lambda}{4}\right] \frac{2\pi}{\lambda} = \frac{\pi}{2}$$

$$\delta(P_2) = \left[(\mu-1)t - \frac{\lambda}{3}\right] \cdot \frac{2\pi}{\lambda} = \frac{-\pi}{6}$$

$$I(P_0) = \left(\frac{\lambda}{2} - \frac{\lambda}{3}\right) \cdot \frac{2\pi}{\lambda} = \frac{\pi}{6} \times \frac{2\pi}{\lambda} = \frac{\pi}{3}$$

(c) → (t)

(d) $\delta(P_0) = \frac{3\pi}{4} \times \frac{2\pi}{\lambda} = \frac{3\pi}{2}$

$I(P_1) = 0$

$I(P_2) > 0,\ I(P_0) > 0$

(d) → (r), (s), (t)

4. (a)→p, r; (b)→q, s, t; (c)→ p, r, t; (d)→q, s;

(p) $\mu_2 = \mu_3$, (Since no deviation in medium 2 and 3)
$\mu_1 < \mu_2$, (Since ray bends towards the normal)

(q) $\mu_1 > \mu_2$, (Since ray goes away from the normal)
$\mu_2 > \mu_3$, for similar reasons

(r) Similar to (p)

(s) Similar to (q)

(t) $\mu_1 > \mu_2$, (Since ray goes away from the normal)
$\mu_2 = \mu_3$, (Since no deviation in Madiun 2 & 3)

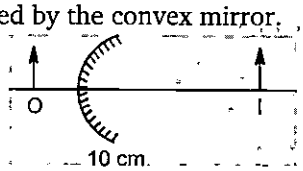
Integer Answer Type Peoblems

1. (6) Magnification,

$$m = \frac{f}{f+u}; \qquad m_{25} = \frac{20}{20-25} = -4$$

$$m_{50} = \frac{20}{20-50} = \frac{-2}{3}; \quad \frac{m_{25}}{m_{50}} = \frac{12}{2} = 6$$

2. (3) Virtual image of object behind the mirror is obtained by the convex mirror.



Using Mirror formula

$$\frac{1}{V} + \frac{1}{u} = \frac{1}{f}$$

For the first object

$$\Rightarrow \quad \frac{1}{+25/3} + \frac{1}{u} = \frac{1}{+10}$$

$$\Rightarrow \quad u = -50 \text{ m}$$

For the second object

$$\Rightarrow \quad \frac{1}{+50/7} + \frac{1}{u} = \frac{1}{+10}$$

$$\Rightarrow \quad u = -25 \text{ m}$$

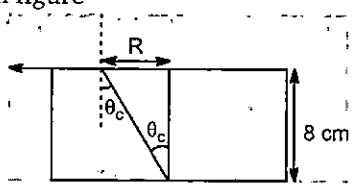
$$\therefore \quad \text{Speed} = \frac{25}{30} \text{ m/s}$$

$$= \frac{25}{30} \times \frac{36}{10} \text{ km/h} = 3 \text{ km/h}$$

3. (6) Since the rays are going out at angle greater than the critical angle θ_c, total internal reflection will result and therefore light will appear only from a circular region of radius R.

$$\sin\theta_c = \frac{1}{\mu} = \frac{3}{5}$$

From figure



$$\sin\theta_c = \frac{R}{\sqrt{R^2+8^2}}$$

Equating

$$\frac{R}{\sqrt{R^2+8^2}} = \frac{3}{5}$$

Solving

$$R = 6\text{cm}$$